On Fuzzy Semi Volterra Spaces

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Abstract. In this paper, the notions of semi Volterra spaces in fuzzy setting is introduced. Few examples are given to illustrate the concepts and several characterizations of fuzzy semi Volterra spaces are also studied in this paper.

Keywords. Fuzzy semi dense set, fuzzy semi nowhere dense set, fuzzy semi G_δ-set, fuzzy semi F_σ-set, fuzzy semi first category set and fuzzy semi Volterra spaces.

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1 Introduction

Till 1965, Mathematicians were concerned only about well-defined things and smartly avoided any other possibilities which are more realistic in nature. In 1965, L.A.Zadeh [12] introduced the concept of fuzzy set to accommodate real life situations by giving partial membership to each element of a situation under consideration. Fuzzy sets allow everyone us to represent vague concepts expressed in natural language. The representation depends not only on the concept but also on the context in which it is used. This inspired mathematicians to fuzzify mathematical structures. General topology is one of the important branches of mathematics in which fuzzy set theory has been applied systematically. Based on the concept of fuzzy sets invented by Zadeh, C.L.Chang [3] introduced the concept of fuzzy topological spaces in 1968 as a generalization of topological spaces. Since then many topologists have contributed to the theory of fuzzy topological spaces. The concepts of Volterra spaces have been studied extensively in classical topology in [4], [5], [6] [7] and [8]. In this paper, the notions of semi Volterra spaces in fuzzy setting is introduced. Few examples are given to illustrate the concepts and several characterizations of fuzzy semi Volterra spaces are also studied in this paper.

2 Preliminaries

In 1965, L.A.Zadeh [12] introduced the concept of fuzzy set λ on a base set X as a function from X into the unit interval $I = [0,1]$. This function is also called a membership function. A membership function is a generalization of a characteristic function.

Definition 2.1 *[3] Let λ and μ be fuzzy sets in X. Then for all $x \in X$,*

- $\lambda = \mu \Leftrightarrow \lambda(x) = \mu(x)$,
- $\lambda \leq \mu \Leftrightarrow \lambda(x) \leq \mu(x)$,
- $\psi = \lambda \vee \mu \Leftrightarrow \psi(x) = max\{\lambda(x), \mu(x)\}$,
- $\delta = \lambda \wedge \mu \Leftrightarrow \delta(x) = min\{\lambda(x), \mu(x)\}$,
- $\eta = \lambda^c \Leftrightarrow \eta(x) = 1 - \lambda(x)$.

For a family $\{\lambda_i / i \in I\}$ of fuzzy sets in X, the union $\psi = \vee_i \lambda_i$ and intersection $\delta = \wedge_i \lambda_i$ are defined by $\psi(x) = sup_i\{\lambda_i(x), x \in X\}$, and $\delta(x) = inf_i\{\lambda_i(x), x \in X\}$.

The fuzzy set 0_X is defined as $0_X(x) = 0$, for all $x \in X$ and the fuzzy set 1_X defined as $1_X(x) = 1$, for all $x \in X$.

Definition 2.2 *[3] A fuzzy topology is a family T of fuzzy sets in X which satisfies the following conditions:*

- $\Phi, X \in T$,
- If $A, B \in T$, then $A \cap B \in T$,
- If $A_i \in T$, for each $i \in I$, then $\cup_{i \in I} A_i \in T$.

T is called a fuzzy topology for X and the pair (X,T) is a fuzzy topological space or fts in short. Every member of T is called a T-open fuzzy set. A fuzzy set is T-closed if and only if its complement is T-open. When no confusion is likely to arise, we shall call a T-open (T-closed) fuzzy set simply an open (closed) fuzzy set.

Lemma 2.1 *[1] Let (X,T) be any fuzzy topological space and λ be any fuzzy set in (X,T). We define the fuzzy semi-closure and the fuzzy semi-interior of λ as follows:*

- $scl(\lambda) = \wedge \{\mu / \lambda \leq \mu, \mu$ is fuzzy semi-closed set of $X\}$
- $sint(\lambda) = \vee \{\mu / \mu \leq \lambda, \mu$ is fuzzy semi-open set of $X\}$.

