The Golden Ratio

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Abstract :

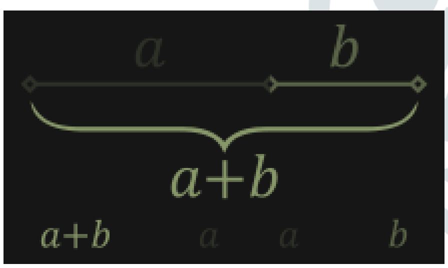
Would you think that why design and logo of i-phone attracts us or when we look at someone face we say that he or she looking attractive. The Great Pyramid Of Giza, Notre Dame in Paris, The Parthenon, The Taj Mahal and many more other thing which are in our mind and we would like to show this all things at lest once in our life because of its beauty. The question that came to your mind is what is it like in these places that we are drawn to them? Any flower or plant in nature we love to see. What is the reason behind this all such things?? The answer is Golden ratio. So let's know a little bit about it and the use of it in all such things.

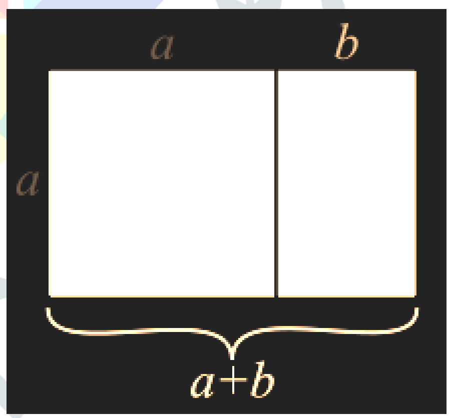
Key Words :

Golden ratio(φ), Fibonacci Sequence, Golden spiral (or Fibonacci spiral), etc.

Introduction :

The golden ratio is a very special number evaluate by dividing a line into two parts so that the bigger part of line divided by the shorter part is also equal to the total length of line divided by the bigger part. it is denoted by 21st letter of Greek alphabet *phi* symbolized as φ.





A golden rectangle with bigger part a and shorter part b as shown in given figure, when placed adjacent to a square with sizes of length a, will produce an equivalent golden rectangle with bigger size $a + b$ and shorter side a.

That is $\frac{a+b}{a} = \frac{a}{b} \equiv \varphi$.

In other words, two quantities are said to be in golden ratio if their ratio is same as the ratio of their sum to larger of the two quantities.

The golden ratio is also known as golden mean or golden section or extreme and mean ratio.

Calculation :

Two numbers a and b are said to be in golden ratio φ if $\frac{a+b}{a} = \frac{a}{b} \equiv \varphi$

To calculate the value of φ we proceed as follows…

$$\frac{a+b}{a} = \frac{a}{a} + \frac{b}{a} = 1 + \frac{b}{a} = 1 + \frac{1}{\varphi}$$

$$\Rightarrow \varphi = 1 + \frac{1}{\varphi}$$

$$\Rightarrow \varphi + 1 = \varphi^2$$

$$\Rightarrow \varphi^2 - \varphi - 1 = 0$$

Solving above equation we get two solution given as

$\varphi = 1.618033 \ldots$ and $\varphi = -0.618033 \ldots$

in which one is negetive, but the value of φ must be positive as value of x and y are positive.So the value of φ is 1.618033 ... which is an irrational number.

History :

It is accepted that Martin Ohm (1792–1872) was the principal individual to utilize the term “golden” " to portray the golden ratio. to utilize the term In 1815, he published “Die reine Elementar-Mathematik” (The Pure Elementary Mathematics). This book is celebrated for containing the principal known use of the term “goldener schnitt” (golden section).

The Renaissance specialists utilized the Golden Mean broadly in their canvases and models to accomplish parity and magnificence. Leonardo Da Vinci, for example, utilized it to characterize all the central proportions of his work of art of "The Last Supper," from the components of the table at which Christ and the disciples sat to the ratios of the walls and windows in the background.

