# IMPATIENT CUSTOMERS IN AN M/M/1 WORKING VACATION QUEUE WITH A WAITING SERVER AND SETUP TIME

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**ABSTRACT**

In this paper, we using probability generating function method, impatient customers in an M/M/1 working vacation queue with a waiting server and set-up time are discussed in this paper. This paper is organized as follows in section we adopt some usual terminology, notations and conventions which will be used later in the section we establish Customers impatience is due to the server's vacation. We obtain the probability generating functions of the stationary state probabilities, performance measures, sojourn time of a customer and stochastic decomposition of the queue length and waiting time have also been presented.

**Key words:** M/M/1 queue; impatient customers; Working vacation; Setup time; Probability generating function

## 1.1 INTRODUCTION

Server vacations were introduced first by Levy and Yechiali[8]. Substantial work in this area has been conducted during the last few decades while Tian and Zhang[13], Doshi[6], Baba[2] and Takagi[12] surveyed the vacation models. Recently, Ke, Wu and Zhang[7] did a survey about the vacation models. These studies help understanding about a variety of vacation models. Queueing systems with impatient customers appear in many real life situations such as those involving impatient telephone switchboard customers, hospital emergency rooms handling critical patients, and inventory systems that store perishable goods. There is growing interest in the analysis of queueing systems with impatient customers. This is due to their potential application in communication systems, call centers, Production-inventory systems and many other related areas, see for[4,3]

Queuing systems with impatient customers have been studied by a number of authors. We mention a few of the more significant works below. Palm's pioneering work[11] seems to be the first to analyze queuing systems with impatient customers by considering the M/M/C queue, where the customers have independent exponentially distributed waiting times.

The situation of impatient customers in a server vacations period was investigated by Altman and Yechialli, Yue *et al.*[14] have analyzed an M/M/1 queueing system with working vacation and impatient customers and obtained the pgf of the number of units in the model when the server is in a working vacation period and in a service period. Padmavathy *et al.*[10] have studied the steady state behaviour of vacation queues with impatient customers and a waiting server.

In this paper we analyze the impatient customers in an M/M/1 working vacation queue with waiting server and setup times. In this model, if the system becomes empty, then the server waits for a random period of time. This option of the server waits reflects many real life queueing systems. It was first introduced by Boxma *et al.*[5] for the ordinary

M/G/1 queue with server vacations.

The rest of the paper is organized as follows. Section 1.2, gives a description of the Impatient customers in an Markovian working vacation queue with waiting server and setup times and analysis of the M/M/1 working vacation model with the stochastic decomposition property. In section 1.3, gives the numerical results.

## 1. 2 MODEL DESCRIPTION

We consider a Impatient customers in an M/M/1 working vacation queue with a waiting server and setup time. The assumption of the model are as follows: Customers arrive according to a Poisson process with arrival rate $\lambda$. The service rate is $\mu_b$ during a normal service period. When the busy period is ended the server waits a random duration of time before beginning a vacation. This waiting time duration follows the exponential distribution with rate $\eta$ is the waiting rate of a server. The server takes a working vacation as soon as the system becomes empty. During the vacation period, the server serves the customers at an exponential rate $\mu_v$, where $\mu_v < \mu_b$. The duration of a vacation is also exponential with rate $\gamma$.

The customers in this system are assumed to be impatient as described below. Whenever arriving customer has to join a queue and finds the system in working vacation, the customer activates an impatient timer 'T', which is exponential with rate $\alpha$ and is independent of the number of customers in the system at that moment. If the server ends the vacation before the time T expires, the customer stays in the system till his service is completed; otherwise the customer leaves the system and will never return. When a vacation ends, if there are customers in the queue, the server changes his service rate from $\mu_v$ to $\mu_b$ and a regular busy period starts, otherwise the server begins a closed-down period. However, a server needs some setup time to be active so as to serve waiting customers. We assume that the setup time follows the, Exponential distribution with mean $\frac{1}{\beta}$

We assume that inter arrival times, service times, working vacation times, impatient times and setup times are mutually independent. In addition the service order is First in First out (FIFO). At time t, let Q(t) denote the total number of customers in the system and J(t) denotes the state of the server with,

$$J(t) = \begin{cases} 0, & \text{at time t the server is busy in working vacation period} \\ 1, & \text{at time t the server is free during in setup period,} \\ 2, & \text{at time t the server is busy is normal service period} \end{cases}$$