Lemma 2.2 *[1] For a fuzzy set λ of a fuzzy space X,*

- $1 - scl(\lambda) = sint(1 - \lambda)$ and
- $1 - sint(\lambda) = scl(1 - \lambda)$.

Lemma 2.3 *[1] For a family $\mathcal{A} = \{\lambda_\alpha\}$ of fuzzy sets of a fuzzy space X. Then, $\vee \; cl \; \lambda_\alpha \leq cl(\vee \lambda_\alpha)$. In case $\mathcal{A}$ is a finite set, $\vee \; cl \; \lambda_\alpha = cl(\vee \lambda_\alpha)$. Also $\vee\, int \; \lambda_\alpha \leq int(\vee \lambda_\alpha)$.*

Definition 2.3 *[2] A fuzzy set λ in a fuzzy topological space X is called fuzzy semi-open if $\lambda \leq clint(\lambda)$ and fuzzy semi-closed if $intcl(\lambda) \leq \lambda$.*

Definition 2.4 *[11] A fuzzy set λ in a fuzzy topological space (X,T) is called a fuzzy semi G_δ-set in (X,T) if $\lambda = \wedge_{i=1}^{\infty} (\lambda_i)$, where (λ_i)'s are fuzzy semi open sets in (X,T).*

Definition 2.5 *[11] A fuzzy set λ in a fuzzy topological space (X,T) is called a fuzzy semi F_σ-set in (X,T) if $\lambda = \vee_{i=1}^{\infty} (\lambda_i)$, where (λ_i)'s are fuzzy semi closed sets in (X,T).*

Definition 2.6 *[10] A fuzzy set λ in a fuzzy topological space (X,T) is called fuzzy semi dense if there exists no fuzzy semi closed set μ in (X,T) such that $\lambda < \mu < 1$. That is, $scl(\lambda) = 1$.*

Definition 2.7 *[9] Let (X,T) be a fuzzy topological space. A fuzzy set λ in (X,T) is called a fuzzy semi nowhere dense set if there exists no non-zero fuzzy semi open set μ in (X,T) such that $\mu < scl(\lambda)$. That is, $sintscl(\lambda) = 0$.*

Definition 2.8 *[9] Let (X,T) be a fuzzy topological space. A fuzzy set λ in (X,T) is called fuzzy semi first category if $\lambda = \vee_{i=1}^{\infty} (\lambda_i)$, where (λ_i)'s are fuzzy semi nowhere dense sets in (X,T). Any other fuzzy set in (X,T) is said to be of fuzzy semi second category.*

Definition 2.9 *[9] If λ is a fuzzy semi first category set in a fuzzy topological space (X,T), then $1 - \lambda$ is called a fuzzy semi residual set in (X,T).*

Theorem 2.1 *[9] If λ is a fuzzy semi nowhere dense set in a fuzzy topological space (X,T), then $1 - \lambda$ is a fuzzy semi dense set in (X,T).*

3 Fuzzy semi Volterra spaces

Definition 3.1 *A fuzzy topological space* (X,T) *is called a fuzzy semi Volterra space if* $scl(\wedge_{i=1}^{N}(\lambda_i)) = 1$, *where* (λ_i)*'s are fuzzy semi dense and fuzzy semi* G_δ*-sets in* (X,T).

Example 3.1 *Let* $X = \{a,b,c\}$. *The fuzzy sets* λ,μ *and* ν *are defined on* X *as follows :*

$\lambda: X \to [0,1]$ *is defined as* $\lambda(a) = 0.8$; $\lambda(b) = 0.6$; $\lambda(c) = 0.7$.
$\mu: X \to [0,1]$ *is defined as* $\mu(a) = 0.6$; $\mu(b) = 0.9$; $\mu(c) = 0.8$.
$\nu: X \to [0,1]$ *is defined as* $\nu(a) = 0.7$; $\nu(b) = 0.5$; $\nu(c) = 0.9$.

