On Fuzzy Semi Weakly Volterra Spaces

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Abstract. The concept of semi weakly Volterra space in fuzzy setting is introduced and studied in this paper, by using fuzzy semi dense and fuzzy semi G_δ-sets. Examples are given for fuzzy semi weakly Volterra spaces. Several characterizations of fuzzy semi weakly Volterra spaces are given. The interrelations among fuzzy semi weakly Volterra spaces and other fuzzy topological spaces such as fuzzy semi σ-second category space, fuzzy almost semi irresolvable space have been investigated. The conditions under which a fuzzy semi second category space becomes a fuzzy semi weakly Volterra space, have also been established in this paper.

Keywords. Fuzzy semi dense set, fuzzy semi nowhere dense set, fuzzy semi G_δ-set, fuzzy semi F_σ-set, fuzzy semi first category set and fuzzy semi weakly Volterra spaces.

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1 Introduction

In 1965, L.A.Zadeh [13] introduced the concept of fuzzy set to accommodate real life situations by giving partial membership to each element of a situation under consideration. Fuzzy sets allow everyone us to represent vague concepts expressed in natural language. The representation depends not only on the concept but also on the context in which it is used. This inspired mathematicians to fuzzify mathematical structures. General topology is one of the important branches of mathematics in which fuzzy set theory has been applied systematically. Based on the concept of fuzzy sets invented by Zadeh, C.L.Chang [3] introduced the concept of fuzzy topological spaces in 1968 as a generalization of topological spaces. Since then many topologists have contributed to the theory of fuzzy topological spaces. The concepts of Volterra spaces have been studied extensively in classical topology in [4], [5], [6] [7] and [8]. Motivated by their works, the concept of semi weakly Volterra space in fuzzy setting is introduced and studied in this paper, by using fuzzy semi dense and fuzzy semi G_δ-sets. Examples are given for fuzzy semi weakly Volterra spaces. Several characterizations of fuzzy semi weakly Volterra spaces are given. The interrelations among fuzzy semi weakly Volterra spaces and other fuzzy topological spaces such as fuzzy semi σ-second category space, fuzzy almost semi irresolvable space have been investigated. The conditions under which a fuzzy semi second category space becomes a fuzzy semi weakly Volterra space, have also been established in this paper.

2 Preliminaries

In 1965, L.A.Zadeh [13] introduced the concept of fuzzy set λ on a base set X as a function from X into the unit interval $I = [0,1]$. This function is also called a membership function. A membership function is a generalization of a characteristic function.

Definition 2.1 *[3] Let λ and μ be fuzzy sets in X. Then for all $x \in X$,*

- $\lambda = \mu \Leftrightarrow \lambda(x) = \mu(x)$,
- $\lambda \leq \mu \Leftrightarrow \lambda(x) \leq \mu(x)$,
- $\psi = \lambda \vee \mu \Leftrightarrow \psi(x) = max\{\lambda(x), \mu(x)\}$,
- $\delta = \lambda \wedge \mu \Leftrightarrow \delta(x) = min\{\lambda(x), \mu(x)\}$,
- $\eta = \lambda^c \Leftrightarrow \eta(x) = 1 - \lambda(x)$.

For a family $\{\lambda_i / i \in I\}$ of fuzzy sets in X, the union $\psi = \vee_i \lambda_i$ and intersection $\delta = \wedge_i \lambda_i$ are defined by $\psi(x) = sup_i\{\lambda_i(x), x \in X\}$, and $\delta(x) = inf_i\{\lambda_i(x), x \in X\}$.

The fuzzy set 0_X is defined as $0_X(x) = 0$, for all $x \in X$ and the fuzzy set 1_X defined as $1_X(x) = 1$, for all $x \in X$.

Definition 2.2 *[3] A fuzzy topology is a family T of fuzzy sets in X which satisfies the following conditions:*

- $\Phi, X \in T$,
- If $A, B \in T$, then $A \cap B \in T$,
- If $A_i \in T$, for each $i \in I$, then $\cup_{i \in I} A_i \in T$.

T is called a fuzzy topology for X and the pair (X, T) is a fuzzy topological space or fts in short. Every member of T is called a T-open fuzzy set. A fuzzy set is T-closed if and only if its complement is T-open. When no confusion is likely to arise, we shall call a T-open (T-closed) fuzzy set simply an open (closed) fuzzy set.

