AN OSCILLATION CRITERION FOR A FIRST ORDER DIFFERENCE EQUATION WITH THE SINGLE DELAY AND GENERAL DELAY ARGUMENT

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ABSTRACT

In this paper, we define difference equation; an oscillation criterion for first order difference equation is various classifications of Basic definitions. This paper is organized as follows in section we adopt some usual terminology, notations and conventions which will be used later in the section we establish the single delay and General delay arguments. The first order difference equation is using basic notations of difference equation and also presents a sufficient condition for the oscillation of all solution of linear difference with general delay argument have also been presented.

Key words: Difference equation, first order difference equation, Single delay, and General delay. Basic notations of difference Equation.

1.1 INTRODUCTION

In the last three decades, the study of difference equation has received significant interest as they provide the first step for developing techniques theory digital signal processing and in particular computer science because of their successful use in computers for solving difficult problem in applications.

In the paper, we study oscillation of solution of the delay difference equation

x(n+1)-x(n)+p(n)x(n-k)=0, for n=0,1,2,……..., (1.1)

Where k ϵ N and {p (n)} n-> ∞ is a non-negative sequence of real numbers

By a solution of equation 1 we mean a sequence{x(n)} n=-k for which x(n+1)=x(n)-p(n)x(n-k) hold for n=0,1,2….., a solution {x(n)} n=-k of equation (1.1) is said to be oscillation if its terms are neither eventually positive nor eventually negative, otherwise the solution {x(n)} is called non-oscillatory

Erbeand Zhang [1] proved that, if p (n) ≥0 then either one of the following conditions

$$\lim_{n\to\infty} inf P(n) > \frac{K^\wedge k}{(K+1)^\wedge(k+1)} \text{ or} \quad C1$$

$$\lim_{n\to\infty} sup \sum_{i=n-k}^{n} P(i) > 1 \quad C2$$

Implies that all solution of equation (1.1) oscillation then Ladas, Philos and Sficas proved that the same conclusion hold if p (n) ≥ 0 and

$$\lim_{n\to\infty} inf \left(\frac{1}{k}\sum_{i=n-k}^{n-1} p(i)\right) > \frac{k^\wedge k}{(k+1)^\wedge(k+1)} \quad C3$$

Therefore they improved the condition (C1) by replacing the p (n) of (C1) by the arithmetic mean of the terms p (n-k)…….,p (n-1) in (C3)

In the paper, we obtain a further improvement of the above condition we also present a sufficient condition under which all solution of (1.1) oscillation without the assumption that p (n)≥ 0 for all n≥ 0 finally we extend our results to difference equation with single delay.

1. 2 .INEQUALITIES AND EQUATIONS WITH A SINGLE DELAY

Consider the difference inequalities

Δx (n) +p (n) x (n-k) ≤0 n=0, 1…… (1.2)

Δx (n) +p (n) x (n-k) ≥0 n=0, 1…… (1.3)

and the difference equation

Δx (n) +p (n) x (n-k) =0 n=0, 1…… (1.4)

Where p (n) is a sequence of real numbers, k is a positive integer and Δ denotes the forward difference operator Δx (n) =x (n+1)-x (n)

By a solution of (1.2), we mean a sequence {x (n)} which is defined for n≥-k and which satisfies (1.2) for n ≥0.Solution of (3) are defined in a similar manner

To obtain our result we need the following lemma which is also very interesting in its own right

LEMMA 1

Assume that {p (n)} is a sequence of non-negative real numbers and that there exists M > 0 such that

$$\lim_{n\to\infty} inf \sum_{i=n-k}^{n-1} p\ (i) > M \qquad (1.5)$$

If {x (n)} is an eventually positive solution of (2) then for every n sufficiently large, there exists an integer n' with n-k ≤ n'≤ n-1 such that

$$\frac{x\ (n'-k)}{x\ (n')} \leq \left(\frac{2}{M}\right)^2 \qquad (1.6)$$

PROOF

Let {x (n)} be an eventually positive solution of (1.2) by (1.5) for n sufficiently large, say for $n\geq n_0$

$$\sum_{i=n-k}^{n-1} p\ (i) \geq M$$

Thus for $n\geq n_0+k$, we can find an integer n' with n-k ≤ n' ≤ n-1 such that

$$\sum_{i=n-k}^{n'} p\ (i) \geq \frac{M}{2} \text{and} \quad \sum_{i=n'}^{n-1} p\ (i) \geq \frac{M}{2} \qquad (1.7)$$