Clearly $T = \{0,\lambda,\mu,\nu,\lambda\vee\mu,\lambda\vee\nu,\mu\vee\nu,\lambda\wedge\mu,\lambda\wedge\nu,\mu\wedge\nu,\lambda\vee(\mu\wedge\nu),\lambda\wedge(\mu\vee\nu),\mu\vee(\lambda\wedge\nu),\mu\wedge(\lambda\vee\nu),\nu\vee(\lambda\wedge\mu),\nu\wedge(\lambda\vee\mu),\lambda\vee\mu\vee\nu,1\}$ *is a fuzzy topology on* X. *Now, consider fuzzy sets*

$$\alpha = \{\mu\wedge(\lambda\vee\mu)\wedge(\mu\wedge\nu)\wedge[\mu\vee(\lambda\wedge\nu)]\wedge[\nu\wedge(\lambda\vee\mu)] \wedge[\mu\wedge(\lambda\vee\nu)]\wedge[\lambda\vee(\mu\wedge\nu)]\},$$
$$\beta = \{\lambda\wedge(\lambda\wedge\mu)\wedge(\lambda\wedge\nu)\wedge[\lambda\wedge(\mu\vee\nu)]\} and$$
$$\delta = \{\nu\wedge(\mu\vee\nu)\wedge(\lambda\vee\nu)\wedge[\lambda\vee(\mu\wedge\nu)]\wedge[\nu\vee(\lambda\wedge\mu)]\}.$$

Then α, β *and* δ *are fuzzy semi* G_δ*-sets in* (X,T). *On computation, we find that* $scl(\alpha) = 1$, $scl(\beta) = 1$ *and* $scl(\delta) = 1$ *and* $scl(\alpha\wedge\beta\wedge\delta) = 1$. *Hence the fuzzy topological space* (X,T) *is a fuzzy semi Volterra space.*

Example 3.2 *Let* $X = \{a,b,c\}$. *The fuzzy sets* λ,μ *and* ν *are defined on* X *as follows :*

$\lambda: X \to [0,1]$ *is defined as* $\lambda(a) = 0.4$; $\lambda(b) = 0.5$; $\lambda(c) = 0.6$.
$\mu: X \to [0,1]$ *is defined as* $\mu(a) = 0.6$; $\mu(b) = 0.4$; $\mu(c) = 0.5$.
$\nu: X \to [0,1]$ *is defined as* $\nu(a) = 0.7$; $\nu(b) = 0.6$; $\nu(c) = 0.4$.

Clearly $T = \{0,\lambda,\mu,\nu,\lambda\vee\mu,\lambda\vee\nu,\mu\vee\nu,\lambda\wedge\mu,\lambda\wedge\nu,\mu\wedge\nu,\lambda\wedge(\mu\vee\nu),\mu\vee(\lambda\wedge\nu),\nu\wedge(\lambda\vee\mu),1\}$ *is a fuzzy topology on* X. *Now there is no fuzzy semi dense and fuzzy semi* G_δ*-set in* (X,T) *such that* $scl(\wedge_{i=1}^{N}(\lambda_i)) = 1$ *and therefore the fuzzy topological space* (X,T) *is* **not** *a fuzzy semi Volterra space.*

4 Characterizations of fuzzy semi Volterra spaces

Proposition 4.1 *Let* (X,T) *be a fuzzy topological space. If* $sint(\vee_{i=1}^{N}(\lambda_i)) = 0$, *where* (λ_i)*'s are fuzzy semi nowhere dense and fuzzy semi* F_σ*-sets in* (X,T), *then* (X,T) *is a fuzzy semi Volterra space.*

Proof. Suppose that $sint(\vee_{i=1}^{N}(\lambda_i)) = 0$. Then $1 - sint(\vee_{i=1}^{N}(\lambda_i)) = 1$ and $scl[1-(\vee_{i=1}^{N}(\lambda_i))] = 1$. This implies that $scl[\wedge_{i=1}^{N}(1-\lambda_i)] = 1$. Since λ_i is a fuzzy semi nowhere dense set, by theorem 2.1, $1-\lambda_i$ is a fuzzy semi dense set in (X,T). Also, since λ_i is a fuzzy semi F_σ-set in (X,T), $1-\lambda_i$ is a fuzzy semi G_δ-set in (X,T). Hence, $scl(\wedge_{i=1}^{N}(1-\lambda_i)) = 1$, where $(1-\lambda_i)$'s are fuzzy semi dense and fuzzy semi G_δ-sets in (X,T). Therefore (X,T) is a fuzzy semi Volterra space.