Lemma 2.1 *[1] Let (X, T) be any fuzzy topological space and λ be any fuzzy set in (X, T). We define the fuzzy semi-closure and the fuzzy semi-interior of λ as follows:*

- $scl(\lambda) = \wedge \{\mu / \lambda \leq \mu, \mu$ is fuzzy semi-closed set of $X\}$
- $sint(\lambda) = \vee \{\mu / \mu \leq \lambda, \mu$ is fuzzy semi-open set of $X\}$.

Lemma 2.2 *[1] For a fuzzy set λ of a fuzzy space X,*

- $1 - scl(\lambda) = sint(1 - \lambda)$ and
- $1 - sint(\lambda) = scl(1 - \lambda)$.

Lemma 2.3 *[1] For a family $\mathcal{A} = \{\lambda_\alpha\}$ of fuzzy sets of a fuzzy space X. Then, $\vee \; cl \; \lambda_\alpha \leq cl(\vee \lambda_\alpha)$. In case $\mathcal{A}$ is a finite set, $\vee \; cl \; \lambda_\alpha = cl(\vee \lambda_\alpha)$. Also $\vee \, int \; \lambda_\alpha \leq int(\vee \lambda_\alpha)$.*

Definition 2.3 *[2] A fuzzy set λ in a fuzzy topological space X is called fuzzy semi-open if $\lambda \leq clint(\lambda)$ and fuzzy semi-closed if $intcl(\lambda) \leq \lambda$.*

Definition 2.4 *[11] A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy semi G_δ-set in (X, T) if $\lambda = \wedge_{i=1}^{\infty} (\lambda_i)$, where (λ_i)'s are fuzzy semi open sets in (X, T).*

Definition 2.5 *[11] A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy semi F_σ-set in (X, T) if $\lambda = \vee_{i=1}^{\infty} (\lambda_i)$, where (λ_i)'s are fuzzy semi closed sets in (X, T).*

Definition 2.6 *[10] A fuzzy set λ in a fuzzy topological space (X, T) is called fuzzy semi dense if there exists no fuzzy semi closed set μ in (X, T) such that $\lambda < \mu < 1$. That is, $scl(\lambda) = 1$.*

Definition 2.7 *[9] Let (X, T) be a fuzzy topological space. A fuzzy set λ in (X, T) is called a fuzzy semi nowhere dense set if there exists no non-zero fuzzy semi-open set μ in (X, T) such that $\mu < scl(\lambda)$. That is, $sintscl(\lambda) = 0$.*

Definition 2.8 *[9] Let (X,T) be a fuzzy topological space. A fuzzy set λ in (X,T) is called fuzzy semi first category if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy semi nowhere dense sets in (X,T). Any other fuzzy set in (X,T) is said to be of fuzzy semi second category.*

Definition 2.9 *[9] If λ is a fuzzy semi first category set in a fuzzy topological space (X,T), then $1-\lambda$ is called a fuzzy semi residual set in (X,T).*

Theorem 2.1 *[9] If λ is a fuzzy semi nowhere dense set in a fuzzy topological space (X,T), then $1-\lambda$ is a fuzzy semi dense set in (X,T).*

Theorem 2.2 *[12] If λ is a fuzzy semi dense and fuzzy semi G_δ-set in a fuzzy topological space (X,T), then $1-\lambda$ is a fuzzy semi first category set in (X,T).*

Theorem 2.3 *[12] In a fuzzy topological space (X,T), a fuzzy set λ is a fuzzy semi σ-nowhere dense set in (X,T) if and only if $1-\lambda$ is a fuzzy semi dense and fuzzy semi G_δ-set in (X,T).*

3 Fuzzy semi weakly Volterra spaces

Definition 3.1 *A fuzzy topological space (X,T) is called a fuzzy semi weakly Volterra space if $cl(\wedge_{i=1}^{N}(\lambda_i)) \neq 0$, where (λ_i)'s are fuzzy semi dense and fuzzy semi G_δ-sets in (X,T).*

It is clear from the definition that every fuzzy semi Volterra space is fuzzy semi weakly Volterra space. The following example shows that the reverse implication **need not** be true.

Example 3.1 *Let $X = \{a,b,c\}$. The fuzzy sets λ,μ and ν are defined on X as follows :*

$\lambda : X \to [0,1]$ is defined as $\lambda(a) = 1$; $\lambda(b) = 0.2$; $\lambda(c) = 0.7$.
$\mu : X \to [0,1]$ is defined as $\mu(a) = 0.3$; $\mu(b) = 1$; $\mu(c) = 0.2$.
$\nu : X \to [0,1]$ is defined as $\nu(a) = 0.7$; $\nu(b) = 0.4$; $\nu(c) = 1$.

