QUASI g*- NORMAL SPACES AND πgg*- CLOSED SETS

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Abstract: In this paper, we introduce a new class of sets called πgg*-closed sets in topological spaces. Also we study and investigate the relationship with other existing closed sets. Moreover, we introduce some functions such as g*-closed, πgg*-closed, almost g*-closed, almost πgg*-closed, πgg*-continuous and almost πgg*-continuous.We also study a new class of normal space called, quasi g*-normal space. The relationships among normal, π-normal, quasi normal, softly normal, mildly normal, α-normal,πα-normal, quasi α-normal, softly α-normal, mildly α-normal, g*-normal, πg*-normal,quasi g*-normal, softly g*-normal and mildly g*-normal spaces are investigated. Further we show that this property is a topological property and it is a hereditary property only with respect to closed domain subspaces. Utilizing πgg*-closed sets and some functions, we obtained some characterizations and preservation theorems for quasi g*-normal spaces.

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1. Introduction

In 1958, Kuratowski, **[13]** introduced the concept of regular open and regular closed sets in topological spaces. In 1968, Zaitsev [**32**] introduced the concept of quasi-normal space in topological spaces and obtained several properties of such a space. In 1970, Levine **[15]** defined generalized closed sets in topological spaces. In 1973, Singal and Singal **[26]** introduced the concept of mildly normal spaces and obtained their properties. In 1989, Nour **[22]** introduced the notion of p-normal spaces and obtained their characterizations and preservation theorems for p-normal spaces. In 1990, Mahmoud and Monsef **[17]** introduced the concept of β-normal spaces. In 2000, M. K. R. S. Veera Kumar **[31]** introduced the concepts of g*-closed sets in topological spaces. In 2007, Ekici **[10]** introduced the concept of γ-normal spaces and obtained their characterizations and preservation theorems for γ-normal spaces. In 2008, Kalantan **[11]** introduced the notion of π-normal spaces and obtained some characterizations. In 2010, Tahiliani **[30]** introduced the notion of πgβ-closed sets and their properties are studied. In 2010, M. C. Sharma and Hamant Kumar **[24]** introduced the notion of πβ-normal spaces and obtained their characterizations. In 2012, Thabit and Kamaruihaili **[28]** introduced the notion of a weaker form of p-normality called quasi p-normality which lies between πp-normality and mild p-normality. In 2013, Thanh and Thinh **[29]** introduced the notion of πgp-normal spaces and prove that πgp-normality is a topological property and it is a hereditary property with respect to π-open, πgp-closed subspaces. Recently, Hamant Kumar and M.C.Sharma **[12]** introduced the concept of πgγ-closed sets as weak form of πg-closed sets due to Dontchev [**9**]. and introduced the concept of quasi γ-normal spaces and by using πgγ-closed sets, we obtained a characterization and preservation theorems for quasi γ-normal spaces.

2. Preliminaries

Throughout in this paper, the spaces (X, τ), (Y, σ) and (Z, γ) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a space X. The closure of A and interior of A are denoted by cl(A) and int(A) respectively.

2.1. Definition. A subset A of a space X is said to be

1. **regular open [13]** if A = int(cl(A)).
2. The finite union of regular open sets is said to be **π-open [32].**
3. **γ-open [3]** if $A \subset cl(int(A)) \cup int(cl(A))$.
4. **p-open [18]** if $A \subset int(cl(A))$.
5. **s-open [14]** if $A \subset cl(int(A))$.
6. **α-open [19]** if $A \subset int(cl(int(A)))$.
7. **β-open [1]** if $A \subset cl(int(cl(A)))$.

The complement of a regular open (resp. π-open, γ-open, p-open, s-open, α-open, β-open) set is said to be **regular closed** (resp. **π-closed, γ-closed, p-closed, s-closed, α-closed, β-closed**).

