# INFLUENCE OF HALL CURRENT AND HEAT SOURCE ON UNSTEADY FREE CONVECTIVE FLOW OF CHEMICALLY REACTING FLUID

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***Abstract:*** The objective of the present study is to investigate the hall current, radiation absorption, heat source and chemical reaction effects on unsteady MHD viscous, incompressible and electrically conducting fluid flow over a vertical porous plate. The governing equations for the flow are solved analytically and the effects of various parameters like chemical reaction parameter, thermal Grashof number, mass Grashof number, hall current parameter, magnetic field parameter, radiation absorption parameter, heat source parameter are studied for the velocity, temperature and concentration fields are analyzed through the graphs. Also skin-friction coefficient, Nusselt number and Sherwood number are discussed with tables.

**Keywords:** Unsteady, MHD, Chemical reaction, hall current, heat source.

## I. INTRODUCTION

In many engineering applications, natural convective flows play an important role and, hence, these have attracted the attention of many research workers. The phenomenon of mass transfer is very common in the theories of stellar structure and observable effects are easily detectable at least on the solar surface. On the other hand, the results of the effects of magnetic field on the flow of an electrically-conducting viscous fluid in the presence of mass transfer are also useful in a stellar atmosphere. Free convective flows with and without heat transfer in electrically conducting fluids have attracted substantial interest in the context of metallurgical fluid dynamics, re-entry aerothermodynamics, astronautics, geophysics, nuclear engineering, and applied mathematics. An early study was presented by Carrier and Greenspan [1] who considered unsteady hydro magnetic flows past a semi-infinite flat plate moving impulsively in its own plane. Gupta [2] considered unsteady magneto-convection under buoyancy forces. Singer [3] further assessed the unsteady free convection heat transfer with magneto hydrodynamic effects in a channel regime. Pop [4] reported on transient buoyancy-driven convective hydro magnetic from a vertical surface. Yu and Yang [5] investigated the influence of channel wall conductance on hydro magnetic convection. Rao [6] analyzed the unsteady magneto hydrodynamic convection heat transfer past an infinite plane. Further excellent studies of unsteady free convection magneto hydrodynamic flows were reported by Antimirov and Kolyshkin [7] for a vertical pipe and Raja ram and Yu for a parallel plate channel [8]. Tokis [9] used Laplace transforms to analyze the three dimensional free-convection hydro magnetic flows of an infinite vertical plate moving in a rotating fluid when the plate temperature undergoes a thermal transient. The influence of oscillatory pressure gradient on transient rotating hydro magnetic flow was considered by Ghosh [10]. Other transient MHD studies include the papers by Sacheti et al. [11], Attia [12] who included viscosity variation effects, Al-Nimr and Alkam [13] who considered open-ended vertical annuli and Takhar et al. [14] who employed a numerical method to study flat-plate magneto hydrodynamic unsteady convection flow. Eswara et al. [15] examined the transient laminar magneto hydrodynamic convection in a cone due to a point sink, with the free stream velocity varying continuously with time and also for the case of an impulsive change either in the strength of the point sink or in the wall temperature. They showed numerically that magnetic field increase the skin friction but decreases heat transfer and that the transient nature of the convection flow is active for short durations with suction present and greater times with injection. Chamkha [16] has analysed the unsteady MHD free three-dimensional convection over an inclined permeable surface with heat generation/absorption. Jha [17] presented exact solutions for transient free convection MHD Couette channel flow with impulsive motion of one of the plates discussed. More recent communications on unsteady hydro magnetic heat transfer flows include the articles by Seddeek [18] incorporating variable viscosity effects, Zakaria [19] who considered a polar fluid, and Ghosh and Pop [20] who included Hall currents. Beg et al. [21] studied the free convective MHD flow from a spinning sphere with impulsive motion using the Blottner difference method. Duwairi et al. [22] analysed the unsteady MHD natural convection for the non-Boussinesq case that is, using a nonlinear density relationship for water at low temperatures. Bozkaya and Tezer-sezgin [23] have presented boundary element numerical solutions for transient magneto hydrodynamic flow in a rectangular duct with insulating walls, showing that *n* increase in Hartmann number causes the formation of boundary layers for both the velocity and the induced magnetic field with the velocity becoming uniform at duct center, with increasing magnetic field, the time for reaching steady-state solution is also reduced. Venkateswarlu et al. [24] investigated the heat and mass transfer characteristics with slip on MHD flow in a channel filled with porous medium. Dharmaiah et al. [25] analysed radiation effect on transient MHD convection stream of fluid past a vertical surface with variable temperature.