Proposition 4.2 *A fuzzy topological space* (X,T) *is a fuzzy semi Volterra space if and only if* $sint(\vee_{i=1}^{N}(1-\lambda_i)) = 0$, *where* (λ_i)*'s are fuzzy semi dense and fuzzy semi* G_δ*-sets in* (X,T).

Proof. Let (X,T) be a fuzzy Volterra space and (λ_i)'s be fuzzy semi dense and fuzzy semi G_δ-sets in (X,T). Then, $scl(\wedge_{i=1}^{N}(\lambda_i)) = 1$. Now $sint(\vee_{i=1}^{N}(1-\lambda_i)) = sint(1-\wedge_{i=1}^{N}(\lambda_i)) = 1 - scl(\wedge_{i=1}^{N}(\lambda_i)) = 1-1 = 0$. Hence $sint(\vee_{i=1}^{N}(1-\lambda_i)) = 0$, where (λ_i)'s are fuzzy semi dense and fuzzy semi G_δ-sets in (X,T).

Conversely, let $sint(\vee_{i=1}^{N} (1-\lambda_i)) = 0$, where (λ_i)'s are fuzzy semi dense and fuzzy semi G_δ-sets in (X,T). Then, $sint(1-[\wedge_{i=1}^{N} (\lambda_i)]) = 0$. This implies that $(1-scl[\wedge_{i=1}^{N} (\lambda_i)]) = 0$ and hence $scl(\wedge_{i=1}^{N} (\lambda_i)) = 1$, where (λ_i)'s are fuzzy semi dense and fuzzy semi G_δ-sets in (X,T). Therefore (X,T) is a fuzzy semi Volterra space.

Proposition 4.3 *If λ is a fuzzy semi dense and fuzzy semi G_δ-set in a fuzzy topological space (X,T), then $1-\lambda$ is a fuzzy semi first category set in (X,T).*

Proof. Let λ be a fuzzy semi dense and fuzzy semi G_δ-set in a fuzzy topological space (X,T). Then $scl(\lambda) = 1$ and $\lambda = \wedge_{i=1}^{\infty} (\lambda_i)$, where the fuzzy sets (λ_i)'s are fuzzy semi open sets in (X,T) and hence $scl(\wedge_{i=1}^{\infty} (\lambda_i)) = 1$. But $scl(\wedge_{i=1}^{\infty} (\lambda_i)) \leq \wedge_{i=1}^{\infty} scl(\lambda_i)$, implies that $1 \leq \wedge_{i=1}^{\infty} scl(\lambda_i)$. That is., $\wedge_{i=1}^{\infty} scl(\lambda_i) = 1$. Then $scl(\lambda_i) = 1$ for each $\lambda_i \in T$ and hence $sclsint(\lambda_i) = 1$. This implies that $1 - sclsint(\lambda_i) = 0$ and hence $sintscl(1-\lambda_i) = 0$. Therefore, $1-\lambda_i$ is a fuzzy semi nowhere dense set in (X,T). Now $1-\lambda = 1 - \wedge_{i=1}^{\infty} (\lambda_i) = \vee_{i=1}^{\infty} (1-\lambda_i)$. Therefore, $1-\lambda = \vee_{i=1}^{\infty} (1-\lambda_i)$ where $(1-\lambda_i)$'s are fuzzy semi nowhere dense sets in (X,T). Hence $1-\lambda$ is a fuzzy semi first category set in (X,T).

Proposition 4.4 *If the fuzzy semi first category sets μ_i, $(i = 1 \quad to \quad N)$ are formed from the fuzzy semi dense and fuzzy semi G_δ-sets in a fuzzy semi Volterra space (X,T), then $sint(\vee_{i=1}^{N} (\mu_i)) = 0$.*

Proof. Let (λ_i)'s be fuzzy semi dense and fuzzy semi G_δ-sets in a fuzzy semi Volterra space (X,T). Then, $scl(\wedge_{i=1}^{N} (\lambda_i)) = 1$. Now $1 - scl(\wedge_{i=1}^{N} (\lambda_i)) = 0$, implies that $sint(\vee_{i=1}^{N} (1-\lambda_i)) = 0$. Since the fuzzy sets (λ_i)'s are fuzzy semi dense and fuzzy semi G_δ-sets in (X,T), by proposition 4.3, $(1-\lambda_i)$'s are fuzzy semi first category sets in (X,T). Let $\mu_i = 1-\lambda_i$. Hence $sint(\vee_{i=1}^{N} (\mu_i)) = 0$, where (μ_i)'s are fuzzy semi first category sets in (X,T).