2.2. Definition. A subset A of a topological space X is said to be

1. **g-closed** [15] if cl(A) $\subset$ U whenever A $\subset$ U and U is open in X.
2. **gp-closed [20]** if p-cl(A) $\subset$ U whenever A $\subset$ U and U is open in X.
3. **gs-closed [6]** if p-cl(A) $\subset$ U whenever A $\subset$ U and U is open in X.
4. **αg-closed [16]** if α-cl(A) $\subset$ U whenever A $\subset$ U and U is open in X
5. **gβ-closed [8]** if β-cl(A) $\subset$ U whenever A $\subset$ U and U is open in X.
6. **gγ-closed [2]** if γ-cl(A) $\subset$ U whenever A $\subset$ U and U is open in X.
7. **πg -closed** [**9**] if cl(A) $\subset$ U whenever A $\subset$ U and U is π-open in X.
8. **πgp-closed [23]** if p-cl(A) $\subset$ U whenever A $\subset$ U and U is π-open in X.
9. **πgs-closed [7]** if s-cl(A) $\subset$ U whenever A $\subset$ U and U is π-open in X.
10. **πgα-closed [5]** if α-cl(A) $\subset$ U whenever A $\subset$ U and U is π-open in X.
11. **πgβ-closed [30]** if β-cl(A) $\subset$ U whenever A $\subset$ U and U is π-open in X.
12. **πgγ-closed [27]** if γ-cl(A) $\subset$ U whenever A $\subset$ U and U is π-open in X.
13. **g-open** (resp. **gp-open, gs-open, αg-open, gβ-open, gγ-open, πg-open, πgp-open, πgs-open, πgβ-open, πgγ-open**) if the complement of A is g-closed (resp. gp-closed, gs-closed, αg-closed, gβ-closed, gγ-closed, πg-closed, πgp-closed, πgs-closed, πgα-closed, πgβ-closed, πgγ-closed).
14. **g*-closed [31]** if cl(A) $\subset$ U whenever A $\subset$ U and U is g-open in X.
15. **πgg*-closed** if g*- cl(A) $\subset$ U whenever A $\subset$ U and U is π-open in X.

The complement of g-closed (resp. α-closed ,g*-closed, gg*-closed, πg-closed ,πgg*-closed) set is called **g-open** (resp. **g*-open**, **gg*-open**, **πg-open**, **πgg*-open**) set and the complement of π-open is called π-closed. The intersection of all g*-closed sets containing A is called the **g*-closure of A** and denoted **g*-cl**(**A**).The union of all g*-open subsets of X which are contained in A is called the **g*-interior of A** and denoted by **g*-int**(**A**).

closed ⇒ g-closed ⇒ πg-closed

⇓ ⇓ ⇓

g*-closed ⇒ gg*-closed ⇒ πgg*-closed

⇓ ⇓ ⇓

α-closed	$\Rightarrow$	αg-closed	$\Rightarrow$	πgα-closed
$\Downarrow$		$\Downarrow$		$\Downarrow$
p-closed	$\Rightarrow$	gp-closed	$\Rightarrow$	πgp-closed
$\Downarrow$		$\Downarrow$		$\Downarrow$
γ-closed	$\Rightarrow$	gγ-closed	$\Rightarrow$	πgγ-closed
$\Downarrow$		$\Downarrow$		$\Downarrow$
β-closed	$\Rightarrow$	gβ-closed	$\Rightarrow$	πgβ-closed

and

closed	$\Rightarrow$	g-closed	$\Rightarrow$	πg-closed
$\Downarrow$		$\Downarrow$		$\Downarrow$
g*-closed	$\Rightarrow$	gg*-closed	$\Rightarrow$	πgg*-closed
$\Downarrow$		$\Downarrow$		$\Downarrow$
α-closed	$\Rightarrow$	αg-closed	$\Rightarrow$	πgα-closed
$\Downarrow$		$\Downarrow$		$\Downarrow$
s-closed	$\Rightarrow$	gs-closed	$\Rightarrow$	πgs-closed
$\Downarrow$		$\Downarrow$		$\Downarrow$
γ-closed	$\Rightarrow$	gγ-closed	$\Rightarrow$	πgγ-closed
$\Downarrow$		$\Downarrow$		$\Downarrow$
β-closed	$\Rightarrow$	gβ-closed	$\Rightarrow$	πgβ-closed

here none of the implications is reversible as can be seen from the following examples.