## II. FORMULATION OF THE PROBLEM

Consider the unsteady two-dimensional MHD free convective flow of a viscous incompressible, electrically conducting fluid past a vertical plate embedded in a porous medium in presence of hall current and heat absorption effects. Let the $x-$axis be taken in vertically upward direction along the plate and $y-$axis is normal to the plate. The following assumptions are made:

- A uniform magnetic field is applied in the direction perpendicular to the plate.
- The viscous dissipation and the Joule heating effects are assumed to be negligible in the energy equation.
- The transverse applied magnetic field and magnetic Reynolds number are assumed to be very small, so that the induced magnetic field is negligible.
- The concentration of the diffusing species in the binary mixture is assumed to be very small in comparison with the other chemical species, which are present, and hence the Soret and Dufour effects are negligible.

- The suction velocity $v' = -U_0$ is considered normal to the plate.
- The plate temperature is constant.
- It is assumed that there exist a homogeneous chemical reaction of first order with constant rate between the diffusing species and the fluid.
- All the fluid properties considered constant except that the influence of the density variation with temperature is considered.

Under the above assumptions as well as Boussinesq's approximation, the equations of conservation of mass, momentum, energy and concentration governing the free convection boundary layer flow over a vertical porous plate in porous medium can be expressed as:

$$\frac{\partial v'}{\partial y'} = 0 \tag{1}$$

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = \nu\frac{\partial^2 u'}{y'^2} + g\beta(T' - T'_\infty) + g\beta'(C' - C'_\infty) - \frac{\nu}{K'}u' - \frac{\sigma B_0^2}{\rho(1+m^2)} \tag{2}$$

$$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = \alpha\frac{\partial^2 T'}{\partial y'^2} - \frac{R}{\rho Cp}\left[C' - C'_\infty\right] - \frac{Q_0}{\rho Cp}\left[T' - T'_\infty\right] \tag{3}$$

$$\frac{\partial C'}{\partial t'} + v\frac{\partial C'}{\partial y'} = D\frac{\partial^2 C'}{\partial y'^2} + k\left[C' - C'_\infty\right] \tag{4}$$

where $u', v'$ are the velocity components in $x', y'$ directions respectively, $t' -$ the time, $p' -$ the pressure, $\rho -$ the fluid density, $g -$ the acceleration due to gravity, $\beta$ and $\beta' -$ the thermal and concentration expansion coefficients respectively, $K' -$ the permeability of the porous medium, $T' -$ the temperature of the fluid in the boundary layer, $\nu -$ the kinematic viscosity, $\sigma -$ the electrical conductivity of the fluid, $T'_\infty -$ the temperature of the fluid far away from the plate, $C' -$ the species concentration in the boundary layer, $C'_\infty -$ the species concentration in the fluid far away from the plate, $B_0 -$ the magnetic induction, $\alpha -$ the fluid thermal diffusivity, $Cp -$ specific heat at constant, $D -$ the coefficient of chemical molecular diffusivity, $K' -$ the chemical reaction.

