Generalization of compact spaces

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***Abstract*– In this paper, we define (i,j)-semi-H-compact spaces and obtain some characterizations and basic properties of such a spaces.**

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I. INTRODUCTION

Connectedness and compactness are powerful tools in topology but they have many dissimilar properties. The concept of Hausdorff spaces is almost an integral part of compactness. Investigations into the properties of cut points of topological spaces which are connected, compact and Hausdorff date back to the 1920s. Connectedness together with compactness with the assumption of Hausdorff has been studied from the view point of cut points.The concept of ideals in topological spaces has been introduced and studied by Kuratowski [5] and Vaidyanathaswamy [11]. A hereditary class H on a topological space (X,τ) is a nonempty collection of subsets of X which satisfies (i) $A \in H$ and $B \subset A$ implies $B \in H$. Given a topological space (X, τ) with a hereditary class H on X and if P(X) is the set of all subsets of X, a set operator $(.)^* : P(X) \to P(X)$, called the local function [11] of A with respect to τ and H, is defined as follows: for $A \subset X$, $A^*(\tau,H) = \{x \in X | U \cap A \notin H \text{ for every } U \in \tau(x)\}$, where $\tau(x) = \{U \in \tau | x \in U\}$. A Kuratowski closure operator $Cl^*(\cdot)$ for a topology $\tau^*(\tau,H)$ called the $*$-topology, finer than τ is defined by $Cl^*(A) = A \cup A^*(\tau,H)$ when there is no chance of confusion, $A^*(H)$ is denoted by A^*. If H is a hereditary class on X, then (X,τ_1,τ_2,H) is called a hereditary bitopological space. The purpose of this paper is to define upper (lower) (i,j)-semi-H-irresolute multifunction and to obtain several characterizations of such a multifunction.

II. PRELIMINARIES

Throughout this paper, (X,τ_1,τ_2,H) and (Y,σ_1,σ_2,K) always mean hereditary bitopological spaces in which no separation axioms are assumed unless explicitly stated. For a subset A of (X,τ), Cl(A) and Int(A) denote the closure of A with respect to τ and the interior of A with respect to τ, respectively. A subset S of a hereditary bitopological space (X,τ_1,τ_2,I) is (i,j)-semi-H-open $S \subset Cl_j^*(Int_i(S))$). The complement of an (i,j)-semi-H-closed set is said to be an (i,j)-semi-H-open set. The (i,j)-semi-H-closure and the (i,j)-semi-H-interior, that can be defined in the same way as Cl(A) and Int(A), respectively, will be denoted by (i,j)-sHCl(A) and (i,j)-sHInt(A), respectively. The family of all (i,j)-semi-H-open (resp.(i,j)-semi-H-closed) sets of (X,τ_1,τ_2,H) is denoted by (i,j)-SHO(X) (resp. (i,j)-SHC(X)). The family of all (i,j)-semi-H-open (resp. (i,j)-semi-H-closed) sets of (X,τ_1,τ_2,H) containing a point $x \in X$ is denoted by (i,j)-SHO(X,x) (resp. (i,j)-SHC(X,x)). A subset U of X is called an (i,j)-semi-H-neighborhood of a point $x \in X$ if there exists $V \in$ (i,j)-SHO(X,x) such that $V \subset U$.

III. PROPERTIES OF (i,j)-SEMI-H-COMPACT SPACES

Definition : (x, τ) is a (i,j)-semi-H-compact space if for every (i,j)-semi-H-open cover $\{w_i : i \in \nabla\}$ there exists a finite ∇_0 of ∇ such that $X \cup \{w_i : i \in \nabla_0\} \in I$

Theorem :

The following are equivalent for a space (X, τ ,I)

(a) (X, τ ,I) is (i,j)-semi-H-compact
(b) (b) For any family $\{F_\lambda : \lambda \in \Delta\}$ of (i,j)-semi-closed sets of X such that $\cap\{F_\lambda : \lambda \in \Delta\} = \emptyset$, there exists a finite subset Δ_0 of Δsuch that $\cap\{F_\lambda : \lambda \in \Delta\} \in I$

Proof: (a) ⇒(b) Let { F_λ : $\lambda\in\Delta$} be a family of (i,j)-semi-H-closed set s of X such that ∩{ F_λ : $\lambda\in\Delta$} =∅ . Then { X-F_λ : $\lambda\in\Delta$} is an (i,j)-semi-open cover of X. By (a), there exists a finite subset Δ_0 of Δ such that X-∪{ X-F_λ : $\lambda\in\Delta_0$ } ∈ I .This implies that ∩{ F_λ : $\lambda\in \Delta_0$ } ∈ I.

(b)⇒(a) Let { U_λ : $\lambda\in\Delta$} be an (i,j)-semi-H-open cover of X ,then ∩{X- U_λ : $\lambda\in\Delta$} is a collection of (i,j)-semi-closed sets and ∩{ X-U_λ : $\lambda\in\Delta$} = ∅. Hence there exists a finite subset Δ_0 and Δ such that ∩{X -U : $\lambda\in\Delta_0$ } ∈ I.This implies that X-∪{ U_λ : $\lambda\in\Delta_0$} ∈ I.Thisshows that (X, τ ,I) is (i,j)-semi-H-compact.