Proposition 4.5 *If $\lambda = \wedge_{i=1}^{N} (\lambda_i)$, where (λ_i)'s are fuzzy semi dense and fuzzy semi G_δ-sets in a fuzzy semi Volterra space (X,T), then λ is not a fuzzy semi closed set in (X,T).*

Proof. Consider the fuzzy set $\lambda = \wedge_{i=1}^{N} (\lambda_i)$, where (λ_i)'s are fuzzy semi dense and fuzzy semi G_δ-sets in (X,T). Since (X,T) is a fuzzy semi Volterra space, $scl(\wedge_{i=1}^{N} (\lambda_i)) = 1$. That is, $scl(\lambda) = 1$. Hence $\lambda \neq scl(\lambda)$. Therefore λ is not a fuzzy semi closed set in (X,T).

Proposition 4.6 *If $\mu = \vee_{i=1}^{N} (\mu_i)$, where (μ_i)'s are fuzzy semi nowhere dense and fuzzy semi F_σ-sets in a fuzzy semi Volterra space (X,T), then μ is not a fuzzy semi open set in (X,T).*

Proof. Let $\mu = \vee_{i=1}^{N} (\mu_i)$, where (μ_i)'s are fuzzy semi nowhere dense and fuzzy semi F_σ-sets in (X,T). Then, $1-\mu = 1 - \vee_{i=1}^{N} (\mu_i) = \wedge_{i=1}^{N} (1-\mu_i)$. Since (μ_i)'s are fuzzy semi nowhere dense sets, by theorem 2.1, $(1-\mu_i)$'s are fuzzy semi dense sets in (X,T). Also, since (μ_i)'s are fuzzy semi F_σ-sets, $(1-\mu_i)$'s are fuzzy semi G_δ-sets in (X,T). Hence, $1-\mu = \wedge_{i=1}^{N} (1-\mu_i)$, where $(1-\mu_i)$'s are fuzzy semi dense and fuzzy semi G_δ-sets in (X,T). Then, by proposition 4.5, $1-\mu$ is not a fuzzy semi closed set in (X,T). Therefore μ is not a fuzzy semi open set in (X,T).

Definition 4.1 *A fuzzy set λ in a fuzzy topological space (X,T) is called a fuzzy semi σ-nowhere dense set if λ is fuzzy semi F_σ-set in X,T such that $sint(\lambda) = 0$.*

Proposition 4.7 *In a fuzzy topological space (X,T), a fuzzy set λ is a fuzzy semi σ-nowhere dense set in (X,T) if and only if $1-\lambda$ is a fuzzy semi dense and fuzzy semi G_δ-set in (X,T).*

Proof. Let λ be a fuzzy semi σ-nowhere dense set in (X,T). Then $\lambda = \vee_{i=1}^{\infty} (\lambda_i)$, where $1-\lambda_i \in T$, for $i \in I$ and $sint(\lambda) = 0$. Then $scl(1-\lambda) = 1 - sint(\lambda) = 1 - 0 = 1$. This implies that $1-\lambda$ is a fuzzy semi dense set in (X,T). Also $1-\lambda = 1-\vee_{i=1}^{\infty}(\lambda_i) = \wedge_{i=1}^{\infty}(1-\lambda_i)$, where $1-\lambda_i \in T$, for $i \in I$. This implies that $1-\lambda$ is a fuzzy semi G_δ-set in (X,T). Therefore $1-\lambda$ is a fuzzy semi dense and fuzzy semi G_δ-set in (X,T).