The boundary conditions for the velocity, temperature, and concentration fields are given as follows:

$$u' = 0, T' = T'_w, C' = C'_w \quad at \quad y' = 0 \tag{5}$$

$$u' \to 0, T' \to T'_\infty, C' \to C'_\infty \ as \ y' \to \infty \tag{6}$$

In order to write the governing equations and the boundary conditions in dimensionless form, the following non-dimensional quantities are introduced.

$$U = \frac{u'}{U_0}, t = \frac{t'U_0^2}{\nu}, y = \frac{y'U_0^2}{\nu}, T = \frac{T' - T'_\infty}{T'_W - T'_\infty}, C = \frac{C' - C'_\infty}{C'_W - C'_\infty}, K = \frac{K'U_0^2}{\nu^2},$$
$$Gr = \frac{\nu g\beta(T'_w - T'_\infty)}{U_0^3}, Gc = \frac{\nu g\beta'(C'_w - C'_\infty)}{U_0^3}, Kr = \frac{k\nu}{U_0^2}, M = \frac{\sigma\nu B_0^2(1+m^2)}{\rho U_0^2}, \tag{7}$$
$$\Pr = \frac{\nu}{\alpha}, Sc = \frac{\nu}{D}, R = \frac{\nu R'(C'_w - C'_\infty)}{\rho Cp U_0^2(T'_w - T'_\infty)}, N = \frac{\beta'(C'_w - C'_\infty)}{\beta(T'_w - T'_\infty)}, Q = \frac{Q_0\nu}{\rho Cp U_0^2}$$

In view of Equations (7), Equations (2), (3) and (4) can be reduced to the following dimensionless form.

$$\frac{\partial U}{\partial t} - \frac{\partial U}{\partial y} = GrT + GcC + \frac{\partial^2 U}{\partial y^2} - \left(M + \frac{1}{K}\right)U \tag{8}$$

$$\frac{\partial T}{\partial t} - \frac{\partial T}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 T}{\partial y^2} + RC - QT \tag{9}$$

$$\frac{\partial C}{\partial t} - \frac{\partial C}{\partial y} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} + KrC \tag{10}$$

The corresponding dimensionless boundary conditions are

$$U = 0, T = 1, C = 1 \quad at \quad y = 0 \tag{11}$$

$$U \to 0, \quad T \to 0, \quad C \to 0 \quad at \quad y \to \infty \tag{12}$$

## III. SOLUTION OF THE PROBLEM

In order to reduce the above system of partial differential equation to a system of ordinary differential equations, the velocity, temperature and concentration are taken as

$$U(y,t) = u_0(y)e^{-nt} \tag{13}$$

$$T(y,t)=T_0(y)e^{-nt} \tag{14}$$

$$C(y,t)=C_0(y)e^{-nt} \tag{15}$$

Substituting equations (13)-(15) in the equations (8)-(10) we get

$$u_0'' + u_0' + (n-M_1)u_0 + GrT_0 + GcC_0 = 0 \tag{16}$$

$$T_0'' + \Pr T_0' + \Pr(n-Q)T_0 + \Pr RC_0 = 0 \tag{17}$$

$$C_0'' + ScC_0' + Sc\left[Kr+n\right]C_0 = 0 \tag{18}$$

Where $M_1 = M + \dfrac{1}{K}$

.The corresponding boundary conditions are

$$u_0 = 0, T_0 = 1, C_0 = 1 \quad at \quad y = 0 \tag{19}$$

$$u_0 \to 0, T_0 \to 0, C_0 \to 0 \quad as \quad y \to \infty \tag{20}$$

Solving the above equations (16)-(18) with (19)-(20), the obtained solutions of velocity, temperature, concentration, skin-friction, Nusselt number and Sherwood number are

$$U(y,t) = u_0(y)e^{-nt} = \left[B_5 e^{-m_3 y} - B_3 e^{-m_2 y} - B_4 e^{-m_1 y}\right] \tag{21}$$

$$\begin{aligned} T(y,t) &= T_0(y)e^{-nt} \\ &= \left[B_2 e^{-m_2 y} - B_2 e^{-m_1 y}\right]e^{-nt} \end{aligned} \tag{22}$$

$$\begin{aligned} C(y,t) &= C_0(y)e^{-nt} \\ &= e^{-m_1 y}e^{-nt} \end{aligned} \tag{23}$$

**Skin Friction Coefficient**:

$$C_f = \left(\frac{\partial U}{\partial y}\right)_{y=0} = \left(m_2 B_3 - m_3 B_5 - m_1 B_4\right)e^{-nt} \tag{24}$$

