Generalization of Regular and Normal spaces

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***Abstract*– In this paper, we define (i,j)-semi-H-regular and (i, j)-normal spaces and obtain some characterizations and basic properties of such a spaces.**

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I. INTRODUCTION

Connectedness and compactness are powerful tools in topology but they have many dissimilar properties. The concept of Hausdorff spaces is almost an integral part of compactness. Investigations into the properties of cut points of topological spaces which are connected, compact and Hausdorff date back to the 1920s. Connectedness together with compactness with the assumption of Hausdorff has been studied from the view point of cut points.The concept of ideals in topological spaces has been introduced and studied by Kuratowski [1] and Vaidyanathaswamy [2]. A hereditary class H on a topological space (X,τ) is a nonempty collection of subsets of X which satisfies (i) A ∈ H and B ⊂ A implies B ∈ H. Given a topological space (X, τ) with a hereditary class H on X and if P(X) is the set of all subsets of X, a set operator (.)* : P(X) → P(X), called the local function [2] of A with respect to τ and H, is defined as follows: for A ⊂ X, A*(τ,H) = {x ∈ X|U ∩ A ∉ H for every U ∈ τ(x)}, where τ(x) = {U ∈ τ|x ∈ U}. A Kuratowski closure operator Cl*(·) for a topology τ*(τ,H) called the ∗-topology, finer than τ is defined by Cl*(A) = A ∪ A*(τ,H) when there is no chance of confusion, A*(H) is denoted by A*. If H is a hereditary class on X, then (X,τ_1,τ_2,H) is called a hereditary bitopological space. The purpose of this paper is to define upper (lower) (i,j)-semi-H-irresolute multifunction and to obtain several characterizations of such a multifunction.

II. PRELIMINARIES

Throughout this paper, (X,τ_1,τ_2, H) and (Y,σ_1,σ_2,K) always mean hereditary bitopological spaces in which no separation axioms are assumed unless explicitly stated. For a subset A of (X,τ), Cl(A) and Int(A) denote the closure of A with respect to τ and the interior of A with respect to τ, respectively. A subset S of a hereditary bitopological space (X,τ_1,τ_2,I) is (i,j)-semi-H-open S ⊂ Clj*(Inti(S))). The complement of an (i,j)-semi-H-closed set is said to be an (i,j)-semi-H-open set. The (i,j)-semi-H-closure and the (i,j)-semi-H-interior, that can be defined in the same way as Cl(A) and Int(A), respectively, will be denoted by (i,j)-sHCl(A) and (i,j)-sHInt(A), respectively. The family of all (i,j)-semi-H-open (resp.(i,j)-semi-H-closed) sets of (X,τ_1,τ_2,H) is denoted by (i,j)-SHO(X) (resp. (i,j)-SHC(X)). The family of all (i,j)-semi-H-open (resp. (i,j)-semi-H-closed) sets of (X,τ_1,τ_2,H) containing a point x ∈ X is denoted by (i,j)-SHO(X,x) (resp. (i,j)-SHC(X,x)). A subset U of X is called an (i,j)-semi-H-neighborhood of a point x ∈ X if there exists V ∈ (i,j)-SHO(X,x) such that V ⊂ U.

III. PROPERTIES OF (i,j)-SEMI-H-REGULAR SPACES

Definition 3.1: A space (X,τ_1,τ_2, H) is (i,j)-semi-H-regular if for each closed set F⊆ X and each x∈X\F there are disjoint H,W ∈(i,j)-SHO(X) such that F⊆ H and x∈ W

Definition 3.2: In a space (X,τ_1,τ_2, H), the (i,j)-semi-H-closure of any A⊆ X is the smallest (i,j)-semi-H-closed set containing A and denoted by (i, j)-sHcl(A).

Proposition 3.3: For (X,τ_1,τ_2, H), and A⊆ X , x∈ (i, j)-sHcl(A) if and only if A∩W≠ ∅ for each W∈∈(i,j)-SH O(X,x)

Theorem 3.4: For any space (X,τ_1,τ_2, H), the following are equivalent :

(a) (X,τ_1,τ_2, H) is (i,j)-semi-H-regular.