From (1.2) taking into account inequalities (1.7) and the fact that the sequence {x (n)} is decreasing we have

$$x\ (n\text{-}k) - x\ (n'\text{-}1) = \sum_{i=n-k}^{n'} (x\ (i) - x\ (i+1)) \geq \sum_{i=n-k}^{n'} p\ (i) x\ (n'-k) \geq \frac{M}{2} x\ (n'\text{-}k)$$

Similarly,

$$x\ (n')\text{-}x\ (n+1) = \sum_{i=n'}^{n} (x\ (i) - x\ (i+1)) \geq \sum_{i=n'}^{n} p\ (i) x (n-k) \geq \sum_{i=n'}^{n-1} p(i) x\ (n-k) \geq \frac{M}{2} x\ (n\text{-}k)$$

Combining the last two inequalities, we obtain

$$x\ (n') \geq \frac{M}{2} x\ (n\text{-}k) \geq \frac{M}{2}\frac{M}{2} x\ (n'\text{-}k) = \left(\frac{M}{2}\right)^2 x\ (n'\text{-}k)$$

that is, inequality (1.6) holds. The proof is complete.

THEOREM 1.1

Assume that there exists a sequence n(m)→∞ such that

$$p(n)\geq 0 \text{ for } n\epsilon[n\ (m)\text{-}(N+1)k,\ n(m)] \text{ and}$$

$$\sum_{i=n-k}^{n-1} p\ (i) \geq c \geq \left(\frac{k}{k+1}\right)^{k+1}, \text{ for } n\epsilon[n\ (m)\text{-}N\ k,\ n\ (m)]\ m=1,2\ldots.. \qquad (1.8)$$

Where

$$N=1+ \left[\frac{log4-2logc}{logc+(k+1)(log(k+1)-logk)}\right] \qquad (1.9)$$

and [.] denotes the greatest integer function then

a. has no eventually positive solution
b. has no eventually negative solution and
c. has oscillatory solution only

PROOF

First we prove that inequality (2) has no eventually positive solutions. To this end assume for the sake of contradiction that {x (n)} is an eventually positive solution of (1.2) then we obtain

$$\left(c \left(\frac{k+1}{k}\right)^{k+1}\right)^N \leq \frac{x\ (n-k)}{x\ (n)} \text{ for } n\epsilon\ [n\ (m)\text{-}N\ k,\ n\ (m)]$$

On the other hand by lemma1 we have

$$\frac{x\ (n-k)}{x\ (n)} \leq \frac{4}{c^2}, \text{ for } n\epsilon\ [n\ (m)\text{-}N\ k,\ n\ (m)]$$

From the above inequalities it follows that

$$\left(c \left(\frac{k+1}{k}\right)^{k+1}\right)^N \leq \frac{4}{c^2}$$

That is $N\leq \frac{log4-2logc}{logc+(k+1)(log(k+1)-logk)}$ which contradicts the definition of N

From the above it follows that (4) has neither positive nor eventually negative solutions and therefore, every solution of (1.4) oscillates. the proof is complete.

THEOREM 1.2

Assume that {p (n)} is a non-negative sequence of real numbers and let k be a positive integers. Assume that there exists M>0 such that

$$\lim_{n\to\infty} inf \sum_{i=n-k}^{n-1} p\ (i) > M \qquad (1.10)$$

and

$$\mu=\lim_{n\to\infty} sup\ p\ (n) > 1\text{-} \left(\frac{M}{2}\right)^2 \qquad (1.11)$$

Then a. has no eventually positive solution
b. has no eventually negative solution
c. has oscillatory solution only

PROOF

First we prove that (1.2) has no eventually positive solutions. To this end assume for the sake of contradiction that {x (n)} is an eventually positive solution of (1.2) then eventually

$$\Delta x\ (n) = x\ (n+1)\text{-}x\ (n) \leq\text{-}p\ (n)\ x\ (n\text{-}k) \leq 0$$

And so {x (n)} is eventually decreasing sequence of positive numbers. Summing (1.2) from n-k to n-1
We have, $x(n)-x(n-k)+\sum_{i=n-k}^{n-1} p(i)\, x(i-k) \le 0$

