A New Dimension of Graph Techniques in Health Care Domain Application

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Abstract

This paper, presents a relation on graphs that induces new dimension of topological structures to the graph and then discusses some of the properties of graph. Also, we have investigated an algorithm to generate the topological structures from different graphs. Finally, some applications in medicine and geographs are discussed and we have verified our results in the real life.

Index Terms : Graph theory,Topological

1.Introduction

Graph theory[1-4] has recently emerged as a subject in its own right as well as an important mathematical tool in such diverse areas such as health care, PERT, sociology, genetics etc. A graph G is a pair (V, E), where V is nonempty set called vertices or nodes and E is 2-element subsets of V called edges or links. The number of vertices in a graph G is the order of G, and the number of edges is the size of G. An edge joining a vertex to itself is called a loop. Two or more edges that join the same pair of distinct vertices are called parallel edges. Let $G = (V(G), E(G))$ be a graph; we call H a subgraph of G if $V(H) \subset V(G)$ and $E(H) \subset E(G)$, in which case we write $H \subset G$. The eccentricity $e(v)$ of a vertex v in a connected graph G is the distance between v and a vertex farthest from v in G, while the radius rad(G) is the smallest eccentricity among the vertices of G. The notions of closure operator is very useful tool in several sections of mathematics, as an example, in algebra,[5,6] topology,[7,8] and computer science theory,[9] the connection between graph theory and different subjects, as in structural analysis,[10] medicine[11] and physics.[12] Topology is the science that deals with the properties of things that does not depend on the dimension, which means that it allows increases and decreases, but without cutting on things. If X is a nonempty set, a collection τ of subsets of X is said to be a topology on X, and if the following condition holds X and ϕ belongs to τ, the finite intersection of any 2 sets in τ belongs to τ and the union of any number of sets in τ belongs to τ.[13] The term topology is also used to refer to a structure imposed upon a set X, a structure that essentially "characterizes" the set X as a topological space by taking proper care of properties such as convergence, connectedness, and continuity, upon transformation. Every element in topology is called an open set, its complement is a closed set. The closure of a subset A (briefly, Cl(A)) is the smallest closed set that contains A . The interior of a subset B (briefly, int(B)) is the greatest open set that is contained in B. The main contribution of the work is that we provide a new definition of a relation to extract a topology from any graph and study some properties. Throughout the paper, we start with the application of abstract topological graph theory. Some ideas in terms of concepts in topological graph theory, which is a branch of mathematics, and in many real-life applications will be investigated. We give an algorithm to generate some topological structural in graphs. Each topological structure on graphs is a topological space. Some properties on closure and interior operators for topological graphs will be studied. Finally, we apply both of a graph and a topology on some of the medical application such as the blood circulation in the human body and geographical application such as a street system of a community.

Definition 1.1. [1] If uv is an edge of G, then u and v are adjacent vertices. Two adjacent vertices are referred to as neighbors (ie, $N(v_i)$) of each other. The number of vertices in a graph G is the order (degree) of G, and the number of edges is the size of G. The degree of a vertex v in a graph G is the number of vertices in G that are adjacent to v, denoted by $\deg_G(v)$.

Definition 1.2. [1] A multigraph is a nonempty set of vertices, every 2 of which are joined by a finite number of edges. Structures that permit both parallel edges and loops are called pseudographs. A graph is simple if it has no loops or parallel edges. A simple graph is called complete graph, if any 2 distinct vertices are joined by an edge.

Definition 1.3. [1] If the vertex set of a graph G can be split into 2 disjoint sets A and B, so that each edge of a graph G joins a vertex of A and a vertex of B, thena graph G is a bipartite graph. A complete bipartite graph is a bipartite graph in which each vertex in A is joined to each vertex in B by just one edge.

Definition 1.4. [1] A spanning subgraph of a graph G is a subgraph obtained by edge deletions only. An induced subgraph of a graph G is a subgraph obtained by vertex deletions together with the incident edges.

Definition 1.5. [2] The path P is called topological open subgraph if the subgraph not contained its end point. The path P is called topological closed subgraph if the subgraph contained its initial and its end points.

Proposition 1.6. [2] *If $G = (V, E)$ is a connected graph and $(V(G), \tau)$ is a topology induced by $\beta_i=\{V(G), \phi, \{v_i\}, \{N(v_i)\}\}$*
as a basis and if P_1 and P_2 are open paths, then

1. $V(P_1) \subseteq Cl(V(P_1))$.
2. *If* $P_1 \subseteq P_2$, *then* $Cl(V(P_1)) \subseteq Cl(V(P_2))$.