2.3. Example. Let X = {a, b, c, d} and τ = {ϕ, X, {a}, {d}, {a, d} {c, d}, {a, c, d}}. Here we show that A = {c} is πgα-closed but not g-closed.

2.4. Example. Let X = {a, b, c, d} and τ = {ϕ, {a}, {b, c}, {a, b, c}, X}. Then A = {a} is gγ-closed as well as gβ-closed but not closed.

2.5. Example. Let X = {a, b, c, d) and τ = {ϕ, {b}, {d}, {b, d}, X}. Then A = {a, b, d} is gγ-closed as well as gβ-closed but it is not closed.

2.6. Example. Let X = {a, b, c, d} and τ = {ϕ, {b, d}, {a, b, d}, {b, c, d}, X}. A = {a, b} is gγ-closed as well as πgγ-closed but it is not closed.

2.7. Example. Let X = {a, b, c} and τ = {ϕ, {b}, {c}, {b, c}, X}. Then the subset A = { b} is g-closed as well as gγ-closed but not closed.

2.8. Example. Let X = {a, b, c} and τ = {ϕ, {a}, X}. Then the subset A = {a, b} is g-closed as well as gγ-closed but not closed.

2.9. Example. Let X = {a, b, c} and τ = {ϕ, {a}, {a, b}, X}. Then the subset A = {a, c} is g-closed as well as gγ-closed but not closed.

2.10. Example. Let X = {a, b, c, d, e} and τ = {ϕ, {a, b}, {b, d}, {a, b, c, d}, X}. Then A = {a, e} is πg-closed as well as πgα-closed but it is not closed.

2.11. Example. Let X = {a, b, c, d} and τ = {ϕ, {a}, {d}, {a, d}, {c, d}, {a, c, d}, X}. Then A ={c} is πgα-closed as well as πgp-closed but it is not closed.

2.12. Example. Let X = {a, b, c, d} and τ = {ϕ, {a}, {b}, {a,b}, {a, b, d}, X}. Then A = {a} is πgs-closed as well as πgγ-closed but it is not closed.

2.13. Example. Let X = {a, b, c, d} and τ = {ϕ, {a}, {d}, {a,d}, {c, d}, {a, c, d}, X }. Then A = {c} is πgp-closed as well as πgβ-closed but it is not closed.

2.14.Example. Let X = { a, b, c, d } and τ = { ϕ, {a}, {c, d}, {a, c, d }, {d}, {a, d}, X }.Let A ={c}.Then A is πgg*-closed set but not πg-closed set in X.

2.15. Example. Let X = { a, b, c, d } and τ = { ϕ, {a},{c}, {a, b}, {a, c},{a, d} , {a, d, c }, {a, b, d}, {a, b, c}, X }. Then the set A = {a} is πgg*-closed set not gg*-closed set in X .

2.16. Theorem. A subset A of a topological space X is πgg*-open iff F $\subset$ g*-int (A) whenever F is π-closed and F $\subset$ A.

3. Quasi g*- Normal Spaces

3.1. Definition. A topological space X is said to be **g*-normal** (resp. **α-normal [4]**) if for every pair of disjoint closed subsets A , B of X, there exist disjoint g*-open (resp.α-open) sets U, V of X such that A $\subset$ U and B $\subset$ V.

3.2. Definition. A topological space X is said to be **πg*-normal** (resp. **π-normal [11], πα-normal**) if for every pair of disjoint closed subsets A, B of X, one of which is π-closed, there exist disjoint g*-open (resp. open, α-open) sets U, V of X such that A ⊂ U and B ⊂ V.