**Nusselt Number:**

$$Nu = -\left(\frac{\partial T}{\partial y}\right)_{y=0} = \left(m_2 B_2 - m_1 B_1\right)e^{-nt} \tag{25}$$

**Sherwood Number:**

$$Sh = -\left(\frac{\partial C}{\partial y}\right)_{y=0} = m_1 e^{-nt} \tag{26}$$

## IV. RESULTS AND DISCUSSION

Numerical evaluation of the analytical results reported in the previous section was performed and a representative of results is reported graphically in Figs. 1 – 8. The influence of different pertinent parameters such as Grashof number (Gr), modified Grashof number (Gc), heat source parameter (Q), chemical reaction parameter (Kr), radiation absorption parameter (R), Prandtl number (Pr), permeability parameter (k) and Hall Current Parameter (m) on velocity distribution have been studied and analyzed with the help of graphs. Also the behavior of rate of heat transfer and rate of mass transfer with respect to various parameters has been studied and results were presented in Tables. Velocity profiles for different values of hall current parameter (m) are shown in figure 1. As m increases, velocity distribution increases. It is seen from figure 2 that increasing by radiation absorption parameter R causes to increase the fluid velocity and hence the maximum peak value is attained at the porous plate. The velocity profiles for different values of heat source parameter are depicted in figure 3. It is noticed that the velocity decreases significantly with the increasing values of Q because when heat is absorbed, the buoyancy force decreases the velocity profiles. The effect of prandtl number Pr on velocity is shown in figure 4. This figure shows that the fluid velocity decreases with increasing values of Pr.