(b) Each member of τ(x) contains the (i,j)-semi-H-closure of member of ∊(i,j)-SHO(X,x)

(c) For each A⊆ X and each U∊ τ suah that A∩U= ∅ there is W ∊ ∊(i,j)-SHO(X,x) having A∩ W = ∅ and (i, j)sHcl(W)⊆ U

(d) For any ∅ ≠ A⊆ X and each closed set F of X with A∩ F = ∅ , then there are disjoint H,W ∊ ∊(i,j)-SHO(X,x) such that A∩H ≠ ∅ and F⊆ W.

Proof: (a)⇒(b) Let U ∊ τ(x) ,then X\U is closed not containing x, so by (a) there are disjoint H,W ∊ ∊(i,j)-SHO(X,x) such thatx∊ H and X\U ⊆ W. Hence x∊ H ⊆ (i, j)-sHCl(H) ⊆ U.

(b)⇒(c) Assume A∊ P(X) having A∩U ≠ ∅ for some U∊ τ, so letting x∊A∩U. By (b), there is W∊ ∊(i,j)-SHO(X,x) such that x∊ W⊆ (i, j)-sHcl(W)⊆ U. Also one can deduce that A∩W ≠ ∅.

(c)⇒(d) Consider A∩F =∅ for a closed set F and for any ∅ ≠ A ⊆ X . This means that X\F ∊ τ

having A ∩ (X\F) ≠ ∅. By (c), there is H ∊ ∊(i,j)-SHO(X, τ) such that A∩ H ≠ ∅ and (i,j)-SHCl(H)⊆ X\F. Putting W∊X\(i,j)-SHCl(H), then F⊆ W∊(i, j)-SH(X, τ).

(d)⇒(a) It follows by taking A = {x}.

Definition 3.5: A space (X,τ_1,τ_2, H) is called (i,j)-semi-H-T_3 if it is (i,j)-semi-H-regular and T_1 space.

Theorem 3.6: Each (i,j)-semi-H-T_3 space is (i,j)-semi-H-Hausdroff

Proof: Let (X,τ_1,τ_2, H) be (i,j)-semi-H-T_3 and distinct x_1 ,x_2 ∊ X. Then each {x_i } i=1,2 is closed and x_j ∊ X\{x_i} for i,j = 1,2 and i,j = 1,2 and i≠j. By (i,j)-semi-H-regularity of (X,τ_1,τ_2, H) there exist disjoint (i,j)-semi-H-open sets H,W such that x_i∊ {x_i} ⊆ H and x_j ∊ W where i,j ∊{1,2} and i≠j. This shows that (X,τ_1,τ_2, H) is (i, j)-semi-H-Hausdroff.

Definition 3.7: A space (X,τ_1,τ_2, H) is (i, j)-semi-H-normal if for each disjoint closed sets F_1, F_2 of X, there exist disjoint (i, j)-semi-H-open sets W_1 , W_2 such that F_i⊆W_i i=1,2.

Theorem 3.8: For a space (X,τ_1,τ_2, H), the following statements are equivalent

(a) (X,τ_1,τ_2, H) is (i, j)-semi-H-normal
(b) For each disjoint closed sets F_1,F_2, there exists H∊(i, j)-SHO(X,x) such that F_1⊆ H and (i,j)-sHcl(H) is disjoint of F_2.
(c) For any closed set F⊆ X and any U∊ τ containing F, there is H ∊ (i, j)-SHO(X) such that F⊆H⊆(i, j)sHcl(H) ⊆U.

Proof : (a)⟶(b): Let F_1,F_2 be nonempty disjoint closed in an normal space (X,τ_1,τ_2, H). Then there are H,W ∊ (i, j)-SHO(X) such that F_1⊆ H ,F_2⊆ W and H∩W = ∅. Thus X\W⊆ X\$F_2$, this implies (i, j)-sHcl(X\W) = X\W ⊆ X\$F_2$ but H and its (i,j)-semi-H-closure are in X\W. Therefore, (i, j)-semi-H-cl(H) ⊆X\$F_2$ hence the result.

(b)⟶(c) Assume F is closed and U∊τ_i such that F⊆ U, then X\U is closed and disjoint of F. By (b), there is H ∊ (i, j)-SHO(X), having F⊆ H and (i, j)-sHcl(H) ∩(X\U) = ∅ .This gives (i,j)-sHcl(H) ⊆ U and so F⊆ H ⊆ (i,j)-sHcl(H) ⊆ U.