Then $T = \{0, \lambda, \mu, \nu, \lambda\vee\mu, \lambda\vee\nu, \mu\vee\nu, \lambda\wedge\mu, \lambda\wedge\nu, \mu\wedge\nu, \lambda\vee(\mu\wedge\nu), \mu\vee(\lambda\wedge\nu), \nu\wedge(\lambda\vee\mu), 1\}$ is a fuzzy topology on X. Now $1-\lambda, 1-\mu, 1-\nu, 1-(\lambda\vee\mu), 1-(\lambda\vee\nu), 1-(\mu\vee\nu), 1-[\lambda\vee(\mu\wedge\nu)]$ and $1-[\mu\vee(\lambda\wedge\nu)]$ are fuzzy semi nowhere dense sets in (X,T). Now the fuzzy sets $\alpha = \{\lambda\wedge(\lambda\vee\mu)\wedge[\lambda\vee(\mu\wedge\nu)]\}$, $\beta = \{\mu\wedge(\mu\vee\nu)\wedge[\mu\vee(\lambda\wedge\nu)]\}$ and $\delta = \{\nu\wedge(\lambda\vee\nu)\wedge[\nu\vee(\lambda\wedge\mu)]\}$ are fuzzy semi dense and fuzzy semi G_δ-sets in (X,T) and $cl(\alpha\wedge\beta\wedge\delta) = 1-[\nu\wedge(\lambda\vee\mu)] \neq 0$ and hence (X,T) is a fuzzy semi weakly Volterra space and $cl(\alpha\wedge\beta\wedge\delta) \neq 1$ and hence (X,T) is **not** a fuzzy semi Volterra space.

Proposition 3.1 *Let (X,T) be a fuzzy topological space. Then the following are equivalent :*

- If (X,T) is a fuzzy semi weakly Volterra space and if $\vee_{i=1}^{N}(\lambda_i) = 1$, where (λ_i)'s are fuzzy semi F_σ-sets in (X,T), then there exists at least one λ_i in (X,T) with $sint(\lambda_i) \neq 0$.

- If $\vee_{i=1}^{N}(\lambda_i) = 1$, where (λ_i)'s are fuzzy semi F_σ-sets in (X,T) and if $sint(\lambda_i) \neq 0$ for at least one i $(i = 1$ to $N)$, then (X,T) is a fuzzy semi weakly Volterra space.

Proof. $(1) \Rightarrow (2)$: Let (X,T) be a fuzzy semi weakly Volterra space and $\vee_{i=1}^{N}(\lambda_i) = 1$, where (λ_i)'s are fuzzy semi F_σ-sets in (X,T). Assume the contrary that $sint(\lambda_i) = 0$ for all i $(i = 1 to$ $N)$ in (X,T). Then $1 - sint(\lambda_i) = 1$ and hence $scl(1-\lambda_i) = 1 - sint(\lambda_i) = 1$. Since (λ_i)'s are fuzzy semi F_σ-sets in (X,T), $(1-\lambda_i)$'s are fuzzy semi G_δ-sets in (X,T). Now $scl(\wedge_{i=1}^{N}(1-\lambda_i)) = scl[1-(\vee_{i=1}^{N}(\lambda_i))] = cl[1-1] = 0$, where $(1-\lambda_i)$'s are fuzzy semi dense and fuzzy semi G_δ-sets in (X,T). Hence (X,T) is not a fuzzy semi weakly Volterra space which is a contradiction to (X,T) being fuzzy semi weakly Volterra space. Therefore $sint(\lambda_i) \neq 0$ for at least one i $(i = 1 \quad to \quad N)$ in (X,T).

(2) ⇒ (1): Let $\vee_{i=1}^{N}(\lambda_i) = 1$, where (λ_i)'s are fuzzy semi F_σ-sets in (X,T) and $sint(\lambda_i) \neq 0$ for at least one i $(i = 1 \quad to \quad N)$. Suppose that $scl(\wedge_{i=1}^{N}(\lambda_i)) = 0$, where (λ_i)'s are fuzzy semi dense and fuzzy semi G_δ-sets in (X,T). Then $1 - scl(\wedge_{i=1}^{N}(\lambda_i)) = 1$ implies that $sint[1 - (\wedge_{i=1}^{N}(\lambda_i))] = 1$. Then $sint[\vee_{i=1}^{N}(1-\lambda_i)] = 1$ and hence $\vee_{i=1}^{N}(1-\lambda_i) = 1$, where $(1-\lambda_i)$'s are fuzzy semi F_σ-sets in (X,T) and $sint(1-\lambda_i) = 0$ [since $scl(\lambda_i) = 1 \quad for \quad all \quad i = 1 \quad to N$] which is a contradiction to the hypothesis. Hence $scl(\wedge_{i=1}^{N}(\lambda_i)) \neq 0$ in (X,T). Therefore (X,T) must be a fuzzy semi weakly Volterra space.