Definition : A subset A of a space (X, τ ,I) is said to be (i,j)-semi-H-compact relative to X if for every cover { U_λ : $\lambda\in\Delta$} of A by (i,j)-semi-open sets of X , there exists a finite subset Δ_0 of Δ such that A- ∪{ U_λ : $\lambda\in\Delta_0$} ∈ I.

Theorem : If A_i i= 1,2 are (i,j)-semi-H-compact subsets relative to a space (X, τ ,I) ,then A_1∪ A_2 is (i,j)-semi-H-compact relative to X.

Proof: Let { V_λ : $\lambda\in\Delta$} be a (i,j)-semi-H-open cover of A_1∪ A_2 .Then it is a (i,j)-semi-open cover of A_i for i= 1,2 Since A_i is (i,j)-semi-compact relative to X , then there exists a finite subset Δ_0 of Δ such that A_i - ∪{ V_λ : $\lambda\in\Delta_0$} ∈ I for i = 1,2 . Therefore A_1∪ A_2 - ∪{ V_λ : $\lambda\in\Delta_0$} ∈ I so A_1∪ A_2 is (i,j)-semi-H-compact relative to X

Theorem : Let (X, τ) be a space with an ideal I on X and A∈ τ . A is (i,j)-semi-H-compact relative to X if and only if (A, τ/A) is qI/A – compact.

Proof: (Necessity) Let (U_λ∩ A) be a τ/A –(i,j)-semi-H-open cover of A, where U_λ ∈QIO(X, τ) for each λ. Now {U_λ } is a τ -(i,j)-semi-H-open cover of A and hence there exists a finite sub family {$U_{\lambda i}$ } such that A-∪{ $U_{\lambda i}$: i= 1,2,3,……n} ∈ I .This implies A∩[A-∪{ $U_{\lambda i}$: i= 1,2,3,……n}]⊆I∩ A and we have A∩[A-∪{ $U_{\lambda i}$: i= 1,2,3,……n}]= A-∪ {(A∩$U_{\lambda i}$) i= 1,2,3……..n }∈ I/A. Thus (A, τ/A) is (i,j)-semi-H-compact. Sufficiency : Let { U_λ : $\lambda\in\Delta$} be a (i,j)-semi-H-open cover of A. Then (U_λ ∩A) is (i,j)-semi-open cover of A.There exists a finite subfamily { $U_{\lambda i}$ ∩A} such that A-∪{ $U_{\lambda i}$ ∩A} : i=1,2,3,…….n} ∈ I\A ⊆ I.Hence A is (i,j)-semi-H-compact .

Definition : A space (X, τ ,I) is said to be countably (i,j)-semi-H-compact if for every countable (i,j)-semi-H-open cover { U_n : n∈N } of X there exists a finite subset N_0 of N such that X -∪{ U_n : n∈N_0} ∈ I ,where N denotes the set of positive integers.

Theorem : The following are equivalent for a space (X, τ ,I)

(a) (X, τ ,I) is countably (i,j)-semi-H-compact.

(b)For any countable family { F_n : n∈N } of (i,j)-semi-H-closed sets of X such that ∩{ F_n : n∈N }= ∅ ,there exists a finite subset N_0 of N such that ∩ { F_n : n∈N_0 } ∈ I.

Proof : (a)⟶(b) Let { F_n : n∈N } be a countable family of qI-closed sets of x such that ∩{ F_n : n∈N } = ∅ .Then ∩{ X- F_n : n∈N } is countable (i,j)-semi-H-open cover of X .By (a) there exists a finite subset N_0 of N such that X- ∪{ X- F_n : n∈N_0 }∈ I .This implies that ∩ {F_n : n∈N_0 } ∈ I.

(b)⟶(a) Let {U_n : n∈N } be a countable (i,j)-semi-H-open cover of X .Then{ X- U_n : n∈N } is a countable collection of (i,j)-semi-H-closed sets and ∩ {X- U_n : n∈N } = ∅ .Hence there exists a finite subset N_0 of N such that ∩ {X- U_n : n∈N_0 }∈ I.Therefore ,we have X-∪ { U_n : n∈N_0 }∈ I .This shows that (X, τ ,I) is countably (i,j)-semi-H-compact.

Theorem : If (X, τ ,I) is countably (i,j)-semi-H-compact and J is an ideal on X such that J⊃I ,then (X, τ ,I)

Is countably (i,j)-semi-H-compact.

Theorem : Let f: (X, τ ,I) ⟶ (Y,***σ***) be an qI-irresolute (resp.(i,j)-semi-H-continuous) surjection. If(X, τ ,I)

Is countably (i,j)-semi-H-compact then (Y, ***σ*** ,f(I))is countably qIf(I)- compact (resp. countably f(I) –compact)

Proof: Let { V_n : n∈N } be a countable qI-open (resp. open) cover of Y. Then { $f^{-1}(V_n)$: n∈N} is countable qI-open cover of X and hence there exists a finite subset N_0 of N such that X-∪{ $f^{-1}(V_n)$: n∈N_0}∈I .Now since f is surjective ,we have Y-∪{ V_n : n∈N_0 } =f [X-∪{ $f^{-1}(V_n)$: n∈N_0}∈f(I)]. Therefore is conntably qI-f(I) compact (resp. countably f(I)-compact)