And because {x (n)} is eventually decreasing it follows that for n sufficiently large

$$x(n)-x(n-k) + \left(\sum_{i=n-k}^{n-1} p(i)\right) x(n-k) \le 0 \text{ or}$$

$$x(n-k)\left(\sum_{i=n-k}^{n-1} p(i) + \frac{x(n)}{x(n-k)} - 1\right) \le 0$$

and using lemma1, for N sufficiently large there exist an integer n' with N-k≤n'≤N-1 such that

$$x(n'-k)\left(\sum_{i=n'-k}^{n'-1} p(i) + \frac{M^2}{4} - 1\right) \le 0$$

now let λ (n) be a sequence such that p (λ (n)) →μ for N=λ (n) +k+1, n' satisfies

$$\lambda(n)+1 \le n' \le \lambda(n)+k \text{ or } n'-k \le \lambda(n) \le n'-1$$

Thus, $x(n'-k)\left(p(\lambda(n)) + \left(\frac{M}{2}\right)^2 - 1\right) \le x(n'-k)\left(\sum_{i=n'-k}^{n'-1} p(i) + \left(\frac{M}{2}\right)^2 - 1\right) \le 0$

Which is view of (1.11) leads to a contradiction. Hence the equation (1.2) has no eventually positive solution equation (1.4) has neither positive nor eventually negative solution and therefore every solution of (1.4) oscillate. The proof is complete

1.3 OSCILLATION CRITERIOR FOR THE GENERAL DELAY ARGUMENT

Our main result is to prove the oscillation solution of the general delay argument. The proof of the theorem is based on the following lemma

LEMMA 2

Assume that the sequence $(\tau(n))_{n\ge 0}$ is increasing. Moreover, assume that $0 < \alpha \le -1+\sqrt{2}$, where α is defined by $\lim_{n\to\infty} \inf \sum_{j=\tau(n)}^{n-1} p(j)$. Then every non-oscillatory solution $(x(n))_{n\ge -k}$ of the delay difference equation

$$\Delta x(n) + p(n)\, x(\tau(n)) = 0 \qquad (1.11a)$$

Where (p (n)) with n≥0 is a sequence of non-negative real numbers and τ (n)with n≥0 is a sequence of integers such that τ (n) ≤ n-1 for all n≥0 and $\lim_{n\to\infty} \tau(n) = \infty$ satisfies

$$\lim_{n\to\infty} \inf \frac{x(n+1)}{x(\tau(n))} \ge \frac{1}{2}\left(1-\alpha-\sqrt{(1-2\alpha-\alpha^2)}\right) \qquad (1.12)$$

PROOF

Define q (t) =p (n) for n ≤ t < n+1 (n=0, 1…..)

Clearly q is a non-negative real-valued function on the interval [0, ∞), which is continuous on each one of the intervals (n, n+1) (n=0, 1…). Note that q (n) = p (n) for every integer n≥0. Furthermore, consider the real-valued function σ defined on the interval [0, ∞) by

$$\sigma(t) = \tau(n) \qquad \text{for } n \le t < n+1 \ (n=0, 1\ldots..)$$

It is obvious that for each n=0, 1… the function σ is continuous on (n, n+1). We notice that σ (n) = τ (n) for all integers n≥0. We can immediately see that

$$\Sigma(t) < t \text{ for all } t \ge 0 \qquad \text{and} \qquad \lim_{n\to\infty} \sigma(t) = \infty$$

Also, as the sequence (τ (n)) with n≥0 is assumed to be increasing, we observe that the function σ is increasing on [0, ∞).

Let (x (n)) with n ≥ -k be a solution of the delay difference equation(1.11a). We define

$$y(t) = x(n) + (\Delta x(n))(t-n) \qquad \text{for } n \le t \le n+1$$

It is clear

$$y(n) = x(n) \qquad \text{for all integers } n \ge -k$$

Moreover, it is easy to verify that the real-valued function *y* is continuous on the interval [-k, ∞). Also, we see that yis continuously differentiable on each one of the intervals (n, n+1)
(n + -k, -k+1…) with

$$y'(t) = \Delta x(n) \qquad \text{for all integers } (n=-k,-k+1,)$$

Furthermore, as *x (n)* satisfies (1.11a) for all integers n≥0, we can easily conclude that the function y satisfies

$$y'(t) + q(t)\, y(\sigma(t)) = 0 \quad \text{for all integers}(n=0,1\ldots\ldots.)$$

Next assume that the solution x (n) with n≥-k is non-oscillatory. Then it is either eventually positive or eventually negative. As (–x (n)) with n ≥ -k is also a solution of (1'). We may restrict ourselves only to the case where x (n) > 0 for all large n.

