A STUDY ON LABELING OF GRAPHS

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Abstract

In this paper is proposed to Cordial labeling and vertex n-magic labeling that two types of methods. Using this cordial labeling is best one because vertex n-magic labeling is used 1 to n value but the cordial labeling is used 0 and 1 only. So, the paper assign 0 and 1 is the best one.

Keywods: Cordial labeling, binary vertex labeling, even vertex magic total graph, apex vertex, pendant vertex.

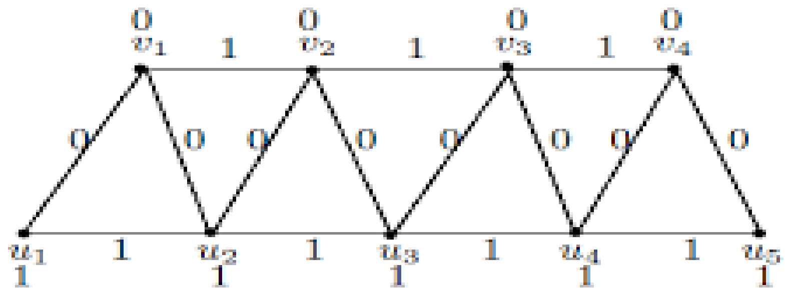
INTRODUCTION:

A graph is cordial if it is possible to label its vertices with 0's and 1's so that when the edges are labeled with the difference of the labels at their endpoints, the number of vertices labeled with ones and zeros differ at most by one. An interesting vertex labeling with numbers is vertex n-magic. Vertex n-magic graphs are graphs labeled with numbers in which every vertex and its incident edges add up to the same number. This number is called the magic number. It is still does not shows that what types of graphs are vertex-magic and which are not. The cycle graph is one type of graph that has interesting vertex-magic properties. The degree of a vertex, denoted f(v) in a graph is the number of edges incident to it. A vertex with degree zero is called an isolated vertex. That is, a vertex which is not an endpoint of any edge. A vertex with degree one is called leaf vertex. A vertex is said to be an apex vertex,if all the chords of the vertices are concurrent.

Definition 1. Cordial labeling

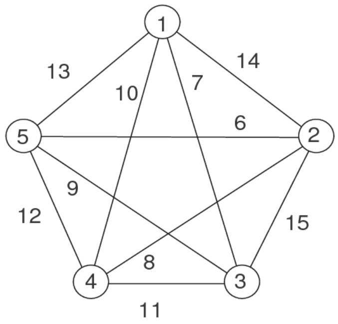
Let G = (V,E) be a simple graph, and f be the function from $V(G) \rightarrow (0,1)$ and for edge uv assign the label

$|f(u) - f(v)|$. f is called a cordial labeling.



Definition 2. Vertex magic graph

A graph whose edges are labelled by positive integers, so that the sum over the edges incident with any vertex is the same, independent of the choice of vertex; or it is a graph that has such a labeling is called magic graph.



Definition 3. Binary vertex labeling

A mapping f : V(G)→ {0,1} is called binary vertex labelling of G and f(V) is called the label of the vertex V of G under f.

Definition 4. Even vertex magic total graph

A labeling is said to be an even vertex N-magic total if f (V(G)) = {2,4,6,..........}. A graph that admits an even vertex N-magic total labeling is called even vertex magic total graph.

Definition 5. Shadow graph

The connected graph G is said to be a Shadow graph, it is constructed by taking two copies of G say G' and G". Join each vertex u' in G' to the neighbours of the corresponding vertex v' in G".

Definition 6. Splitting graph

A graph G is said to be the Splitting graph S'(G) of a graph G is obtained a new vertex V' corresponding to each vertex V of G such that N(V)=N(V').

Theorem.1

$S'(k_1,n)$ is cordial graph.

Proof:

Let $v_1,v_2,\ldots\ldots\ldots\ldots v_n$ be the pendant vertices and v be the apex vertex of $k_{1,n}$ and $u_1,u_2,\ldots\ldots\ldots\ldots u_n$ are added vertices corresponding to $v_1,v_2,\ldots\ldots\ldots\ldots v_n$ to obtain $S'(k_1,n)$.

Let G be the Splitting graph $S'(k_{1,n})$ then

$|V(G)| = 2n + 2$ and

$|E(G)| = 3n$.