Then the pair $\{(Q(t), J(t)); t \geq 0\}$ defines a two dimensional continuous time Markov chain with state space $E = \{\{(0,0), (0,1), (0,2)\} \cup \{i, j\}, i = 1,2, \ldots, j = 0,1,2\}$. Let $P_{i,j} = \lim_{t\to\infty} P\{Q(t) = i, J(t) = j\}, \quad (i, j) \in E$. Let $P_{i,0}$, $i \geq 0$, be the probability that there are i customers in the system when the server is in working vacation period, $P_{i,1}$, $i \geq 0$, be the probability that there are i customers in the system when the server is in setup period and $P_{i,2}$, $i \geq 0$, be the probability that there are i customers in the system when the server is in normal busy period. Therefore, using the theory of Markov process, we obtain the following set of balance equations.

$$(\lambda + \gamma)\, p_{0,0} = \eta p_{0,2} + \mu_v p_{1,0}\,, \tag{1.1}$$

$$(\lambda + (n-1)\alpha + \mu_v + \gamma)\, p_{n,0} = \lambda p_{n-1,0} + (\mu_v + n\alpha)p_{n+1,0},\ n \geq 1, \tag{1.2}$$

$$\lambda p_{0,1} = \gamma p_{0,0}\,, \tag{1.3}$$

$$(\lambda + \beta)p_{n,1} = \lambda p_{n-1,1},\ n \geq 1, \tag{1.4}$$

$$(\lambda + \eta)p_{0,2} = \mu_b p_{1,2} + \beta P_{0,1}, \tag{1.5}$$

$$(\lambda + \mu_b)p_{n,2} = \lambda p_{n-1,2} + \beta p_{n,1} + \gamma p_{n,0} + \mu_b p_{n+1,2}, \quad n \geq 1\,. \tag{1.6}$$

Define the Probability generating functions (PGFs)

$$Q_0(z) = \sum_{n=0}^{\infty} P_{n,0} z^n, Q_1(z) = \sum_{n=0}^{\infty} P_{n,1} z^n, Q_2(z) = \sum_{n=0}^{\infty} P_{n,2} z^n \tag{1.7}$$

$$Q_0(1)+Q_1(1)+Q_2(1) \quad and \quad Q_0'(z)=\sum_{n=0}^{\infty}P_{n,2}z^{n-1}$$

Multiplying (1.2) by $z^n$ and summing over n and using (1.1), we get

$$(\lambda+\mu_0+\gamma)\sum_{n=1}^{\infty}P_{n,0}z^n+\alpha\sum_{n=1}^{\infty}P_{n,0}z^n(n-1)+(\lambda+\gamma)p_{00}$$

$$=\lambda z\sum_{n=1}^{\infty}P_{n-1,0}z^n+\alpha\sum_{n=1}^{\infty}nP_{n+1,0}z^n+np_{0,2}+\mu_v p_{1,0}$$

$$\alpha z(1-z)Q_0'(z)-Q_0(z)\left[(\lambda z-\mu_v+\alpha)(1-z)+\gamma z\right]+(\alpha-\mu_v)(1-z)p_{0,0}+\eta zp_{0,2}=0 \qquad (1.8)$$

Putting Z=1 in (1.8) we get

$$\gamma Q_0(1)=\eta p_{0,2}-Q_0(z)\left[(\lambda z-\mu_0)(1-z)+\gamma z\right]+(\alpha-\mu_v)(1-z)p_{0,0}+\eta zp_{0,2}=0$$

$$Q_0(z)=\frac{\mu_v(1-z)p_{0,0}+\eta zp_{0,2}}{\lambda z-\lambda^2-\mu_v+\mu_v z+\gamma z}$$

Equation (1.8) can be written as

$$Q_0(z)-\left(\frac{\lambda z-\mu_v+\alpha}{\alpha z}+\frac{\gamma}{\alpha(1-z)}\right)Q_0(z)+\frac{(\alpha-\mu_0)}{\alpha z}p_{0,0}+\frac{\eta}{\alpha(1-z)}p_{0,2}=0$$

Integrating factor

$$e^{-\int\left(\frac{\lambda z-\mu_v+\alpha}{\alpha z}+\frac{\gamma}{\alpha(1-z)}\right)dz}=e^{-\frac{\lambda}{\alpha}z}z^{\frac{\mu_v}{\alpha}-1}(1-z)^{\frac{\gamma}{\alpha}}$$