Proposition 3.2 *If $\wedge_{i=1}^{N}(\lambda_i)$ is a fuzzy somewhere semi nowhere dense set in a fuzzy topological space (X,T), where (λ_i)'s are fuzzy semi dense and fuzzy semi G_δ-sets in (X,T), then (X,T) is a fuzzy semi weakly Volterra space.*

Proof. Let $\wedge_{i=1}^{N}(\lambda_i)$ be a fuzzy somewhere semi nowhere dense set in (X,T), where (λ_i)'s are fuzzy semi dense and fuzzy semi G_δ-sets in (X,T). Then, $sintcl(\wedge_{i=1}^{N}(\lambda_i)) \neq 0$. Suppose that $scl(\wedge_{i=1}^{N}(\lambda_i)) = 0$, where (λ_i)'s are fuzzy semi dense and fuzzy semi G_δ-sets in (X,T). But $sintcl(\wedge_{i=1}^{N}(\lambda_i)) \leq scl(\wedge_{i=1}^{N}(\lambda_i))$ implies that $sintscl(\wedge_{i=1}^{N}(\lambda_i)) = 0$ which is a contradiction to $\wedge_{i=1}^{N}(\lambda_i)$ being a fuzzy somewhere semi nowhere dense set in (X,T). Hence $scl(\wedge_{i=1}^{N}(\lambda_i)) \neq 0$, where (λ_i)'s are fuzzy semi dense and fuzzy semi G_δ-sets in (X,T). Therefore (X,T) is a fuzzy semi weakly Volterra space.

4 Fuzzy semi weakly Volterra spaces and fuzzy semi second category spaces

Proposition 4.1 *If λ is a fuzzy semi dense and fuzzy semi open set in (X,T), then $1-\lambda$ is fuzzy semi nowhere dense set in (X,T).*

Proof. Let λ is a fuzzy semi open set in (X,T) such that $scl(\lambda) = 1$. Now $sintscl(1-\lambda) = 1 - sclsint(\lambda) = 1 - scl(\lambda) = 1 - 1 = 0$. Hence $1-\lambda$ is a fuzzy semi nowhere dense set in (X,T).

Proposition 4.2 *If each fuzzy semi first category set is a fuzzy semi closed set in a fuzzy semi second category space (X,T), then (X,T) is a fuzzy semi weakly Volterra space.*

Proof. Let (λ_i)'s $(i = 1 \quad to \quad N)$ be fuzzy semi dense and fuzzy semi G_δ-sets in (X,T). Then, by theorem 2.2, $(1-\lambda_i)$'s are fuzzy semi first category sets in (X,T). By hypothesis, $(1-\lambda_i)$'s are fuzzy semi closed sets and hence (λ_i)'s are fuzzy semi open sets in (X,T). Now (λ_i)'s are fuzzy semi dense and fuzzy semi open sets in (X,T). Then, by proposition 4.1, $(1-\lambda_i)$'s are fuzzy semi nowhere dense sets in (X,T). Since (X,T) is a fuzzy semi second category space, $\vee_{\alpha=1}^{\infty}(\mu_\alpha) \neq 1$, where (μ_α)'s are fuzzy semi nowhere dense sets in (X,T). Let the first $N(\mu_\alpha)$'s be $(1-\lambda_i)$ in (X,T). But $\vee_{i=1}^{N}(1-\lambda_i) \leq \vee_{\alpha=1}^{\infty}(\mu_\alpha)$ and $\vee_{\alpha=1}^{\infty}(\mu_\alpha) \neq 1$, implies that $\vee_{i=1}^{N}(1-\lambda_i) \neq 1$. This implies that $1 - \wedge_{i=1}^{N}(\lambda_i) \neq 1$. Then $\wedge_{i=1}^{N}(\lambda_i) \neq 0$ and hence $scl[\wedge_{i=1}^{N}(\lambda_i)] \neq 0$, where (λ_i)'s are fuzzy semi dense and fuzzy semi G_δ-sets in (X,T). Therefore (X,T) is a fuzzy semi weakly Volterra space.