The labeling is a function from $f : V(G) \rightarrow \{0,1\}$ is defined as per the following cases

Case (i)

$n \equiv 0 \pmod 6$ $n = 6,12,18,\ldots..$

$f(u) = 1,$

$f(u_i) = 0$; if $i \equiv 1, 3, \pmod 6$ $i = 7,9,\ldots$

$= 1$; if $i \equiv 0, 2 \pmod 6$ $i = 6,8,\ldots.$

$f(v) = 0,$

$f(v_i) = 0$; if $i \equiv 3, 5 \pmod 6$ $i = 9.11,\ldots$

$= 1$; if $i \equiv 2, 4 \pmod 6$ $i = 8,10,\ldots$

Case (ii)

$n \equiv 1 \pmod 8$ $n=9,17,25,\ldots.$

$f(u) = 1$

$f(u_i) = 0$: if $i \equiv 5, 7 \pmod 8$ $i = 13,15,\ldots.$

$=1$: if $i \equiv 4, 6 \pmod 8$ $i = 12,14,\ldots..$

$f(v) = 0,$

$f(v_i) = 0$; if $i \equiv 1, 3 \pmod 8$ $i = 17,19$

$=1$; if $i \equiv 0, 2 \pmod 8$ $i = 16,18,\ldots.$

Case (iii)

$n \equiv 2,3 \pmod 8$ $n=10,11,\ldots..$

$f(u) = 1,$

$f(u_i) = 0;$ if $i \equiv 0, 1 \pmod 8$ $i = 8,9,\ldots.$

$=1;$ if $i \equiv 2, 3 \pmod 8$ $i = 10,11,\ldots$

$f(v) = 0;$

$f(v_i) = 0;$ if $i \equiv 2, 3 \pmod 8$ $i = 18,19,\ldots.$

$=1;$ if $i \equiv 0, 1 \pmod 8$ $i = 16,17,\ldots...$

The condition satisfies the labeling pattern in all the above cases.

$|v_f(0) - v_f(1)| \leq 1$ and

$|e_f(0) - e_f(1)| \leq 1$

in each case which is shown in table.

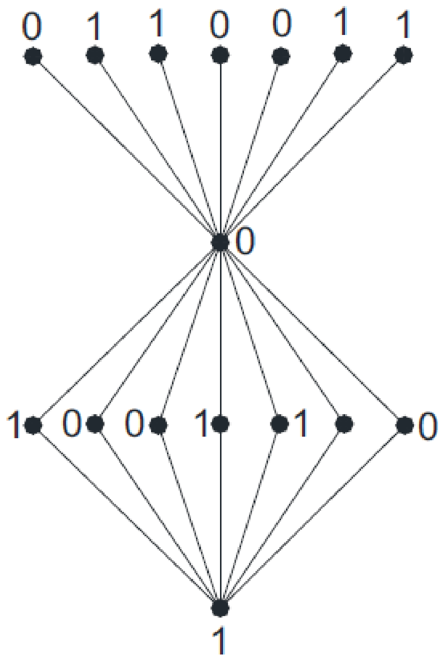
Hence, $S'(k_1,n)$ is cordial graph

Let $n = 4a + b$, where $n \in N$

Table

b	Vertex condition	Edge condition
0,2	$v_f(0) = v_f(1)$	$e_f(0) = e_f(1)$
1	$v_f(0) = v_f(1)$	$e_f(0) = e_f(1) + 1$
3	$v_f(0) = v_f(1)$	$e_f(0) + 1 = e_f(1)$

Example



Theorem.2

For each m, n $\geq$ 3 and m, n odd, there exists an even vertex magic total labeling of $C_m \times C_n$ with the magic constant $k = \frac{17}{2}mn + \frac{5}{2}$.

Proof.

Let G $\cong C_m \times C_n$ have vertices $v_{i,j}$ vertical edges $v_{i,j}\, v_{i+1,j}$ and horizontal edges $v_{i,j}v_{i,j+1}$

where $i = 0, 1, \ldots\ldots, m-1$,

$j = 0, 1, \ldots, n-1$ and m and n are odd integers greater than 1.