Consider an arbitrary real number ε with 0<ε<α then we can choose an integer n_0>r such that τ (n) ≥ r for n ≥ n_0 such that

$$\sum_{j=t(n)}^{n-1} p(j) > \alpha - \varepsilon \qquad \text{for every } n \ge n_0$$

For any point τ≥ n_0, there exists an integer n ≥ n_0 such that n ≤ t< n+1, and consequently

$$\int_{\sigma(t)}^{t} q(s)ds = \int_{\tau(n)}^{t} q(s)ds \ge \int_{\tau(n)}^{n} q(s)ds = \sum_{j=\tau(n)}^{n-1} p(j) > \alpha - \varepsilon.$$

So we have

$$\int_{\sigma(t)}^{t} q(s)ds > \alpha - \varepsilon \text{ for all } t \ge n_0 \qquad (1.13)$$

Furthermore, we will establish the following claim

For each point $t \geq n_0$, there exists α $t^* > t$ such that $\sigma(t^*) < t$ and

$$\int_t^{t^*} q(s)ds = \alpha - \varepsilon \tag{1.14}$$

To prove this claim, let us consider an arbitrary point $t \geq n_0$ set

$$f(v) = \int_t^v q(s)ds \quad \text{for } v \geq t$$

We see that f (t) = 0. Moreover, it is not difficult to show that (1.13) guarantees that

$$\int_0^\infty q(s)ds = \infty$$

And in particular,

$$\int_t^\infty q(s)ds = \infty$$

That is $\lim_{n\to\infty} f(v) = \infty$. Thus, as the function f is continuous on the interval [t,∞), there exists α point $t^* > t$ so that $f(t^*) = \alpha - \varepsilon$,that is (14) is satisfied. By using (1.13), it follows that

$$y(t) = y(t^*) + \int_t^{t^*} q(s)y(\sigma(s))ds \tag{1.15}$$

Let s be any point with $t \leq s \leq t^*$.

As the function σ is increasing on [0, ∞), we have $n_0 \leq \sigma(t) \leq \sigma(t^*) < t$,

and $r \leq \sigma(u) \leq \sigma(t)$ for every u with $\sigma(s) \leq u$ with $\sigma(s) \leq u \leq t$. thus, by taking into account the fact that the function y is decreasing on [r, ∞), from (1.12) we obtain

$$y(\sigma(s)) = y(t) + \int_{\sigma(s)}^t q(u)y(\sigma(u))du$$

$$\geq y(t) + \left[\int_{\sigma(s)}^t q(u)du - \int_t^s q(u)du\right] y(\sigma(t))$$

So by applying (1.13), we get

$$y(\sigma(s)) > y(t) + \left[(\alpha - \varepsilon) - \int_t^s q(u)\right] y(\sigma(t))$$

As the inequality holds true for all s with $t \leq s \leq t^*$, it follows from (1.15) that

$$y(t) \geq y(t^*) + \int_t^{t^*} q(s)\{y(t) + [(\alpha - \varepsilon) - \int_t^s q(u)du]\, y(\sigma(t))\}\, ds$$

$$= y(t^*) + [\int_t^{t^*} q(s)ds]\, y(t) + \{(\alpha - \varepsilon)\int_t^{t^*} q(s)ds - \int_t^{t^*} q(s)[\int_t^s q(u)du]\, ds\}\, y(\sigma(t))$$

And consequently, in view of (1.14),

$$y(t) \geq y(t^*) + (\alpha - \varepsilon)\, y(t) + \{(\alpha - \varepsilon)^2 - \int_t^{t^*} q(s)[\int_t^s q(u)du]ds\}\, y(\sigma(t)) \tag{1.16}$$

By the known formula

$$\int_t^{t^*} q(s)[\int_t^s q(u)du]\, ds = \frac{1}{2}\{\int_t^{t^*} q(s)[\int_t^s q(u)du]\, ds + \int_t^{t^*} q(s)[\int_s^{t^*} q(u)du]\, ds\}$$

$$= \frac{1}{2}\int_t^{t^*} q(s)[\int_t^s q(u)du + \int_s^{t^*} q(u)du]\, ds$$

$$= \frac{1}{2}\int_t^{t^*} q(s)[\int_t^{t^*} q(u)du]\, ds = \frac{1}{2}[\int_t^{t^*} q(s)ds]^2$$

And therefore, by (1.14),

$$\int_t^{t^*} q(s)[\int_t^s q(u)du]\, ds = \frac{1}{2}(\alpha - \varepsilon)^2$$