Euclid (365 BC – 300 BC), in “Elements,” alluded to isolating a line at the 0.6180399… point as “dividing a line in the extreme and mean ratio.” This later offered ascend to the utilization of the term mean in the golden ratio. He additionally connected this number to the development of a pentagram.

Leonardo Da Vinci gave outlines to a thesis distributed by Luca Pacioli in 1509 entitled "De Divina Proportione" (1), perhaps the most punctual reference in writing to another of its names, the “Divine Proportion”. This book contains drawings made by Leonardo da Vinci of the five Platonic solids.

The term "Phi" was not utilized until the 1900's that American mathematician Mark Barr utilized the Greek letter phi (φ) to assign this ratio. This showed up in "The Curves of Life" (page 420) in 1914 by Theodore Andrea Cook . At this point this ubiquitous ratio was known as the golden mean, golden section and golden ratio as well as the Divine ratio. Phi is the first letter of Phidias (1), who used the golden ratio in his sculptures, as well as the Greek equivalent to the letter “F,” the first letter of Fibonacci. Phi is additionally the 21st letter of the Greek letters in order, and 21 is one of numbers in the Fibonacci series.

It creates the impression that the Egyptians may have utilized phi in the structure of the Great Pyramids. The Greeks are thought by some to have based the plan of the Parthenon on this proportion, however this is liable to some guess.

Johannes Kepler (1571-1630), pioneer of the circular idea of the circles of the planets around the sun, additionally went on about the "Divine Ratio," saying this regarding it:

“Geometry has two great treasures: one is the theorem of Pythagoras; the other, the division of a line into extreme and mean ratio. The first we may compare to a measure of gold; the second we may name a precious jewel.”

Relation between Golden ratio and Fibonacci Sequence :

There is a special relationship between the Golden Ratio and the Fibonacci Sequence.

Fibonacci Sequence given as 1, 1, 2, 3, 5, 8, 13, 21, 34, ...

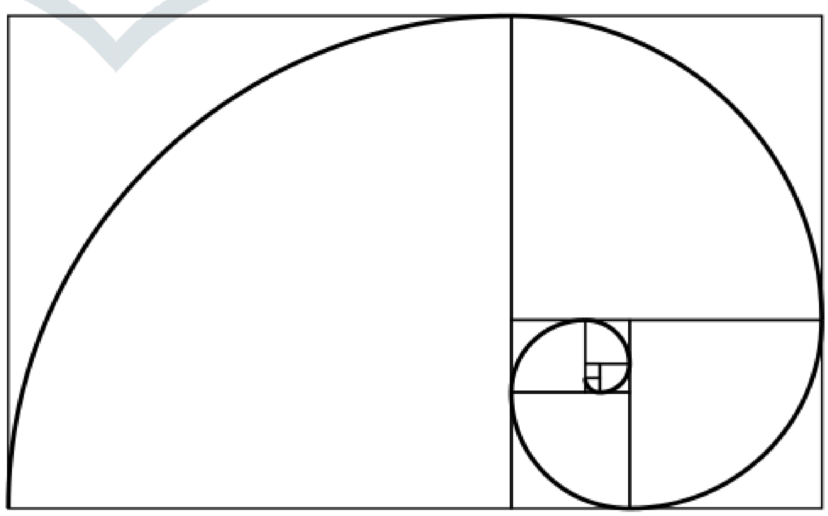
In which after the initial two terms the following term is found by including the two numbers before it.

What's more, the amazing thing here is that when we take any two progressive Fibonacci Numbers, their proportion is near the Golden Ratio.