3.3. Definition. A topological space X is said to be **quasi g*-normal** (resp. **quasi normal [32], quasi α-normal [5])** if for every pair of disjoint π-closed subsets H, K, there exist disjoint g*-open sets U, V of X such that H ⊂ U and K ⊂ V.

normal	⇒	π-normal	⇒	quasi-normal
⇓		⇓		⇓
g*-normal	⇒	πg*-normal	⇒	quasi g*-normal
⇓		⇓		⇓
α-normal	⇒	πα-normal	⇒	quasi α-normal
⇓		⇓		⇓
p-normal	⇒	πp-normal	⇒	quasi p-normal
⇓		⇓		⇓
γ -normal	⇒	πγ-normal	⇒	quasi γ-normal
⇓		⇓		⇓
β-normal	⇒	πβ-normal	⇒	quasi β

and

normal	⇒	π-normal	⇒	quasi-norma
⇓		⇓		⇓
g*-normal	⇒	πg*-normal	⇒	quasi g*-normal
⇓		⇓		⇓

α-normal	$\Rightarrow$	$\pi\alpha$-normal	$\Rightarrow$	quasi α-normal
$\Downarrow$		$\Downarrow$		$\Downarrow$
s-normal	$\Rightarrow$	πs-normal	$\Rightarrow$	quasi s-normal
$\Downarrow$		$\Downarrow$		$\Downarrow$
γ -normal	$\Rightarrow$	$\pi\gamma$-normal	$\Rightarrow$	quasi γ-normal
$\Downarrow$		$\Downarrow$		$\Downarrow$
β-normal	$\Rightarrow$	$\pi\beta$-normal	$\Rightarrow$	quasi β-normal

3.4. Example. Let X = {a, b, c, d, e} and τ = {ϕ, {e}, {a, b}, {c, d}, {a, b, e}, {c, d, e}, {a, b, c, d}, X}. The pair of disjoint π-closed subsets of X are A = {a, b} and B = {c, d}. Also U = {a, b, e} and V ={c, d} are γ-open sets such that A $\subset$ U and B $\subset$ V. Hence X is quasi γ-normal but not quasi-normal, since U and V are not open sets.

3.5. Example. Let X = {a, b, c} and τ = {ϕ, X, {a}, {a, b}, {a, c}} is quasi p-normal but not p-normal space.

3.6. Example. Let X = {a, b, c, d} and τ = {ϕ, {a}, {c}, {a, c}, { b, d}, {a, b, d}, {b, c, d}, X}. The pair of disjoint π-closed subsets of X are A = {a} and B = {c}. Also U = {a} and V = {b, c, d} are disjoint open sets such that A $\subset$ U and B $\subset$ V. Hence X is quasi-normal as well as quasi g*-normal because every open set is g*-open set.

3.7. Theorem. For a topological space X, the following are equivalent :

(a) X is quasi g*-normal.

(b) For any disjoint π-closed sets H and K, there exist disjoint gg*-open sets U and V such that H $\subset$ U and K $\subset$ V.

(c) For any disjoint π-closed sets H and K, there exist disjoint πgg*-open sets U and V such that H $\subset$ U and K $\subset$ V.

(a) For any π-closed set H and any π-open set V containing H, there exists a gg*-open set U of X such that H $\subset$U $\subset$ g*-cl(U) $\subset$ V.

(b) For any π - closed set H and any π - open set V containing H, there exists a πgg*- open set U of X such that H $\subset$ U $\subset$ g*-cl(U) $\subset$ V.

Proof. (a) $\Rightarrow$ (b), (b) $\Rightarrow$ (c), (d) $\Rightarrow$ (e) , (c) $\Rightarrow$ (d), and (e) $\Rightarrow$ (a). (a) $\Rightarrow$ (b). Let X be quasi g*-normal. Let H, K be disjoint π - closed sets of X. By assumption, there exist disjoint g*-open sets U, V such that $H \subset U$ and $K \subset V$. Since every g*-open set is gg*-open, U ,V are gg*-open sets such that $H \subset U$ and $K \subset V$.