Hence, (1.15) is written as

$$y(t) \geq y(t^*) + (\alpha - \varepsilon)\, y(t) + \frac{1}{2}(\alpha - \varepsilon)^2 y(\sigma(t)) \tag{1.17}$$

This gives

$$y(t) > (\alpha - \varepsilon)\, y(t) + \frac{1}{2}(\alpha - \varepsilon)^2 y(\sigma(t))$$

$$y(t) > \frac{(\alpha - \varepsilon^2)}{2[1-(\alpha-\varepsilon)]}\, y(\sigma(t))$$

We have thus proved that

$$y(t) > \lambda_1 y(\sigma(t)) \quad \text{for all } t \geq N, \tag{1.18}$$

Where $\lambda_1 = > \frac{(\alpha - \varepsilon^2)}{2[1-(\alpha-\varepsilon)]}$

Now in view of (1.18), we have

$$y(t^*) > \lambda_1 y(\sigma(t^*))$$

but since $\sigma(t^*) < t$ and the function y is decreasing on [r, ∞), we also have

$$y(\sigma(t^*)) \geq y(t)$$

Combining the last two inequalities, we obtain

$$y(t^*) > \lambda_1 y(t)$$

and hence (1.17) yields

$$y(t) > \lambda_1 y(t) + (\alpha - \varepsilon)\, y(t) + \frac{1}{2}(\alpha - \varepsilon)^2 y(\sigma(t))$$

Or $$[1-(\alpha - \varepsilon) - \lambda]\, y(t) > \frac{1}{2}(\alpha - \varepsilon)^2 y(\sigma(t))$$

this implies, in particular, that

$$1-(\alpha-\varepsilon)-\lambda_1 > 0$$

Consequently,

$$y(t) > \frac{(\alpha-\varepsilon^2)}{2[1-(\alpha-\varepsilon)-\lambda_1]}\, y(\sigma(t))$$

Thus, it has been shown that

$$y(t) > \lambda_2 y(\sigma(t)) \qquad \text{for all } t \geq N,$$

$$\lambda_2 = \frac{(\alpha-\varepsilon^2)}{2[1-(\alpha-\varepsilon)-\lambda_1]}$$

Following the above procedure, we can inductively construct a sequence of positive real numbers (λ_v) as $v \geq 1$ with $1-(\alpha-\varepsilon)-\lambda_v > 0\ (v=1, 2\ldots.)$

and $\lambda_{v+1} = \frac{(\alpha-\varepsilon^2)}{2[1-(\alpha-\varepsilon)-\lambda_v]}$ for all $(v = 1, 2\ldots.)$

such that $y(t) > \lambda_v\, y(\sigma(t))$ for all $(v = 1, 2\ldots.)$ (1.19)

as $\lambda_1 > 0$, we obtain

$$\lambda_2 = \frac{(\alpha-\varepsilon^2)}{2[1-(\alpha-\varepsilon)-\lambda_1]} > \frac{(\alpha-\varepsilon^2)}{2[1-(\alpha-\varepsilon)]} = \lambda_1,$$

by an easy induction, one can immediately see that the sequence (λ_v) as $v \geq 1$ is strictly increasing. Furthermore, by taking into account the fact that the function y is decreasing on $[r, \infty)$ and using (1.19)

We get, $y(N) > \lambda_v y(\sigma(N)) \geq \lambda_v y(N)$ for all $(v = 1,2\ldots.)$

Therefore, for each integer $v \geq 1$, we have $\lambda_v<1$.this ensures that the sequence is bounded

Hence it follows that

$$\Lambda = \lim_{v\to\infty} \lambda_v$$

Then (1.19) gives

$$y(t) \geq \Lambda\, y(\sigma(t)) \qquad \text{for all } t \geq N \qquad (1.20)$$

Because of the definition of (λ_v) as $v \geq 1$, it holds

$$\Lambda = \frac{(\alpha-\varepsilon^2)}{2[1-(\alpha-\varepsilon)-\Lambda]},$$

$$\Lambda^2-[1-(\alpha-\varepsilon)]\,\Lambda + \frac{1}{2}(\alpha-\varepsilon)^2 = 0$$

Hence either

$$\Lambda = \frac{1}{2}[1-(\alpha-\varepsilon)-\sqrt{(1-2(\alpha-\varepsilon)}-(\alpha-\varepsilon)^2)]$$

Or

$$\Lambda = \frac{1}{2}[1-(\alpha-\varepsilon)+\sqrt{(1-2(\alpha-\varepsilon)}-(\alpha-\varepsilon)^2)]$$