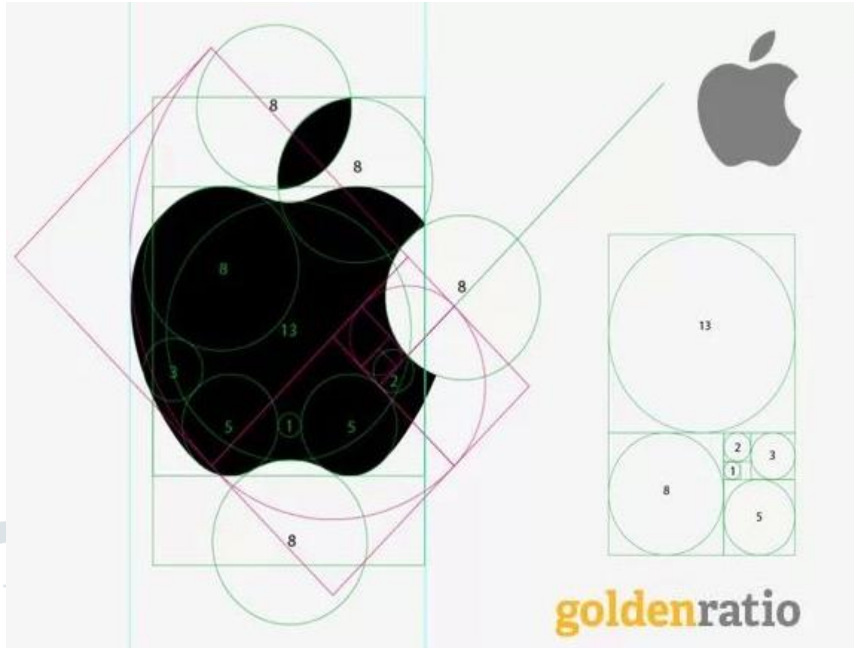
That is the sequence $\frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}, \frac{21}{13}, \frac{34}{21}$, ... which is generated using terms of Fibonacci sequence and it’s converges to φ.

Fibonacci spiral :

Fibonacci spiral is constructed slightly differently. it's begin with a square shape divided into 2 squares. In each progression, a square the length of the square shape's longest side is added to the square shape. Since the ratio between back to back Fibonacci numbers approaches the golden ratio as the Fibonacci numbers approach infinity, so too does this winding get progressively like the past estimate the more squares are included, as shown by the picture.



Examples in real life :



There are numerous other captivating numerical connections and peculiarities in both Phi and the Fibonacci series that can be investigated in more profundity, be that as it may, for the time being we should now remove a stage from the absolutely numerical and adventure into nature, where Phi and the Fibonacci series show themselves unavoidably, yet not generally. Fibonacci numbers much of the time show up in the quantities of petals in a blossom and in the spirals of plants. The positions and ratios of the key components of numerous creatures depend on Phi. Models incorporate the body areas of ants and different creepy crawlies, the wing measurements and area of eye-like spots on moths, the spirals of ocean shells and the situation of the dorsal blades on porpoises. Indeed, even the spirals of human DNA encapsulate phi proportions.

Apple is one of those not many organizations that don't have the organization name in their logo. However, the Apple logo is one of the most perceived corporate images on the planet. The logo is flawlessly adjusted, and the layouts that guide the logo are hovers with breadths proportion ate to the Fibonacci series.

The spirals most usually found in nature are equi-precise spirals. This essentially implies the winding extends at a steady rate. This happens on the grounds that it makes an even progression of vitality or conveyance of pressure. This has nothing at all to do with the golden ratio.

spiral galaxies also follow the familiar Fibonacci pattern. The Milky Way has several spiral arms, each of them a logarithmic spiral of about 12 degrees

Countenances, both human and nonhuman, swarm with instances of the Golden Ratio. The mouth and nose are each situated at golden section of the separation between the eyes and the base of the jaw line. Comparable ratio can been seen from the side, and even the eye and ear itself.

References :

[1] H. E. Huntley, ed. The Divine Ratio : A Study in Mathematical Beauty

[2] Livio, Mario . The Golden Ratio: The Story of Phi, The World's Most Astonishing Number

[3] Wikipedia, “Golden ratio” <https://en.m.wikipedi.org/wiki/golden_ratio>

[4] Online sources ,

<https://en.wikipedia.org/wiki/Golden_ratio> , <scienceabc.com/ >, <cultofmac.com/227961/ >, <blog.prototype.io/ > (credits : totem learning)