(b) $\Rightarrow$ (c). Let H, K be two disjoint π -closed sets. By assumption, there exist gg*-open sets U and V such that $H \subset U$ and $K \subset V$. Since gg*-open set is πgg*-open, U and V are πgg*-open sets such that $H \subset U$ and $K \subset V$.

(d) $\Rightarrow$ (e). Let H be any π-closed set and V be any π-open set containing H. By assumption, there exists a gg*-open set U of X such that

$H \subset U \subset$ g*-cl(U) $\subset V$. Since every gg*-open set is πgg*-open, there exists a πgg*-open set U of X such that $H \subset U \subset$ g*-cl(U) $\subset V$.

(c) $\Rightarrow$ (d). Let H be any π-closed set and V be any π-open set containing H. By assumption, there exist πgg*-open sets U and W such that $H \subset U$ and

$X - V \subset W$. By **Theorem 2.16**, we get $X - V \subset$ g*-int(W) and

g*-cl(U) $\cap$ g*- int(W) = ϕ.Hence $H \subset U \subset$ g*-cl(U) $\subset X -$ g*-int(W) $\subset V$.

(e) $\Rightarrow$ (a). Let H, K be any two disjoint π -closed set of X. Then $H \subset X - K$ and $X - K$ is π-open. By assumption, there exists a πgg*-open set G of X such that $H \subset G \subset$ g*-cl(G) $\subset X - K$. Put U = g*-int(G),V = X – g*-cl(G). Then U and V are disjoint g*-open sets of X such that $H \subset U$ and $K \subset V$.

3.8. Definition. A function $f : X \rightarrow Y$ is said to be

1. **g*- closed** (resp. **gg*- closed ,πgg*- closed)** if f (F) is g*-closed (resp. gg*-closed ,πgg*-closed) in Y for every closed set F of X
2. **rc - preserving** [21](resp. **almost closed** [25], **almost g*- closed, almost gg*-closed**, **almost πgg*- closed)** if f (F) is regularly closed (resp. closed, g*-closed, gg*- closed, πgg*- closed) in Y for every $F \in RC(X)$.
3. **π-continuous [9]** (resp. **almost π-continuous [9]** if f^{-1}(F) is π-closed in X for every closed (resp. regular closed) set F of Y.

4.**almost πgg*-continuous** if f^{-1}(F) is πgg*-closed in X for every regular closed set F of Y.

From the definitions stated above, we obtain the following diagram:

closed	$\Rightarrow$	g*- closed	$\Rightarrow$	gg*- closed	$\Rightarrow$	πgg*- closed
$\Downarrow$		$\Downarrow$		$\Downarrow$		$\Downarrow$
al. closed	$\Rightarrow$	al. g*-closed	$\Rightarrow$	al. gg*-closed	$\Rightarrow$	al. πgg*-closed

here al. = almost.

here none of the reverse implications are true as can be seen from the following examples :

3.9. Example. X = { a, b, c, d }, τ ={ ϕ , {c}, {a, b, d},X } and σ = { ϕ, {a}, {d}, {c, d}, {a, d},{a, c, d}, X }. Let f : (X, τ) → (X, σ) be the identity function. Then f is πgg*-closed but not πg-closed. Since A= {c} is not πg-closed in (X, σ).

3.10. Example . Let X = { a, b, c, d } τ = {ϕ, {c}, {b, d}, {a, b, d},{b, c, d}, X } and σ = { ϕ, X, {a}, {d},{a, d}, {c, d}, {a, c, d}}. Let f : (X, τ) → (X, σ) be the identity function Then f is almost πgg*-closed but not πgg*-closed. Since A= {c} is not πgg*-closed.

3.11.Theorem. A surjection f : X → Y is almost πgg*-closed if and only if for each subset S of Y and each U ∈ RO(X) containing $f^{-1}(S)$, there exists a πgg*-open set V of Y such that S ⊂ V and $f^{-1}(V) \subset U$.