Proposition 4.3 *If the fuzzy topological space (X,T) is a fuzzy semi second category space and if every fuzzy semi nowhere dense set in (X,T) is a fuzzy semi F_σ-set in (X,T), then (X,T) is a fuzzy semi weakly Volterra space.*

Proof. Let the fuzzy topological space (X,T) be a fuzzy semi second category space. Then, $\vee_{i=1}^{\infty}(\lambda_i) \neq 1$, where (λ_i)'s are fuzzy semi nowhere dense sets in (X,T) and hence $sint(\vee_{i=1}^{\infty}(\lambda_i)) \neq 1$ implies that $1 - sint(\vee_{i=1}^{\infty}(\lambda_i)) \neq 0$. Then $scl(\wedge_{i=1}^{\infty}(1-\lambda_i)) \neq 0$. Now (λ_i)'s are fuzzy semi F_{σ}-sets in (X,T) implies that $(1-\lambda_i)$'s are fuzzy semi G_{δ}-sets in (X,T) and (λ_i)'s are fuzzy semi nowhere dense sets in (X,T) implies that $(1-\lambda_i)$'s are fuzzy semi dense sets in (X,T). Now $scl(\wedge_{i=1}^{\infty}(1-\lambda_i)) \leq scl(\wedge_{i=1}^{N}(1-\lambda_i))$. Then $scl(\wedge_{i=1}^{N}(1-\lambda_i)) \neq 0$, where $(1-\lambda_i)$'s are fuzzy semi dense and fuzzy semi G_{δ}-sets in (X,T) [otherwise $scl(\wedge_{i=1}^{N}(1-\lambda_i)) = 0$, will imply that $scl(\wedge_{i=1}^{\infty}(1-\lambda_i)) = 0$, a contradiction]. Hence (X,T) is a fuzzy semi weakly Volterra space.

Definition 4.1 *A fuzzy topological space (X,T) is called a fuzzy semi open hereditarily irresolvable space if $sintscl(\lambda) \neq 0$, then $sint(\lambda) \neq 0$ for any non-zero fuzzy set λ in (X,T).*

Proposition 4.4 *If the fuzzy topological space (X,T) is a fuzzy semi open hereditarily irresolvable and fuzzy semi second category space, then (X,T) is a fuzzy semi weakly Volterra space.*

Proof. Let (X,T) be a fuzzy semi open hereditarily irresolvable and fuzzy semi second category space. Suppose that $scl(\wedge_{i=1}^{N}(\lambda_i)) = 0$, where (λ_i)'s are fuzzy semi dense and fuzzy semi G_{δ}-sets in (X,T). Then, $1 - scl(\wedge_{i=1}^{N}(\lambda_i)) = 1$. This implies that $sint(1-(\wedge_{i=1}^{N}(\lambda_i))) = 1$. Then $sint(\vee_{i=1}^{N}(1-\lambda_i)) = 1$ and hence $\vee_{i=1}^{N}(1-\lambda_i) = 1$. But $\vee_{i=1}^{N}(1-\lambda_i) \leq \vee_{i=1}^{\infty}(1-\lambda_i)$ implies that $\vee_{i=1}^{\infty}(1-\lambda_i) = 1$. Since (λ_i)'s are fuzzy semi dense sets in (X,T), $scl(\lambda_i) = 1$, and hence $1 - scl(\lambda_i) = 0$. Then, $sint(1-\lambda_i) = 0$. Since (X,T) is a fuzzy semi open hereditarily irresolvable, $sint(1-\lambda_i) = 0$ implies that $sintscl(1-\lambda_i) = 0$ and hence $(1-\lambda_i)$'s are fuzzy semi nowhere dense sets in (X,T). Hence $\vee_{i=1}^{\infty}(1-\lambda_i) = 1$, where $(1-\lambda_i)$'s are fuzzy semi nowhere dense sets in (X,T), implies that (X,T) will be a fuzzy semi first category space, a contradiction to (X,T) being a fuzzy semi second category space. Hence the assumption that $scl(\wedge_{i=1}^{N}(\lambda_i)) = 0$ is not true. Thus, $scl(\wedge_{i=1}^{N}(\lambda_i)) \neq 0$, where (λ_i)'s are fuzzy semi dense and fuzzy semi G_{δ}-sets in (X,T). Therefore (X,T) is a fuzzy semi weakly Volterra space.