2.RELATIONS ON GRAPHS

Let x be a vertex in a graph G with m loops and n multiple edges, then $(\deg_G(x))_x = (2m_x + n_x)_x$. In simple graphs, we represent the vertex x only in the form $(\deg_G(x))_x$.

Definition 2.1. A relation R on a graph G is defined as $R = \{((2m_x + n_x)_x, (2m_y + n_y)_y) : x, y \in V\}$, where m_x and m_y are the number of loops of vertices x and y, respectively, and n_x and n_y are the number of multiple edges of vertices x and y, respectively. If G is a simple graph, R on G takes the form $R=\{((\deg_G(x))_x, (\deg_G(y))_y) : x, y \in V\}$.

From Definition 2.1, if $m = 0$, ie, there is no loop in G, then the relation in G can be written as $R = \{((n_x)_x, (n_y)_y) : x, y \in V\}$, where n_x and n_y are the number of multiple edges of vertices x and y, respectively. If $n = 1$ and $m = 0$, ie, there is no multiple edges and no loops in G, then the relation in G can be written as $R = \{(1_x, 1_y) : x, y \in V\}$. If G is directed and simple, then $R = \{(1_x, 1_y) = (x, y) : x, y \in V\}$, and in case G is undirected, then the relation can be written as $R = \{(1_x, 1_y) = (x, y) \text{ or } (y, x) : x, y \in V\}$.

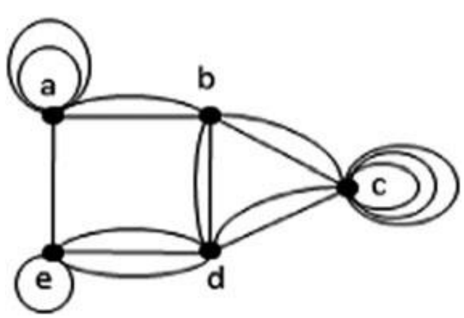

FIGURE 1 A pesudograph G

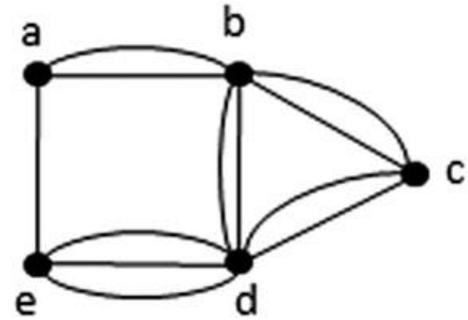

FIGURE 2 A multigraph graph *G*

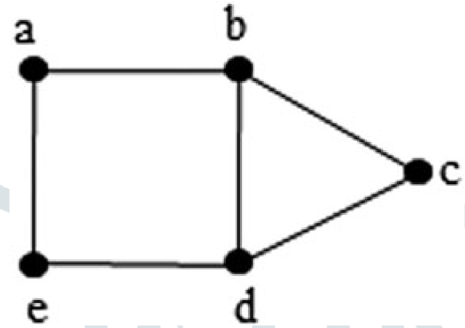

FIGURE 3 A simple graph *G*

Example 2.2. Let *G* be an undirected graph as shown in Figure 1.
The relation *R* on *G* has the form $R = \{(7_a, 6_b), (7_a, 5_e), (6_b, 10_c), (10_c, 6_d), (6_b, 6_d), (6_d, 5_e), (7_a, 7_a), (5_e, 5_e), (10_c, 10_c)\}$.

Example 2.3. Let *G* be a graph that has multiple edges and no loops, as shown in Figure 2. The relation *R* on *G* has the form $R = \{(3_a, 6_b), (3_a, 4_e), (6_b, 4_c), (4_c, 7_d), (6_b, 7_d), (7_d, 4_e)\}$.

Example 2.4. Let *G* be simple graph that has no loops and no multiple edges, as shown in Figure 3. The relation *R* on *G* has the form $R = \{(a, b), (a, e), (b, c), (c, d), (d, b), (d, e)\}$.

It is clear that the relation *R* in Example 2.4 is a special case from the one given in Example 2.2 and also in Example 2.3 and the relation *R* in Examples 2.3 and 2.4 represents a special case of Example 2.2.