Proof. Necessity. Suppose that f is almost πgg*-closed. Let S be a subset of Y and U ∈ RO(X) containing $f^{-1}(S)$. If V = Y– f (X – U) , then V is a πgg*-open set of Y such that S ⊂ V and $f^{-1}(V) \subset U$.

Sufficiency. Let F be any regular closed set of X. Then f^{-1} (Y– f (F)) ⊂ X – F and X – F ∈ RO(X). There exists a πgg*- open set V of Y such that Y – f (F) ⊂ V and f^{-1} (V) ⊂ X – F. Therefore, we have f (F) ⊃ Y – V and F ⊂ X – f^{-1} (V) ⊂ f^{-1} (Y – V) . Hence we obtain f (F) = Y – V and f (F) is πgg*-closed in Y which shows that f is almost πgg*-closed.

4. Preservation Theorems

4.1.Theorem. If f : X → Y is an almost πgg*-continuous rc-preserving injection and Y is quasi g*-normal then X is quasi g*-normal.

Proof. Let A and B be any disjoint π-closed sets of X. Since f is a rc-preserving injection, f (A) and f (B) are disjoint π-closed sets of Y. Since Y is quasi g*-normal, there exist disjoint g*-open sets U and V of Y such that f (A) ⊂ U and f (B) ⊂ V. Now if G = int(cl(U)) and H = int(cl(V)). Then G and H are regular open sets such that f (A) ⊂ G and f (B) ⊂ H. Since f is almost πgg*-continuous, $f^{-1}(G)$ and $f^{-1}(H)$ are disjoint πgg*-open sets containing A and B respectively which shows that X is quasi g*-normal.

4.2.Theorem. If $f : X \to Y$ is π-continuous almost g*-closed surjection and X is quasi g*-normal space then Y is g*-normal.

Proof. Let A and B be any two disjoint closed sets of Y. Then $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint π-closed sets of X. Since X is quasi g*-normal, there exist disjoint g*-open sets of U and V such that $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$. Let G = int(cl(U)) and H = int(cl(V)). Then G and H are disjoint regular open sets of X such that $f^{-1}(A) \subset G$ and $f^{-1}(B) \subset H$. Set $K = Y - f(X - G)$, $L = Y - f(X - H)$.Then K and L are g*-open sets of Y such that $A \subset K$, $B \subset L$, $f^{-1}(K) \subset G$, $f^{-1}(L) \subset H$. Since G and H are disjoint, K and L are disjoint. Since K and L are g*-open and we obtain $A \subset$ g*-int(K), $B \subset$ g*-int(L) and g*-int(K) $\cap$ g*- int(L) = ϕ. Therefore Y is g* - normal.

4.3.Theorem. Let $f : X \to Y$ be an almost π-continuous and almost πgg*-closed surjection. If X is quasi g*-normal space then Y is quasi g*-normal.

Proof. Let A and B be any disjoint π-closed sets of Y. Since f is almost π-continuous, $f^{-1}(A)$, $f^{-1}(B)$ are disjoint closed subsets of X. Since X is quasi g*-normal, there exist disjoint g*-open sets U and V of X such that $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$. Put G = int(cl(U)) and H = int(cl(V)).Then G and H are disjoint regular open sets of X such that $f^{-1}(A) \subset G$ and $f^{-1}(B) \subset H$. By **Theorem 3.11**, there exist πgg*-open sets K and L of Y such that $A \subset K$, $B \subset L$, $f^{-1}(K) \subset G$ and $f(L) \subset H$. Since G and H are disjoint. So are K and L by **Theorem 2.16**, $A \subset$ g*-int(K), $B \subset$ g*-int(L) and g*-int(K) $\cap$ g*-int(L) = ϕ . Therefore, Y is quasi g*-normal.

4.4.Corollary. If $f : X \to Y$ is an almost continuous and almost closed surjection and X is a normal space, then Y is quasi g*-normal.

Proof. Since every almost closed function is almost πgg*-closed so Y is quasi g*-normal.

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