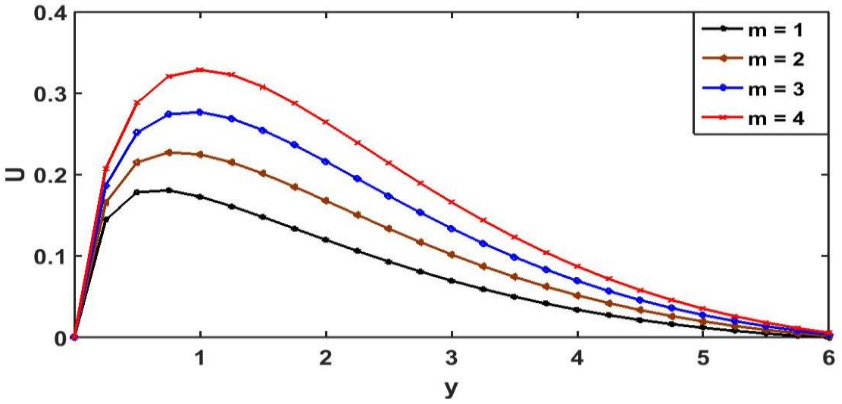

**Figure 1: Effect of hall parameter m over the velocity profile**

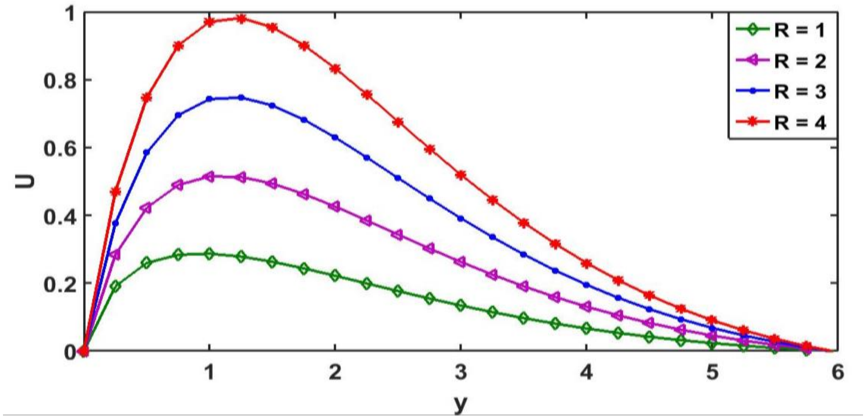

Figure 2: Effect of Radiation absorption parameter R over the velocity profile

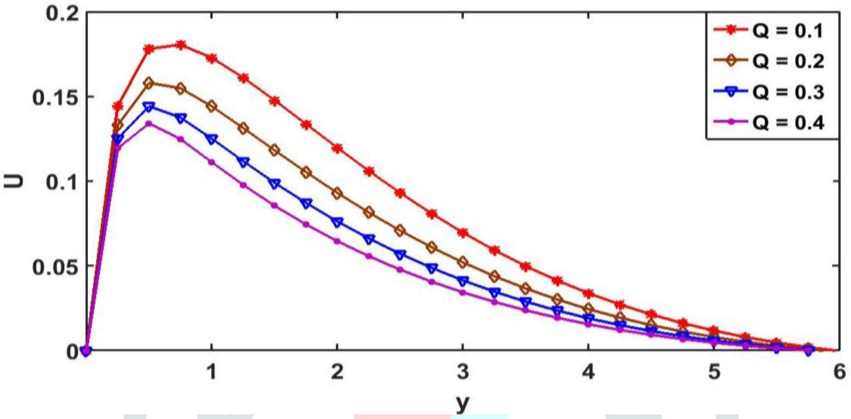

Figure 3: Effect of heat source parameter Q over the velocity profile

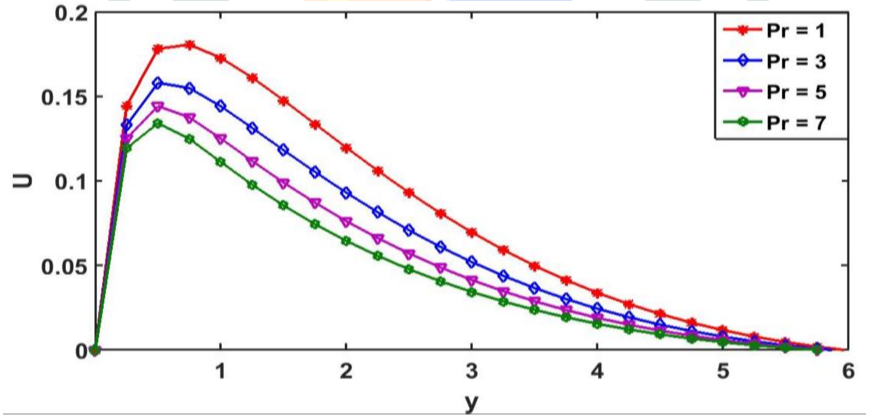

Figure 4: Effect of Prandtl number Pr over the velocity profile

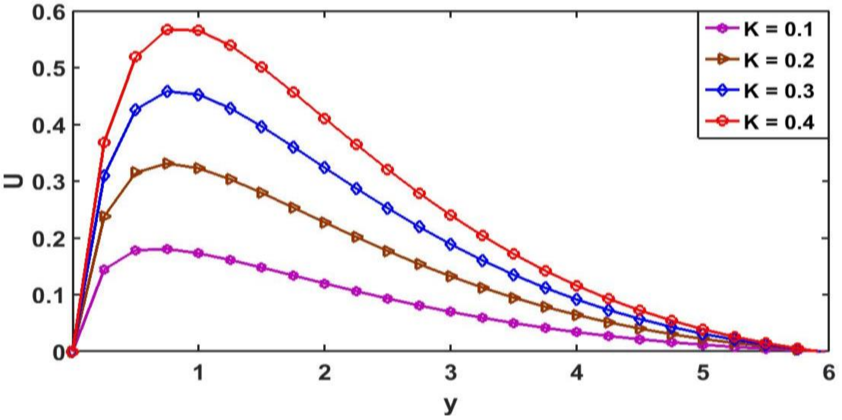

Figure 5: Effect of Permeability parameter K over the velocity profile

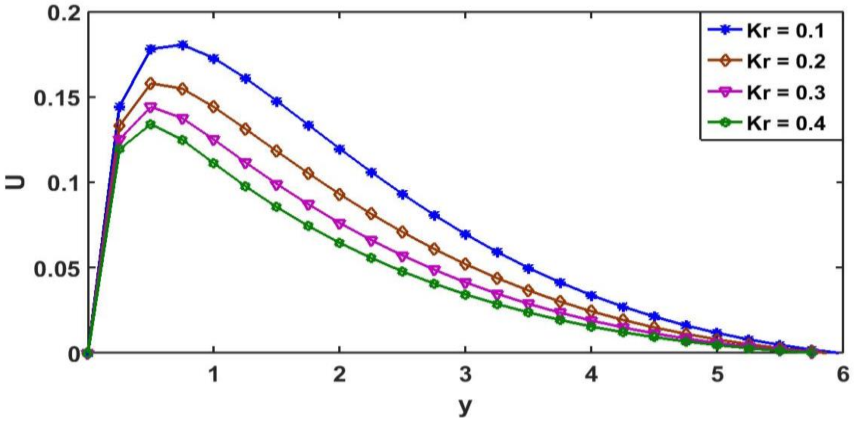

**Figure 6: Effect of Chemical reaction parameter Kr over the velocity profile**

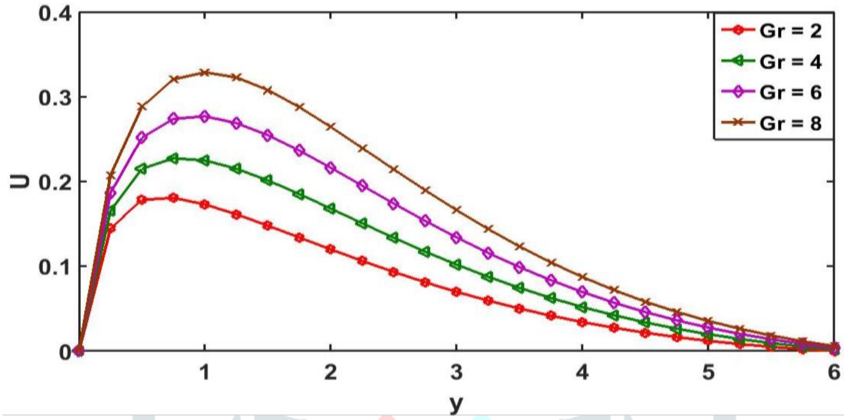

**Figure 7: Effect of thermal Grashof number Gr over the velocity profile**

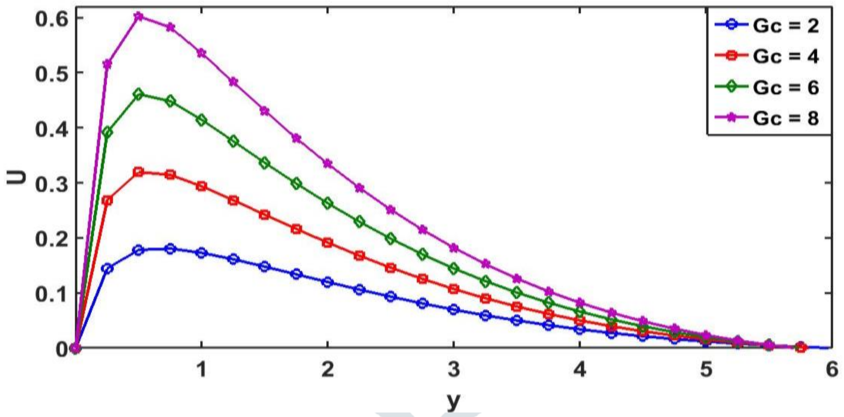

**Figure 8: Effect of modified Grashof number Gc over the velocity profile**

The influence of the permeability parameter K on velocity is shown in figure 5, it is seen that maximum peak value attains for K=0.4 and minimum peak value is observed for K=0.1, also it is clear that the velocity increases significantly with the increasing values of K. Figure 6 elucidates the effect of the velocity profiles for different values of the chemical reaction parameter (Kr). It is seen that the velocity increases with reducing chemical reaction effect. The effect of thermal Grashof number Gr on velocity is shown in figure 7. This figure shows that the fluid velocity increases with increasing values of Gr. The effect of solutal Grashof number Gc on velocity is shown in figure 8. This figure shows that the velocity increases with increasing values of Gc. Table1 and Table2 show the influences of the Nusselt number and Sherwood number respectively. Table 1 depicts that the Nusselt number (Nu) decreases with the increasing values of Sc and R but the effects of Q, Pr, and Kr are to be increase the Nusselt number. Further it is found from table2 that Sherwood number increases with increase in Schmidt number and opposite phenomenon is observed in the case of chemical reaction parameter i.e., as chemical reaction parameter Kr increases, Sherwood number decreases.