The general solution to the differential equation is given by

$$Q_o(z)=e^{\frac{\lambda}{\alpha}z}z^{\frac{\mu_v}{\alpha}-1}(1-z)^{\frac{\gamma}{\alpha}}\left(\left(\frac{\mu_v}{\alpha}-1\right)p_{0,0}\int_0^z e^{-\frac{\lambda}{\alpha}x}x^{\frac{\mu_v}{\alpha}-2}(1-x)^{\frac{\gamma}{\alpha}}dx-\frac{\eta}{\alpha}p_{0,2}\int_0^z e^{-\frac{\lambda}{\alpha}x}x^{\frac{\mu_v}{\alpha}-2}(1-x)^{\frac{\gamma}{\alpha}-1}dx\right)$$

$$Q_o(z)=z^{\frac{\mu_v}{\alpha}-1}(1-z)^{\frac{\gamma}{\alpha}}\left(\left(\frac{\mu_v}{\alpha}-1\right)p_{0,0}k_1(z)-\frac{\eta}{\alpha}p_{0,2}k_2(z)\right) \qquad (1.9)$$

$$k_1(z)=\int_0^z e^{-\frac{\lambda}{\alpha}x}x^{\frac{\mu_v}{\alpha}-2}(1-x)^{\frac{\gamma}{\alpha}}dx$$

$$k_2(z)=\int_0^z e^{-\frac{\lambda}{\alpha}x}x^{\frac{\mu_v}{\alpha}-2}(1-x)^{\frac{\gamma}{\alpha}-1}dx$$

Taking $\lim_{z\to 1}$ in equation (1.8) we get

$$\lim_{z\to 1}Q_o(z)=Q_0(1)=\left[\left(\frac{\mu_v}{\alpha}-1\right)p_{0,0}k_1(z)-\frac{\eta}{\alpha}p_{0,2}k_2(z)\right]\lim_{z\to 1}(1-z)^{-\frac{\gamma}{\alpha}}$$

$$\left(\frac{\mu_v}{\alpha}-1\right)p_{0,0}k_1(1)=\frac{\eta}{\alpha}p_{0,2}k_2(1)$$

$$p_{02} = \left( \frac{\mu_v - \alpha}{\alpha} \frac{\alpha}{\eta} p_{0,0} \frac{k_1(1)}{k_2(1)} \right)$$

$$p_{02} = \left( \frac{\mu_v - \alpha}{\eta} p_{0,0} \frac{k_1(1)}{k_2(1)} \right) \tag{1.10}$$

Multiplying (1.3) and (1.4) by $z^n$ and summing over n, we get

$$(\lambda + \beta) \qquad \sum_{n=1}^{\infty} p_{n,0} z^n = \lambda \qquad \sum_{n=1}^{\infty} p_{n-1,1} z^n$$

$$(\lambda + \beta)Q_1(z) - (\lambda + \beta)p_{0,1} = \lambda z Q_1(z)$$

$$(\lambda + \beta - \lambda z)Q_1(z) = (\lambda + \beta)p_{0,1} \tag{1.11}$$

Similar way (1.5) and (1.6) with power of $z^n$ and summing over all possible values of n, we get

$$(\lambda + \mu_b)(Q_2(z) - p_{0,2}) + (\lambda + \eta)p_{0,2} = \lambda z Q_2(z) + \beta(Q_1(z) - p_{0,1}) + \frac{\mu_b}{z}(Q_2(z) - P_{1,2}z - P_{0,2})$$
$$+ \gamma(Q_0(z) - P_{0,0}) + \mu_b P_{1,2} + \beta P_{0,1}$$

$$(1 - z)(\lambda z - \mu_b)Q_2(z) = \gamma z Q_0(z) + \beta z Q_1(z) - \mu_b(1 - z)P_{0,2} - (\eta p_{02} + \gamma p_{0,0})z$$

$$Q_2(z) = \frac{\beta z Q_1(z) + \gamma Q_0(z) - (\eta p_{02} + \gamma p_{0,0})z - \mu_b(1-z)p_{0,2}}{(\lambda z - \mu_b)(1-z)} \tag{1.12}$$

Adding (1.5) and (1.6) over all possible values of n, we get

$$\beta Q_1(1) + \gamma Q_0(1) = \eta p_{0,2} + \gamma p_{0,0} \tag{1.13}$$

Equation (1.12) can be written as,

$$Q_2(1) = \frac{\beta Q_1(z) + \gamma Q_0'(1)}{\mu_b - \lambda} - \frac{\mu_b p_{0,2}}{\mu_b - \lambda} \tag{1.14}$$