Conversely, let λ be a fuzzy semi dense and fuzzy semi G_δ-set in (X,T). Then $\lambda = \wedge_{i=1}^{\infty}(\mu_i)$ where $\mu_i \in T$, for $i \in I$ and $scl(\lambda) = 1$. Now $1-\lambda = 1-\wedge_{i=1}^{\infty}(\mu_i) = \vee_{i=1}^{\infty}(1-\mu_i)$, where $(1-\mu_i)$'s are fuzzy semi closed sets in (X,T). This implies that $1-\lambda$ is a fuzzy semi F_σ-set in (X,T). Now $sint(1-\lambda) = 1 - scl(\lambda) = 1-1 = 0$. Hence $1-\lambda$ is a fuzzy semi F_σ-set in (X,T) and $sint(1-\lambda) = 0$. Therefore $1-\lambda$ is a fuzzy semiσ-nowhere dense set in (X,T).

Proposition 4.8 *If the fuzzy semi σ-nowhere dense sets (λ_i)'s are fuzzy semi closed sets in a fuzzy semi Volterra space (X,T), then $sclsint(\wedge_{i=1}^{N}(\lambda_i)) = 1$.*

Proof. Let (λ_i)'s be fuzzy semi dense and fuzzy semi G_δ-sets in (X,T). Then, by proposition 4.7, $(1-\lambda_i)$'s are fuzzy semi σ-nowhere dense sets in (X,T). By hypothesis, the fuzzy semi σ-nowhere dense sets $(1-\lambda_i)$'s are fuzzy semi closed and hence (λ_i)'s are fuzzy semi open in (X,T). Then $\wedge_{i=1}^{N}(\lambda_i)$ is also fuzzy semi open in (X,T). Now $sint(\wedge_{i=1}^{N}(\lambda_i)) = \wedge_{i=1}^{N}(\lambda_i)$. Then $sclsint(\wedge_{i=1}^{N}(\lambda_i)) = scl(\wedge_{i=1}^{N}(\lambda_i))$. Since (X,T) is a fuzzy semi Volterra space, $scl(\wedge_{i=1}^{N}(\lambda_i)) = 1$, where (λ_i)'s are fuzzy semi dense and fuzzy semi G_δ-sets in (X,T). This implies that $sclsint(\wedge_{i=1}^{N}(\lambda_i)) = 1$.

Proposition 4.9 *If (λ_i)'s are fuzzy semi σ-nowhere dense sets in a fuzzy semi Volterra space (X,T), then $sint(\vee_{i=1}^{N}(\lambda_i)) = 0$.*

Proof. Let (λ_i)'s be fuzzy semi σ-nowhere dense sets in (X,T). We claim that $sint(\vee_{i=1}^{N}(\lambda_i)) = 0$. Assume the contrary that $sint(\vee_{i=1}^{N}(\lambda_i)) \neq 0$. Then $1 - sint(\vee_{i=1}^{N}(\lambda_i)) \neq 1$. This will imply that $scl(\wedge_{i=1}^{N}(1-\lambda_i)) \neq 1$. Since (λ_i)'s are fuzzy semi σ-nowhere dense sets in (X,T), by proposition 4.7, $(1-\lambda_i)$'s are fuzzy semi dense and fuzzy semi G_δ-sets in (X,T). This implies that $scl(\wedge_{i=1}^{N}(1-\lambda_i)) \neq 1$, for the fuzzy semi dense and fuzzy semi G_δ-sets $(1-\lambda_i)$'s in (X,T). This in turn will imply that (X,T) is not a fuzzy semi Volterra space, a contradiction to our hypothesis. Hence, our assumption is wrong. Therefore, we must have $sint(\vee_{i=1}^{N}(\lambda_i)) = 0$.

Proposition 4.10 *If (λ_i)'s are fuzzy semi nowhere dense and fuzzy semi F_σ-sets in a fuzzy semi Volterra space (X,T), then $sint(\vee_{i=1}^{N}(\lambda_i)) = 0$.*

Proof. Let (λ_i)'s be fuzzy semi nowhere dense and fuzzy semi F_σ-sets in (X,T). Since (λ_i)'s are fuzzy semi nowhere dense sets in (X,T), $sintscl(\lambda_i) = 0$. But, $sint(\lambda_i) \leq sintscl(\lambda_i) = 0$, implies that $sint(\lambda_i) = 0$. Hence (λ_i)'s are fuzzy semi F_σ-sets in (X,T) with $sint(\lambda_i) = 0$. Then (λ_i)'s are fuzzy semi σ-nowhere dense sets in (X,T) and hence, by proposition 4.9, $sint(\vee_{i=1}^{N}(\lambda_i)) = 0$.