Table 1: Nusselt Number

| Sc | R | Q | Pr | Kr | Nu |
|---|---|---|---|---|---|
| **1.0** | **1** | **0.1** | **0.71** | **1** | **1.1082** |
| 1.1 | 1 | 0.1 | 0.71 | 1 | 1.0276 |
| 1.2 | 1 | 0.1 | 0.71 | 1 | 0.9868 |
| 1.0 | **2** | 0.1 | 0.71 | 1 | 0.7775 |
| 1.0 | **3** | 0.1 | 0.71 | 1 | 0.5663 |
| 1.0 | 1 | **0.2** | 0.71 | 1 | 1.1354 |
| 1.0 | 1 | **0.3** | 0.71 | 1 | 1.2574 |
| 1.0 | 1 | 0.1 | **0.9** | 1 | 1.3313 |
| 1.0 | 1 | 0.1 | **1.0** | 1 | 1.5218 |
| 1.0 | 1 | 0.1 | 0.71 | **0.1** | 0.9335 |
| 1.0 | 1 | 0.1 | 0.71 | **0.5** | 1.0253 |

Table 2: Sherwood Number

| Sc | Kr | Sh |
|---|---|---|
| **1.0** | **1** | **1.3031** |
| **1.5** | 1 | 2.2024 |
| **2.0** | 1 | 3.0646 |
| **2.5** | 1 | 3.9184 |
| 1.0 | **0.2** | 2.3843 |
| 1.0 | **0.3** | 2.2024 |
| 1.0 | **0.4** | 1.9547 |
| 1.0 | **0.5** | 1.4851 |

## V. CONCLUSIONS

In this paper studied the chemical reaction, heat source and radiation absorption effects on an unsteady MHD free convective flow past a vertical plate with hall current. From the present investigation the following conclusions can be drawn.

- The velocity increases with decreasing heat source parameter or Prandtl number or chemical reaction parameter.
- The velocity increases with increasing hall current parameter or radiation absorption parameter or permeability parameter or thermal Grashof number or solutal Grashof number.
- The Nusselt number is found that increasing with the increase of heat source parameter or Prandtl number or chemical reaction parameter while it decreasing with the increase values of Schmidt number or radiation absorption parameter.
- The Sherwood number is noticed to increase with an increment Schmidt number while it decreases with an increment chemical reaction parameter.

## REFERENCES

**[1]** Carrier., G.F. and Greenspan, H.P. 1960. The time-dependent magneto hydro dynamic flow past a flat plate, Journal of Fluid Mechanics 7: 22–32.

**[2]** Gupta, A.S., 1960. Steady and transient free convection of an electrically conducting fluid from a vertical plate in the presence of a magnetic field, Applied Scientific Research, 9(1): 319–333.

**[3]** Singer, R.M., 1965. Transient magneto hydrodynamic flow and heat transfer, Zeitschriftf ¨*ur* Angewandte Mathematik und Mechanik, 16(4): 483–494.

**[4]** Pop, I., 1969. Unsteady hydro magnetic free convection flow from a vertical infinite flat plate, Zeitschriftf ¨ur Angewandte Mathematik und Mechanik, 49(12): 756–757, 1969.

**[5]** Yu C.P., and Yang, H.K., 1969. Effect of wall conductance's on convective magneto hydrodynamic channel flow, Applied Scientific Research, 20(1): 16–24.

**[6]** Rao, V.V.R., 1971. Transient hydro magnetic free convection flow, Indian Journal of Physics, 45(5) 233–235.

**[7]** Antimirov M.Y., and Kolyshkin, A.A., 1978. Unsteady MHD convection in a vertical circular channel, Magneto hydrodynamics, 14(3): 279–284.