Where , $Q_0'(1) = E[L_0]$, $Q_2(1) = 1 - Q_0(1) - Q_1(1)$ substituting in (1.13), we gives

$$1 - Q_0(1) - Q_1(1) = \frac{\beta E[L_1] + \gamma E[L_0]}{\mu_b - \lambda} - \frac{\mu_b p_{0,2}}{\mu_b - \lambda}$$

$$E[L_0] = \frac{(\mu_b - \lambda)}{\gamma}(1 - Q_0(1) - Q_1(1)) - \frac{\beta}{\gamma} E[L_1] - \frac{\mu_b}{\gamma} p_{0,2} \tag{1.15}$$

So that can be obtained . Adding (1.2), (1.4), (1.6) and rearranging the term , we get

$$\lambda p_{n,0} + \lambda p_{n,1} + \lambda p_{n,2} - \left[ (\mu_v + n\alpha) p_{n+1,0} + \mu_b p_{n+1,2} \right]$$

$$= \lambda p_{n-1,0} + \lambda p_{n-1,1} + \lambda p_{n-1,2} - \left[ \left( \mu_b + (n-1)\alpha \right) p_{n,0} + \mu_b p_{n,2} \right], n \geq 1 \qquad (1.16)$$

Using (1.16) in (1.2), (1.4), (1.6) we get

$$\lambda p_{n,0} + \lambda p_{n,1} + \lambda p_{n,2} = \left( \mu_v + n\alpha \right) p_{n+1,0} + \mu_b p_{n+1,2}, n \geq 0 \qquad (1.17)$$

Adding over all possible values n in (1.17), we get

$$\lambda Q_0(1) + \lambda Q_1(1) + \lambda Q_2(1)$$

$$= \mu_b \left( Q_2(1) - p_{0,2} \right) + \mu_b \left( Q_0(1) - p_{0,0} \right) + \alpha \sum_{n=0}^{\infty} (n+1) p_{n+1,0} - \alpha \left( Q_0(1) - p_{0,0} \right), n \geq 0$$

$$E[L_0] = \sum_{n=0}^{\infty} (n+1) p_{n+1,0} z^n \text{ and } Q_2(1) = 1 - Q_0(1) - Q_1(1), E[L_0]$$

in (1.15) and we get

$$\lambda\gamma - \alpha(\mu_b - \lambda) = \left( (\mu_v - \mu_b)\gamma - \alpha(\gamma + \mu_b - \lambda) \right) Q_0(1) - \left( \alpha(\mu_b - \lambda) + \mu_b \gamma \right) Q_1(1)$$

$$-\gamma(\mu_v - \alpha) p_{0,0} - \mu_b(\gamma + \alpha) p_{0,2} - \alpha\beta E[L_1] \qquad (1.18)$$

For z =1 , from equation (1.9), we get

$$p_{0,0} = \frac{\gamma}{(\mu_v - \alpha)} \frac{k_2(1)}{k_1(1)} Q_0(1) \qquad (1.19)$$

$$\lambda\gamma - \alpha(\mu_b - \lambda) = \left( (\mu_v - \mu_b)\gamma - \alpha(\gamma + \mu_b - \lambda) \right) Q_0(1) - \left( \alpha(\mu_b - \lambda) + \mu_b \gamma \right) \frac{\gamma}{\beta} p_{0,0}$$

$$-\frac{\alpha}{\beta} \gamma\lambda p_{0,0} - \frac{\alpha}{\beta} p_{0,0} \lambda\gamma - \gamma(\mu_v - \alpha) p_{0,0} - \mu_b(\gamma + \alpha) \frac{\gamma}{\eta} Q_0(1)$$

$$Q_0(1) = \lambda\gamma - \alpha(\mu_b - \lambda) \left( \left( (\mu_v - \mu_b)\gamma - \alpha(\gamma + \mu_b - \lambda) - \mu_b(\gamma + \alpha) \frac{\gamma}{\eta} \right) - \frac{\gamma}{(\mu_v - \alpha)} \frac{k_2(1)}{k_1(1)} \left( \frac{\lambda\gamma\alpha}{\beta} + \gamma(\mu_v - \alpha) + \left( \alpha(\mu_b - \lambda) + \mu_b \gamma \right) \frac{\gamma}{\beta} \right)^{-1} \right) \qquad (1.20)$$