Remark. In view of propositions 4.1 and 4.10, one will have the following result:

A fuzzy topological space (X,T) is a fuzzy semi Volterra space if and only if $sint(\vee_{i=1}^{N}(\lambda_i)) = 0$, where (λ_i)'s are fuzzy semi nowhere dense and fuzzy semi F_σ-sets in (X,T).

Proposition 4.11 *If $sint(\vee_{i=1}^{N}(1-\lambda_i)) = 0$, where (λ_i)'s are fuzzy semi G_δ-sets in a fuzzy topological space (X,T), then (X,T) is a fuzzy semi Volterra space.*

Proof. Let (λ_i)'s be fuzzy semi G_δ-sets in a topological space (X,T) such that $sint(\vee_{i=1}^{N}(1-\lambda_i)) = 0$. Now $\vee_{i=1}^{N}(sint(1-\lambda_i)) \leq sint(\vee_{i=1}^{N}(1-\lambda_i)) = 0$ implies that $\vee_{i=1}^{N}(sint(1-\lambda_i)) = 0$. Then, $sint(1-\lambda_i) = 0$ for some i, $(i = 1$ to $N)$. This implies that $1 - scl(\lambda_i) = 0$ and hence $scl(\lambda_i) = 1$. Therefore (λ_i)'s are fuzzy semi dense sets in (X,T). Now $sint(\vee_{i=1}^{N}(1-\lambda_i))=0$ implies that $sint(1-\wedge_{i=1}^{N}(\lambda_i))=0$ and hence $1-scl(\wedge_{i=1}^{N}(\lambda_i))=0$. Thus, $scl(\wedge_{i=1}^{N}(\lambda_i))=1$, where (λ_i)'s are fuzzy semi dense and fuzzy semi G_δ-sets in (X,T). Therefore (X,T) is a fuzzy semi Volterra space.

Proposition 4.12 *If $scl(\wedge_{i=1}^{N}(\lambda_i))=1$, where (λ_i)'s are fuzzy semi G_δ-sets in a fuzzy topological space (X,T), then (X,T) is a fuzzy semi Volterra space.*

Proof. Let (λ_i)'s be fuzzy semi G_δ-sets in (X,T) such that $scl(\wedge_{i=1}^{N}(\lambda_i))=1$. Then $1-scl(\wedge_{i=1}^{N}(\lambda_i))=0$. This implies that $sint(1-\wedge_{i=1}^{N}(\lambda_i)=0$. Hence $sint(\vee_{i=1}^{N}(1-\lambda_i)=0$. Then, by proposition 4.11, (X,T) is a fuzzy semi Volterra space.

Proposition 4.13 *If a fuzzy topological space (X,T) is a fuzzy semi Volterra space, then there exists a fuzzy semi F_σ-set μ in (X,T) such that $sint(\mu)\neq 0$.*

Proof. Let $\lambda=\wedge_{i=1}^{N}(\lambda_i)$, where (λ_i)'s are fuzzy semi dense and fuzzy semi G_δ-sets in (X,T). Since (X,T) is a fuzzy semi Volterra space, $scl(\lambda)=scl(\wedge_{i=1}^{N}(\lambda_i))=1$. Then $scl(\lambda)=1.\dots.(1)$. Now $1-sint(\lambda_i)$ is a fuzzy semi closed set in (X,T). Let $\mu=\vee_{i=1}^{\infty}(\mu_i)$, where (μ_i)'s are fuzzy semi closed sets in (X,T) in which the first N fuzzy semi closed sets be $1-sint(\lambda_i)$. Then, μ is a fuzzy semi F_σ-set in (X,T). Now $\vee_{i=1}^{N}(1-sint(\lambda_i))\leq\vee_{i=1}^{\infty}(\mu_i)$. Then, $1-\wedge_{i=1}^{N}(sint(\lambda_i))\leq\vee_{i=1}^{\infty}(\mu_i)$. Now $1-\wedge_{i=1}^{N}(\lambda_i)<1-\wedge_{i=1}^{N}(sint(\lambda_i))\leq\vee_{i=1}^{\infty}(\mu_i)$. Then $1-\lambda<\mu$. This implies that $sint(1-\lambda)<sint(\mu)$. Then $1-scl(\lambda)<sint(\mu)$. From (1), $1-1<sint(\mu)$. Therefore $sint(\mu)\neq 0$. Hence if (X,T) is a fuzzy semi Volterra space, then there exists a fuzzy semi F_σ-set μ in (X,T) such that $int(\mu)\neq 0$.