3.SOME APPLICATIONS

In this section, we give an algorithm to evaluate the topological structure from a graph based on the topological notions of subbase and base. We represent a graph by its adjacency matrix in our algorithm.

Now, we give 2 different examples. The first one shows the blood circulation of a human body and the second shows the street system of a community.

Example 4.1. In this example, we will apply all the above on our medical application. We conclude that the graph must be connected to modifying the medical state. Figure 4 shows a graph *G*; we can classify the heart into a set of vertices and set of edges. So it is easy to generate the topology τ_G on it. The post classes of the vertices are the following:

$$v_1R = \{v_3\},\ v_2R = \{v_3\},\ v_3R = \{v_4\},\ v_4R = \{v_5\},\ v_5R = \{v_6, v_7\},\ v_6R = \{v_8\},$$
$$v_7R = \{v_8\},\ v_8R = \{v_9\}, v_9R = \{v_{10}\}.$$

The subbase has a form

$$S_G = \{\{v_3\}, \{v_4\}, \{v_5\}, \{v_6, v_7\}, \{v_8\}, \{v_9\}, \{v_{10}\}\}.$$

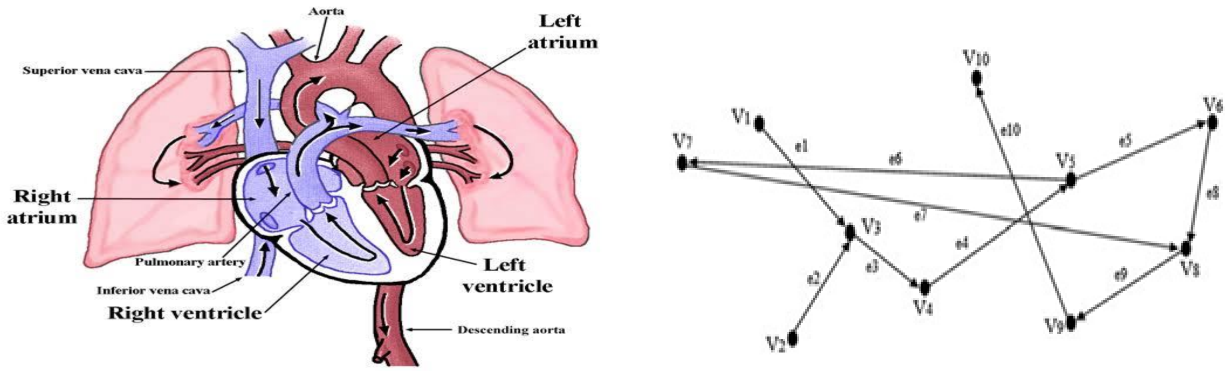

FIGURE 4 The blood circulation of human body[2] [Colour figure can be viewed at wileyonlinelibrary.com]

Algorithm.

Input: A number of vertices of a graph G, Adjacency matrix of V(G).

Output: A topology of G (τ_G).

1.Insert a number of vertices of *G*.

2.**for** $i \in n$

Enter the name of vertices of a graph *G*.

end

3.**for** $i \in n$

for $j \in n$

Enter the of adjacency matrix of $V(G)$.

end

end

4.**for** $i \in n$

for $j \in n$

Calculate a degree of $V(G)$.

end

end

5.**for** $i \in n$

for $j \in n$

if(degree != 0)

class(i,j)=x(j).

R=$(degree(x(i))_{x(i)}, degree(x(j))_{x(j)})$.

end

end

end

6.**for** $i \in n$

```
            for j ∈ n
                if(class(i,j) != 0)
                  subbase=class(j).
                end
        end

            end
    7.for v_i ∈ n
        for v_j ∈ n
            base=subbase + intersection(v_i, v_j).
        end

            end
    8.for v_i ∈ n
        for v_j ∈ n
            if(union(v_i, v_j) != (v_i||v_j))
              v_i, v_j ∈ union.
              union(v_i, v_j) ∈ union.
            end

                topology = base + union(v_i, v_j).
              end

            end
```

The base has a form

$$\beta_G = \{X, \phi, \{v_3\}, \{v_4\}, \{v_5\}, \{v_6, v_7\}, \{v_8\}, \{v_9\}, \{v_{10}\}\}.$$