Let us consider the following labeling, where i and j are the subscripts taken modulo m and n respectively.

$$f\,(v_{i,j}) = 2\,(mn - jm - i)$$

$$f\left(v_{i,j}v_{i+1,j}\right) = \begin{cases} 2m(n+j+1) - 1 - i & \text{if } i \text{ is even} \\ 2m(n+j) + m - i - 1 & \text{if } i \text{ is odd} \end{cases}$$

$$f(\,v_{i,j}v_{i,j+1}) = \begin{cases} 2(mn - m + i) - jm + 1 & \text{if } j \text{ is even} \\ 2(i-m) - jm + mn + 1 & \text{if } j \text{ is odd} \end{cases}$$

The magic constant k is obtained from the following cases.

Case (i)

If both i and j are even then the magic constant k is given by

When i=2, j=2

$$k = f(v_{i,j}) + f(v_{i-1,j}v_{i,j}) + f(v_{i,j}, v_{i+1,j}) + f(v_{i,j-1}v_{i,j}) + f(v_{i,j}, v_{i,j+1})$$

$$k = f(v_{2,2}) + f(v_{2-1,2}v_{2,2}) + f(v_{2,2}, v_{2+1,2}) + f(v_{2,2-1}v_{2,2}) + f(v_{2,2}, v_{2,2+1})$$

$$= 2(mn - 2m - 2) + 2m(n + 2) + m - (2 - 1) - 1 + 2m(n + 2 + 1)$$
$$-1 - 2 + 2(2 - m) - (2 - 1)m + mn + 1 + 2(mn - m + 2) - 2m + 1$$

$$k = 9mn + 1$$

Case (ii)

If i is even and j is odd then the magic constant k is given by

When i=2, j=1

$$k = f(v_{i,j}) + f(v_{i-1,j}v_{i,j}) + f(v_{i,j}, v_{i+1,j}) + f(v_{i,j-1}v_{i,j}) + f(v_{i,j}, v_{i,j+1})$$

$$k = f(v_{2,1}) + f(v_{2-1,1}v_{2,1}) + f(v_{2,1}, v_{3,1}) + f(v_{2,0}v_{2,1}) + f(v_{2,1}, v_{2,2})$$
$$= 2(mn - m - 2) + 2m(n + 1) + m - 2 + 2m(n + 1 + 1) - 1 - 2$$

$$+ 2(mn - m + 2) - (1 - 1)m + 1 + 2(2 - m) - m + mn + 1$$

$$k = 9mn + 1$$

Similarly, we can prove for i is odd,

j is even and for both i and j are odd.

The magic constants are same in all the cases and $f\big(V(G)\big) = \{2, 4, \ldots, 2p\}$.

Therefore G is even vertex magic total with the magic constant $k = 9mn + 1$.

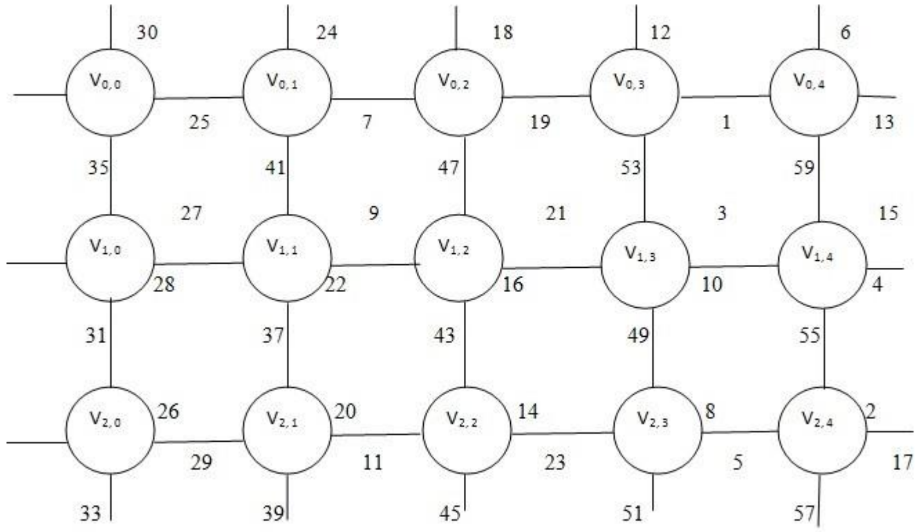


Fig : Even vertex-magic total labeling of $C_3 \times C_5$ with magic constant h = 136.

Conclusion:

From this,we conclude cordial labeling is best one because vertex n-magic labeling is used 1 to n value but the cordial labeling is used 0 and 1 only. So, the paper assign 0 and 1 is the best one.

References

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