Definition 4.2 *Let (X,T) be a fuzzy topological space. A fuzzy set λ in (X,T) is called fuzzy semi σ-first category if $\lambda=\vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy semi σ-nowhere dense sets in (X,T). Any other fuzzy set in (X,T) is said to be of fuzzy semi σ-second category.*

Proposition 4.14 *If each fuzzy semi σ-first category set is a fuzzy semi σ-nowhere dense set in a fuzzy topological space (X,T), then (X,T) is a fuzzy semi Volterra space.*

Proof. Let (λ_α)'s are fuzzy semi dense and fuzzy semi G_δ-sets in (X,T). Then, by theorem 2.1, $(1-\lambda_\alpha)$'s are fuzzy semi σ-nowhere dense sets and hence $\vee_{\alpha=1}^{\infty}(1-\lambda_\alpha)$ is a fuzzy semi σ-first category set in (X,T). By hypothesis, $\vee_{\alpha=1}^{\infty}(1-\lambda_\alpha)$ is a fuzzy semi σ-nowhere dense set in (X,T). Then $\vee_{\alpha=1}^{\infty}(1-\lambda_\alpha)$ is a fuzzy semi F_σ-set such that $sint(\vee_{\alpha=1}^{\infty}(1-\lambda_\alpha))=0$. Then $sint(1-\wedge_{\alpha=1}^{\infty}(\lambda_\alpha))=0$. This implies that $1-scl(\wedge_{\alpha=1}^{\infty}(\lambda_\alpha))=0$. Hence $scl(\wedge_{\alpha=1}^{\infty}(\lambda_\alpha))=1$. But $scl[\wedge_{\alpha=1}^{\infty}(\lambda_\alpha)]\leq scl[\wedge_{\alpha=1}^{N}(\lambda_\alpha)]$ and hence $1\leq scl[\wedge_{\alpha=1}^{N}(\lambda_\alpha)]$. That is, $scl[\wedge_{\alpha=1}^{N}(\lambda_\alpha)]=1$, where (λ_α)'s are fuzzy semi dense and fuzzy semi G_δ-sets in (X,T). Therefore (X,T) is a fuzzy semi Volterra space.

Proposition 4.15 *If each fuzzy semi G_δ-set has a fuzzy semi dense interior in a fuzzy topological space (X,T), then (X,T) is a fuzzy semi Volterra space.*

Proof. Let (λ_α)'s be fuzzy semi dense and fuzzy semi G_δ-sets in (X,T). Then, $\lambda=\wedge_{\alpha=1}^{N}(\lambda_\alpha)$ is a fuzzy semi G_δ-set. By hypothesis, λ has a fuzzy semi dense interior and hence $sclsint(\lambda)=1$. Now $sint(\lambda)\leq\lambda$, implies that $sint(\lambda)\leq\wedge_{\alpha=1}^{N}(\lambda_\alpha)$. Then $sclsint(\lambda)\leq scl[\wedge_{\alpha=1}^{N}(\lambda_\alpha)]$. Hence $1\leq scl[\wedge_{\alpha=1}^{N}(\lambda_\alpha)]$. This implies that $cl(\wedge_{\alpha=1}^{N}(\lambda_\alpha))=1$, where (λ_α)'s are fuzzy semi dense and fuzzy semi G_δ-sets in (X,T). Therefore (X,T) is fuzzy semi Volterra space.

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