From equation (1.11), we get

$$E[L_0] = Q_1'(1) = \frac{\gamma\lambda}{\beta^2} p_{0,0}$$

$$Q_1''(1) = \frac{\gamma\lambda^2}{\beta^2} p_{0,0}$$

We find the values $p_{0,1}$ $p_{0,2}$ and $p_{1,0}$ in the equation (1.3), (1.5) and (1.1), we get

$$p_{0,1} = \frac{\gamma}{\lambda+\beta} p_{0,0}$$

$$(\lambda+\eta) p_{0,2} = \mu_b p_{1,2} + \frac{\beta\gamma}{\lambda+\beta} p_{0,0} \left( (\lambda+\eta) \frac{(\mu_v-\alpha) k_1(1)}{\eta k_2(1)} - \frac{\beta\gamma}{\lambda+\beta} p_{0,0} = \mu_b p_{1,2} \right) \tag{1.22}$$

$$p_{1,2} = \frac{1}{\mu_b} \left( (\lambda+\eta) \frac{(\mu_v-\alpha) k_1(1)}{\eta k_2(1)} - \frac{\beta\gamma}{\lambda+\beta} \right)$$

$$p_{1,0} = \frac{1}{\mu_v} \left( (\lambda+\eta) - (\mu_v-\alpha) \frac{k_1(1)}{\eta k_2(1)} p_{0,0} \right)$$

Differentiating (1.12) with respect to z and using L hospital rule as follows.

$$E(L_2) = Q_2'(1) = \frac{\left[ (\mu_b-\lambda)\left(2\gamma Q_0'(1) + 2\beta Q_1'(1) + \gamma Q_0''(1) + \beta Q_1''(1)\right) \right]}{2(\mu_b-\lambda)^2} \tag{1.23}$$

Differentiating equation (1.8) twice and putting z=1, we get.

$$Q_0''(1) = \frac{\lambda-\gamma-2\alpha}{2\alpha} Q_0'(1) + \frac{\lambda}{\gamma} Q_0(1) \tag{1.24}$$

Substituting (1.15), (1.21) and (1.24) in (1.23) , we get $E$[L(2)] The expected number of customers in the system can be computed as $E[L] = E[L_0] + E[L_1] + E[L_2]$ Let W denote the total sojourn time of a customer in the system.

$$E[w] = \frac{1}{\lambda} \left( E[L_0] + E[L_1] + E[L_2] \right)$$

## 1.3 NUMERICAL RESULTS

In this section, we obtain the mean system length and the mean waiting time of an arbitrary customer. In this section, we illustrate the results obtained above numerically and discuss the effect of system parameters on system performance indices. We assume that the service rate $\mu_b$ in a regular busy period equals 4.0, arrival rate $\mu_v$ equals 2.0, at the same time, we assume that working vacation is an exponential distributed random variable with mean $\gamma$ =0.8, $\beta$ =1.0, $\eta$ =0.4. We plot the values of mean queue length E(Q) and mean waiting time E(W) by fixed the arrival rate $\lambda$ and impatient rate $\alpha$ increases.

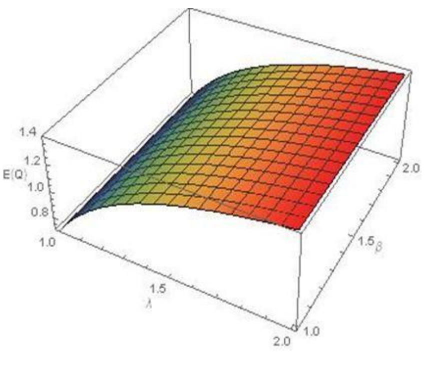


**Figure.1 The changing curve of E(Q)**

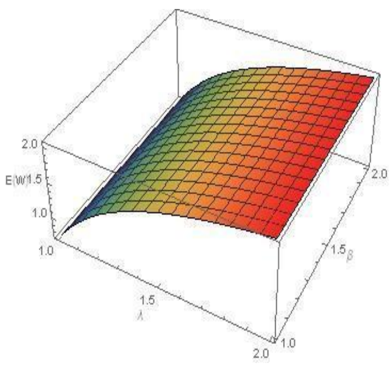


**Figure.2 .The changing curve of E(W)**

## CONCLUSION

**In this paper** we Impatient Customers in an M/M/1 Working Vacation Queue is introduced. This M/M/1 Working Vacation Queue is effectives and easy to understand because of its natural similarity to classical method of Waiting Server and Setup Time. The method of M/M/1 Working Vacation Queue is shows in this paper guarantees the correctness and effectiveness of the working produce of the method. This method of time of a customer and stochastic decomposition of the queue length and waiting time is also easy to apply and understand.

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