**[8]** Rajaram, S., and Yu, C.P., 1984. MHD channel flow in a transient magnetic field, ASME Symposium Forum on Unsteady Flow, Publication no. G00259.

**[9]** Tokis, J.N., 1986. Unsteady magneto hydrodynamic free-convection flows in a rotating fluid, Astrophysics and Space Science, 119(2): 305–313.

**[10]** Ghosh, S.K., 1993. Unsteady hydro magnetic flow in a rotating channel with oscillating pressure gradient, Journal of the Physical Society of Japan, 62(11):3893–3903.

**[11]** Sacheti, N.C., Chandran, P., and Singh, A.K., 1994. An exact solution for unsteady magneto hydrodynamic free convection flow with constant heat flux, International Communications in Heat and Mass Transfer, 21(1): 131–142.

**[12]** Attia, H.A., 1999. Transient MHD flow and heat transfer between two parallel plates with temperature dependent viscosity, Mechanics Research Communications, 26(1): 115–121.

**[13]** Al-Nimr M.A., and Alkam, M.K., 1999. Magneto hydrodynamics transient free convection in open-ended vertical annuli, Journal of thermo physics and heat transfer, 13(2): 256–265.

**[14]** Takhar, H.S., Chamkha, A.J., and Nath, G., 1999. Unsteady flow and heat transfer on a semi-infinite flat plate with an aligned magnetic field, International Journal of Engineering Science, 37(13): 1723–1736.

**[15]** Eswara, A.T., Roy, S., and Nath, G., 2000. Unsteady MHD forced flow due to a point sink, Acta Mechanica, 145(1–4):159–172.

**[16]** Chamkha, A.J., 2000. Transient hydro magnetic three-dimensional natural convection from an inclined stretching permeable surface, Chemical Engineering Journal, 76(2): 159–168.

**[17]** Jha, B.K., 2001. Natural convection in unsteady MHD couette flow, Heat and Mass Transfer, 37(4-5), 329–331, 2001.

**[18]** Seddeek, M.A., Effects of radiation and variable viscosity on an MHD free convection flow past a semi-infinite flat plate with an aligned magnetic field in the case of unsteady flow, International Journal of Heat and Mass Transfer, 45(4): 931–935.

**[19]** Zakaria, M., 2003. Magneto hydrodynamic unsteady free convection flow of a couple stress fluid with one relaxation time through a porous medium, Applied Mathematics and Computation, 146(2-3): 469–494.

**[20]** Ghosh S.K., and Pop, I., 2003. Hall effects on unsteady hydro magnetic flow in a rotating system with oscillating pressure gradient, International Journal of Applied Mechanics and Engineering, 8(1): 43–56.

**[21]** Beg, O., Anwar, H.S., Takhar, G.N., and Chamkha, A.J., 2006. Mathematical Modeling of hydro magnetic convection from a rotating sphere with impulsive motion and buoyancy effects, Nonlinear Analysis: Modeling and Control, 11(3): 227–245.

**[22]** Duwairi, H.M., Damseh, R.A., and Tashtoush, B., 2006. Transient non-boussinesq magneto hydrodynamic free convection flows over a vertical surface, International Journal of Fluid Mechanics Research, 33(2): 137–152.

**[23]** Bozkaya C., and Tezer-Sezgin, M., 2006. Boundary element solution of unsteady magneto hydrodynamic duct flow with differential quadrature time integration scheme, International Journal for Numerical Method in Fluids, 51(5): 567–584.

**[24]** Venkateswarlu, M., Venkata Lakshmi, D., and Dharmaiah, G., 2016. Influence of slip condition on radiative MHD flow of a viscous fluid in a parallel porous plate channel in presence of heat absorption and chemical reaction, Journal of the Korean Society for Industrial and Applied Mathematics, 20(4),:333-354.

**[25]** Dharmaiah, G., Baby Rani, CH., Kranti Kumar, K., and Sridevi, S., 2019. Radiation effect on transient free convection MHD flow with exponentially decaying wall temperature, International Journal of Pure and Applied Mathematics, 120(6): 4139-4152.