Definition 4.2 *A fuzzy topological space (X,T) is called a fuzzy semi D-Baire space if every fuzzy semi first category set in (X,T) is a fuzzy semi nowhere dense set in (X,T). That is, (X,T) is a fuzzy semi D-Baire space if $sintscl(\lambda) = 0$, for each fuzzy semi first category set λ in (X,T).*

Proposition 4.5 *If the fuzzy semi D-Baire space (X,T) is a fuzzy semi second category space, then (X,T) is a fuzzy semi weakly Volterra space.*

Proof. Let the fuzzy semi D-Baire space (X,T) be a fuzzy semi second category space and (λ_i)'s $(i = 1 \ to \ N)$ be fuzzy semi dense and fuzzy semi G_{δ}-sets in (X,T). Then, by theorem 2.2, $(1-\lambda_i)$'s are fuzzy semi first category sets in (X,T). Since (X,T) is a fuzzy semi D-Baire space, $(1-\lambda_i)$'s are fuzzy semi nowhere dense sets in (X,T). Since (X,T) is a fuzzy semi second category space, $\vee_{\alpha=1}^{\infty}(\mu_{\alpha}) \neq 1$, where (μ_{α})'s are fuzzy semi nowhere dense sets in (X,T). Let the first $N(\mu_{\alpha})$'s be $(1-\lambda_i)$ in (X,T). But $\vee_{i=1}^{N}(1-\lambda_i) \leq \vee_{\alpha=1}^{\infty}(\mu_{\alpha})$ and $\vee_{\alpha=1}^{\infty}(\mu_{\alpha}) \neq 1$. Then $\vee_{i=1}^{N}(1-\lambda_i) \neq 1$, implies that $1 - \wedge_{i=1}^{N}(\lambda_i) \neq 1$. That is, $\wedge_{i=1}^{N}(\lambda_i) \neq 0$ and hence $scl[\wedge_{i=1}^{N}(\lambda_i)] \neq 0$, where (λ_i)'s are fuzzy semi dense and fuzzy semi G_{δ}-sets in (X,T). Therefore (X,T) is a fuzzy semi weakly Volterra space.

Definition 4.3 *A fuzzy topological space (X,T) is called a fuzzy semi P-space if countable intersection of fuzzy semi open sets in (X,T) is fuzzy semi open. That is, every non-zero fuzzy semi G_{δ}-set in (X,T) is fuzzy semi open in (X,T).*

Proposition 4.6 *If the fuzzy topological space (X,T) is a fuzzy semi second category space and fuzzy semi P-space, then (X,T) is a fuzzy semi weakly Volterra space.*

Proof. Let (X,T) be a fuzzy semi second category space and fuzzy semi P-space. Let (λ_i)'s $(i = 1 \quad \text{to} N)$ be fuzzy semi dense and fuzzy semi G_δ-sets in (X,T). Since (X,T) is a fuzzy semi P-space, the fuzzy semi G_δ-sets (λ_i)'s are fuzzy semi open sets in (X,T). Then, (λ_i)'s are fuzzy semi dense and fuzzy semi open sets (X,T). By proposition 4.1, $(1-\lambda_i)$'s are fuzzy semi nowhere dense sets in (X,T). Since (X,T) is a fuzzy semi second category space, $\vee_{\alpha=1}^{\infty}(\mu_\alpha) \neq 1$, where (μ_α)'s are fuzzy semi nowhere dense sets in (X,T). Let us take the first $N(\mu_\alpha)$'s as $(1-\lambda_i)$ in (X,T). But $\vee_{i=1}^{N}(1-\lambda_i) \leq \vee_{\alpha=1}^{\infty}(\mu_\alpha)$ and $\vee_{\alpha=1}^{\infty}(\mu_\alpha) \neq 1$. Then, $\vee_{i=1}^{N}(1-\lambda_i) \neq 1$, implies that $1 - \wedge_{i=1}^{N}(\lambda_i) \neq 1$. That is, $\wedge_{i=1}^{N}(\lambda_i) \neq 0$ and hence $scl[\wedge_{i=1}^{N}(\lambda_i)] \neq 0$, where (λ_i)'s are fuzzy semi dense and fuzzy semi G_δ-sets in (X,T). Therefore (X,T) is a fuzzy semi weakly Volterra space.

Definition 4.4 *A fuzzy topological space (X,T) is called a fuzzy semi submaximal space if for each fuzzy set λ in (X,T) such that $scl(\lambda) = 1$, then $\lambda \in T$ in (X,T).*

Proposition 4.7 *If the fuzzy topological space (X,T) is a fuzzy semi second category space and fuzzy semi submaximal space, then (X,T) is a fuzzy semi weakly Volterra space.*