And the topology in a graph has a form

$$\tau_G = \{X, \phi, \{v_3\}, \{v_4\}, \{v_5\}, \{v_6, v_7\}, \{v_8\}, \{v_9\}, \{v_{10}\}, \{v_3, v_4\}, \{v_3, v_5\}, \{v_3, v_6, v_7\}, \{v_3,v_8\},\{v_3,v_9\},\{v_3,v_{10}\},\{v_4,v_5\},\{v_4,v_6,v_7\},\{v_4,v_8\},\{v_4,v_9\},\{v_4,v_{10}\}, \{v_5,v_6,v_7\},\{v_5,v_9\},\{v_5,v_{10}\},\{v_6,v_7,v_8\}\{v_6,v_7,v_9\},\{v_6,v_7,v_{10}\},\{v_8,v_9\}, \{v_8, v_{10}\}, \{v_9, v_{10}\}, \{v_3, v_4, v_5\}, \{v_3, v_4, v_6, v_7\}, \{v_3, v_4, v_8\}, \{v_3, v_4, v_9\}, \{v_3, v_4, v_{10}\}, \{v_3, v_4, v_5, v_6, v_7\}, \{v_3, v_4, v_5, v_8\}, \{v_3, v_4, v_5, v_9\}, \{v_3, v_4, v_5, v_{10}\}, \{v_3, v_4, v_5, v_6, v_7, v_8\}, \{v_3, v_4, v_5, v_6, v_7, v_9\}, \{v_3, v_4, v_5, v_6, v_7, v_{10}\}, \{v_3, v_4, v_5, v_6, v_7, v_8, v_9\}, \{v_3, v_4, v_5, v_6, v_7, v_8, v_{10}\}, \{v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}\}, \{v_4, v_5, v_6, v_7\}, \{v_4, v_5, v_8\}, \{v_4, v_5, v_9\}, \{v_4, v_5, v_{10}\}, \{v_4, v_5, v_6, v_7, v_8\}, \{v_4, v_5, v_6, v_7, v_9\}, \{v_4, v_5, v_6, v_7, v_{10}\}, \{v_4, v_5, v_6, v_7, v_8, v_9\}\{v_4, v_5, v_6, v_7, v_8, v_{10}\}, \{v_4, v_5, v_6, v_7, v_8, v_9, v_{10}\}, \{v_5, v_6, v_7, v_8\}, \{v_5, v_6, v_7, v_9\}, \{v_5, v_6, v_7, v_{10}\}, \{v_5, v_6, v_7, v_8, v_9\}, \{v_5, v_6, v_7, v_8, v_{10}\}, \{v_5, v_6, v_7, v_8, v_9, v_{10}\}, \{v_6, v_7, v_8, v_9\}, \{v_6, v_7, v_8, v_{10}\}, \{v_6, v_7, v_8, v_9, v_{10}\}, \{v_8, v_9, v_{10}\}\}.$$

Firstly, we can get the closure of the graph. If we assign a subgraph $H = \{v_1, v_2, e_1, e_2, e_3\}$, which $V(H) = \{v_1, v_2\}$. We can conclude from the definition of the closure that the resultant closure of the subgraph H is $\text{cl}(V(H)) = \{v_1, v_2, v_3\}$. Medically, when we apply this example in the heart will find is true. Because the blood flow in a heart in a directed path that means that the blood must be pass through each successive point until it completes its cycle. But, if the blood stops, it will cause many problems as heart failure that occurs if the heart cannot pump enough blood to the lungs to pick up oxygen. Left-side heart failure occurs if the heart cannot pump enough oxygen-rich blood to the rest of the body.

Secondly, also we can get the interior of the graph by assigning subgraph $H = \{v_4, e_4, v_5,$

$e_6, v_6\}$. We can conclude from the definition of the interior that the resultant interior of the subgraph H is the $\text{int}(V(H)) = \{v_4, v_5\}$. In this case,
we notice that the end point does not exist. And this is a contradiction for our medical application. But there is one part only we can apply this example to it. This is the lung. Medically, some people have medical problems in lungs like cancer, pulmonary edema, and tuberculosis. So these people have to remove one of their lungs surgically. This surgical operation called a pneumonectomy obviously major lung surgery. Those who do not have heart/respiratory problems are candidates. It is a gift from God to us that we can live with only one lung.

4. CONCLUSIONS

We can find out the topological structure from any graph by using a relation defined above and from this relation, we understand the type of a graphs that are used. We studied the connectedness in both general topology and topological graph and the relation between them. Also, from topological properties, we can deduce the solution of some problems in healthcare and in geography.

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