(c)⟶(a) Consider any two nonempty closed disjoint sets F_1,F_2 of X, then $F_1 \subseteq X \setminus F_2 \in \tau$. Applying (c), there exists H∈ (i, j)-SHO(X) such that F_1⊆H⊆(i,j)-sHcl(H)⊆X\$F_2$ .Therefore F_2 ⊆X\(i,j)-sHcl(H) ∈ (i, j)-SHO(X) and H is disjoint of X\(i,j)-sHcl(H).

Theorem 3.9: If (X,τ_1,τ_2, H) is (i,j)-semi-H-T_3, then it is (i,j)-semi-H-normal.

Proof : Let F_1 ,F_2 be disjoint closed sets, and for every x ∈ F_1⊆X\$F_2$∈ τ,there exists H_x ∈ (i, j)-SHO(X), since ∪ H_x ∈(i, j)-SHO(X) and we get F_1 =∪{x} ⊆∪H_x⊆∪((i,j)-sHcl(H_x))⊆ (i,j)-sHcl(∪H_x)⊆ X\$F_2$. Hence (X,$\tau_1$,$\tau_2$, H) is (i, j)-semi-H-normal.

Definition 3.10: An (i,j)-semi-H-normal space which is T_1 is (i,j)-semi-H-T_4.

Theorem 3.11: Each (i,j)-semi-H-T_4 is (i,j)-semi-H-T_3.

Proof : Let (X,τ_1,τ_2, H) be (i,j)-semi-H-T_4. F⊆X be closed and disjoint of F. (i,j)-semi-H-normality of (X,τ_1,τ_2, H) gives disjoint H,W∈(i, j)-SHO(X) having x∈{x}⊆ H and F⊆W.

Definition 3.12: A space (X,τ_1,τ_2, H) is completely (i,j)-semi-H-normal if for any two separated subsets A,B of X, there are disjoint H,W ∈ (i, j)-SHO(X) such that A⊆H and B⊆W, while a completely (i,j)-semi-H-normal space, which is a T_1- space is an (i, j)-semi-H-T_5 space.

Theorem 3.13: Every completely (i, j)-semi-H-normal space is (i, j)-semi-H-normal.

Proof: This is obviously by the fact that each pair of closed disjoint sets is separated.

Proposition 3.14: If H and J are hereditary classes on X having H ⊆ J .Then (X,τ_1,τ_2, H) is (i,j)-semi-H-regular ((i,j)-semi-H-normal) if (X,τ_1,τ_2, J) is (i,j)-semi-J-regular ((i,j)-semi-J-normal).

Proof: It follows by the fact that (i, j)-SHO(X)⊆ (i, j)-SJO(X) when H⊆J.

Lemma 3.15: In (X, τ), if U∈ τ then U∩A^*(H) ⊆$(U\cap A)^*$(H) for any A⊆ X.

Theorem 3.16: Every open subspace of (i, j)-semi-H-regular (resp (i,j)-semi-H-normal, completely (i,j)-semi-H-normal) is (i, j)-semi-H-regular (resp. (i,j)-semi-H-normal, completely (i,j)-semi-H-normal).

Proof : Let (X,τ_1,τ_2, H) be (i,j)-semi-H-regular and y∈ τ. Let K⊆Y be closed and y∈Y\K. This shows that, there exists closed F⊆X with K= Y∩F. Then (i, j)-semi-H-regularity of (X,τ_1,τ_2, H) means that there are disjoint H,W ∈(i, j)-SHO(X) having F⊆X and y ∈ W. Above proposition illustrates that Y∩H, Y∩W are (i,j)-semi-H-open sets which are containing A and y, respectively.

Corollary 3.17: Any open subspace of (i, j)-semi-H-T_i space i= 3,4,5 is (i, j)-semi-H-T_i i=3,4,5.

REFERENCES

[1] K. Kuratowski, Topology, Academic presemiss, New York, 1966.
[2] R. Vaidyanathaswamy, The localisation theory in set topology, Proc. Indian Acad. Sci., 20(1945), 51-61