Proof. Let (X,T) be a fuzzy semi second category space and fuzzy semi submaximal space. Let (λ_i)'s $(i = 1 \quad to \quad N)$ be fuzzy semi dense and fuzzy semi G_δ-sets in (X,T). Since (X,T) is a fuzzy semi submaximal space, the fuzzy semi dense sets (λ_i)'s are fuzzy semi open sets in (X,T). Then, (λ_i)'s are fuzzy semi dense and fuzzy semi open sets (X,T). By proposition 4.1, $(1-\lambda_i)$'s are fuzzy semi nowhere dense sets in (X,T). Since (X,T) is a fuzzy semi second category space, $\vee_{\alpha=1}^{\infty}(\mu_\alpha) \neq 1$, where (μ_α)'s are fuzzy semi nowhere dense sets in (X,T). Let the first $N(\mu_\alpha)$'s be $(1-\lambda_i)$ in (X,T). But $\vee_{i=1}^{N}(1-\lambda_i) \leq \vee_{\alpha=1}^{\infty}(\mu_\alpha)$ and $\vee_{\alpha=1}^{\infty}(\mu_\alpha) \neq 1$. Then $\vee_{i=1}^{N}(1-\lambda_i) \neq 1$, implies that $1 - \wedge_{i=1}^{N}(\lambda_i) \neq 1$. That is, $\wedge_{i=1}^{N}(\lambda_i) \neq 0$ and hence $scl[\wedge_{i=1}^{N}(\lambda_i)] \neq 0$, where (λ_i)'s are fuzzy semi dense and fuzzy semi G_δ-sets in (X,T). Therefore (X,T) is a fuzzy semi weakly Volterra space.

5 Fuzzy semi weakly Volterra spaces and other fuzzy topological spaces

Definition 5.1 *A fuzzy topological space (X,T) is called a fuzzy semi σ-first category space if the fuzzy set 1_X is a fuzzy semi σ-first category set in (X,T). That is, $1_X = \vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy semi σ-nowhere dense sets in (X,T). Otherwise, (X,T) will be called a fuzzy semi σ-second category space.*

Proposition 5.1 *If the fuzzy topological space (X,T) is a fuzzy semi σ-second category space, then (X,T) is a fuzzy semi weakly Volterra space.*

Proof. Let the fuzzy topological space (X,T) be a fuzzy semi σ-second category space and (λ_i)'s $(i = 1 to \quad N)$ be fuzzy semi dense and fuzzy semi G_δ-sets in (X,T). Then, by theorem 2.3, $(1-\lambda_i)$'s are fuzzy semi σ-nowhere dense sets in (X,T). Let $\mu_\alpha(\alpha = 1 \quad to \quad \infty)$ be fuzzy semi σ-nowhere dense sets in (X,T) in which the first $N(\mu_\alpha)$'s be $(1-\lambda_i)$. Since (X,T) is a fuzzy semi σ-second category space, $\vee_{\alpha=1}^{\infty}(\mu_\alpha) \neq 1$. Then, $1 - [\vee_{\alpha=1}^{\infty}(\mu_\alpha)] \neq 0$. This implies that $\wedge_{\alpha=1}^{\infty}(1-\mu_\alpha) \neq 0$. Then, $scl(\wedge_{\alpha=1}^{\infty}(1-\mu_\alpha)) \neq 0$. But $scl(\wedge_{\alpha=1}^{\infty}(1-\mu_\alpha)) \leq scl(\wedge_{\alpha=1}^{N}(1-\mu_\alpha))$ and hence $scl(\wedge_{\alpha=1}^{N}(1-\mu_\alpha)) \neq 0$. Then $scl(\wedge_{i=1}^{N}(\lambda_i)) \neq 0$, where (λ_i)'s are fuzzy semi dense and fuzzy semi G_δ-sets in (X,T). Therefore (X,T) is a fuzzy semi weakly Volterra space.

Proposition 5.2 *If the fuzzy topological space (X,T) is not a fuzzy semi weakly Volterra space, then (X,T) is a fuzzy semi σ-first category space.*