In both cases, we have

$$\Lambda \geq \frac{1}{2}[1-(\alpha-\varepsilon)-\sqrt{(1-2(\alpha-\varepsilon)}-(\alpha-\varepsilon)^2)]$$

And consequently (1.20) yields

$$y(t) \geq \frac{1}{2}[1-(\alpha-\varepsilon)-\sqrt{(1-2(\alpha-\varepsilon)}-(\alpha-\varepsilon)^2)]\, y(\sigma(t)) \text{ for } n \leq t < n+1 \qquad (1.21)$$

but, $y(\sigma(t)) = y(\tau(n)) = x(\tau(n))$ for $n \leq t < n+1$. So,

$$y(t) \geq \frac{1}{2}[1-(\alpha-\varepsilon)-\sqrt{(1-2(\alpha-\varepsilon)}-(\alpha-\varepsilon)^2)]\, x(\tau(n))$$

and therefore

$$\lim_{t\to(n+1)-0} y(t) \geq \frac{1}{2}[1-(\alpha-\varepsilon)-\sqrt{(1-2(\alpha-\varepsilon)}-(\alpha-\varepsilon)^2)]\, x(\tau(n))$$

Note that $\lim_{t\to(n+1)-0} y(t) = y(n+1) = x(n+1)$. We have thus proved that

$$x(n+1) \geq \frac{1}{2}[1-(\alpha-\varepsilon)-\sqrt{(1-2(\alpha-\varepsilon)}-(\alpha-\varepsilon)^2)]\, x(\tau(n)) \text{ for all } n \geq N \qquad (1.22)$$

Hence we get the inequality

$$\lim_{n\to\infty} \inf \frac{x(n+1)}{x(\tau(n))} \geq \frac{1}{2}[1-(\alpha-\varepsilon)-\sqrt{(1-2(\alpha-\varepsilon)}-(\alpha-\varepsilon)^2)]$$

The last inequality holds true for all real numbers ε. Hence, we can obtain the required inequality

The proof of our lemma is proved.

THEOREM 1. 3

Let the assumptions of lemma 2 hold then the condition

$$\lim_{n\to\infty} \sup \sum_{j=\tau(n)}^{n} p(j) > 1-\frac{1}{2}(1-\alpha-\sqrt{(1}-2\alpha-\alpha^2)) \qquad (1.23)$$

PROOF

Assume, for the sake of contradiction, that there exists α non-oscillatory solution $(x(n))_{n\geq -k}$ of the delay difference equation (1'). Since $(-x(n))_{n\geq -k}$ is also a solution of (1.11a), we can confine our solution to the case where solution $(x(n))_{n\geq -k}$ is eventually positive.

Now we choose an integer $n_0 > r$ such that $\tau(n) \geq r$ for $n \geq n_0$. Furthermore, we consider an integer and the sequence $(x(n))_{n \geq r}$ is decreasing, it follows from (1.11a) that, for every $n \geq N$,

$$x(\tau(n)) - x(n+1) = \sum_{j=\tau(n)}^{n} p(j) x(\tau(j)) \geq \left[\sum_{j=\tau(n)}^{n} p(j)\right] x(\tau(n))$$

this gives
$$\sum_{j=\tau(n)}^{n} p(j) \leq 1 - \frac{x(n+1)}{x(\tau(n))} \quad \text{for all } n \geq N$$

Hence,
$$\lim_{n\to\infty} \sup \sum_{j=\tau(n)}^{n} p(j) \leq 1 - \lim_{n\to\infty} \inf \frac{x(n+1)}{x(\tau(n))}$$

but in view of lemma 2 inequality (1.12) holds. So we obtain

$$\lim_{n\to\infty} \sup \sum_{j=\tau(n)}^{n} p(j) \leq 1 - \frac{1}{2}\left(1 - \alpha - \sqrt{(1 - 2\alpha - \alpha^2)}\right)$$

Which contradicts condition (1.23) hence the proof of the theorem is complete.

CONCLUSION

In this paper we obtain an oscillation criterion for first order Difference Equation is introduced. This single delay and general delay is effectives and easy to understand because of its natural similarity to classical method of single delay and general delay arguments. The method of first order difference equation and arguments e is shows in this paper guarantees the correctness and effectiveness of the working produce of the method. Since the difference equation with delay does possess oscillatory solution and also by the fact that the mathematical models of most real-world problems lead to a difference equation with α constant and the variable argument. This method of delay and general arguments is also easy to apply and understand.

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