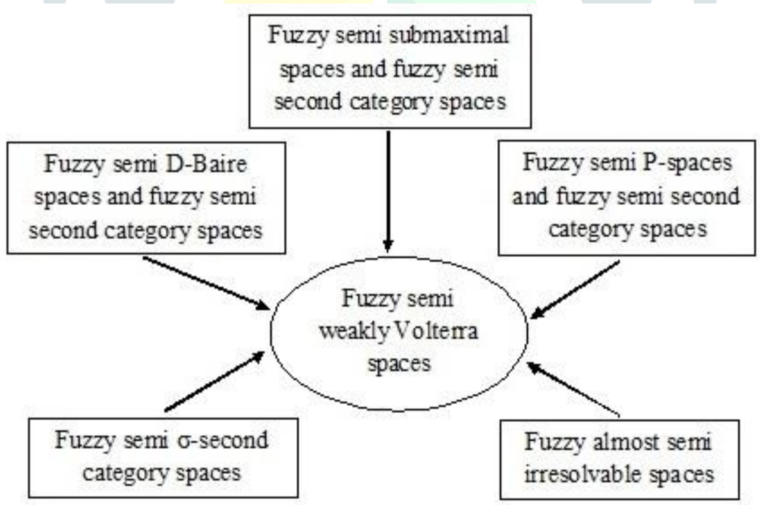
Proof. Let (μ_α)'s $(\alpha = 1 \quad to \infty)$ be fuzzy semi σ-nowhere dense sets in (X,T) which is not a fuzzy semi weakly Volterra space. Now it is to be proved that $\vee_{\alpha=1}^{\infty}(\mu_\alpha) = 1$. Assume the contrary that $\vee_{\alpha=1}^{\infty}(\mu_\alpha) \neq 1$. Then, $\wedge_{\alpha=1}^{\infty}(1-\mu_\alpha) \neq 0$. Since (μ_α)'s are fuzzy semi σ-nowhere dense sets in (X,T), by theorem 2.3, $(1-\mu_\alpha)$'s are fuzzy semi dense and fuzzy semi G_δ-sets in (X,T). But $\wedge_{\alpha=1}^{\infty}(1-\mu_\alpha) \leq \wedge_{\alpha=1}^{N}(1-\mu_\alpha)$ implies that $\wedge_{\alpha=1}^{N}(1-\mu_\alpha) \neq 0$. Let $\lambda_\alpha = 1-\mu_\alpha$. Then (λ_α)'s are fuzzy semi dense and fuzzy semi G_δ-sets in (X,T) and $\wedge_{\alpha=1}^{N}(\lambda_\alpha) \neq 0$. This implies that $scl[\wedge_{\alpha=1}^{N}(\lambda_\alpha)] \neq 0$, a contradiction to (X,T) being a fuzzy non-semi weakly Volterra space for which $scl[\wedge_{\alpha=1}^{N}(\lambda_\alpha)] = 0$. Hence, it must be $\vee_{\alpha=1}^{\infty}(\mu_\alpha) = 1$ in (X,T). Therefore (X,T) is a fuzzy semi σ-first category space.

Definition 5.2 *A fuzzy topological space (X,T) is called a fuzzy almost semi resolvable space if $\vee_{i=1}^{\infty}(\lambda_i) = 1$, where (λ_i)'s in (X,T) are such that $sint(\lambda_i) = 0$. Otherwise, (X,T) is called a fuzzy almost semi irresolvable space.*

Proposition 5.3 *If the fuzzy topological space (X,T) is a fuzzy almost semi irresolvable space, then (X,T) is a fuzzy semi weakly Volterra space.*

Proof. Let (λ_i)'s $(i = 1 \quad to \quad N)$ be fuzzy semi dense and fuzzy semi G_δ-sets in (X,T). Now $scl(\lambda_i) = 1$, implies that $sint(1-\lambda_i) = 0$. Since (X,T) is a fuzzy almost semi irresolvable space, $\vee_{\alpha=1}^{\infty}(\mu_\alpha) \neq 1$, where (μ_α)'s in (X,T) are such that $sint(\mu_\alpha) = 0$. Let the first $N(\mu_\alpha)$'s be $(1-\lambda_i)$ in (X,T). Now $\vee_{\alpha=1}^{\infty}(\mu_\alpha) \neq 1$, implies that $1-[\vee_{\alpha=1}^{\infty}(\mu_\alpha)] \neq 0$. Then, $\wedge_{\alpha=1}^{\infty}(1-\mu_\alpha) \neq 0$ and hence $scl(\wedge_{\alpha=1}^{\infty}(1-\mu_\alpha)) \neq 0$. But $scl(\wedge_{\alpha=1}^{\infty}(1-\mu_\alpha)) \leq scl(\wedge_{\alpha=1}^{N}(1-\mu_\alpha))$ implies that $scl[\wedge_{\alpha=1}^{N}(1-\mu_\alpha)] \neq 0$. That is, $scl[\wedge_{i=1}^{N}(\lambda_i)] \neq 0$, where (λ_i)'s are fuzzy semi dense and fuzzy semi G_δ-sets in (X,T). Therefore (X,T) is a fuzzy semi weakly Volterra space.

Remark. The relationship among the classes of fuzzy semi σ-second category spaces, fuzzy almost semi irresolvable spaces, fuzzy semi D-Baire spaces, fuzzy semi second category spaces, fuzzy semi P-spaces and fuzzy semi weakly Volterra spaces can be summarized as follows:



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