Some Properties of Intuitionistic Fuzzy Near Algebras over a Fuzzy Field

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Abstract

The concept of intuitionistic fuzzy near-algebra over a fuzzy field is studied and using this notion we obtained some results on fuzzy near-algebra. We study the "necessity" and "possibility" operators on intuitionistic fuzzy near-algebra over a fuzzy field and study the nature of intuitionistic fuzzy near-algebra.

1. Introductory Concepts

In [2] Brown introduced the concept of Near-algebras. Nanda [7] studied the notion of fuzzy algebras over fuzzy fields and then redefined by Wenxiang Gu and Tu Lu in [4]. Srinivas and Narasimha swamy introduced the concept of a fuzzy near-algebra over a fuzzy field and investigated the properties of this notion in [11]. In addition, many attempts has been made in the field of near-algebra according to the physical situation was studied in the finite dimensional continuous field by Irish [6] and Yamamuro [13]. The applications of near-algebra was studied by Srinivas and Narasimha Swamy [10,11]. In this paper we introduce the concept of intuitionistic fuzzy near-algebra over a fuzzy field and some results on fuzzy near-algebra were obtained.

Definition 1.1 Let X be the collection of objects denoted generally by x. Then a fuzzy set A in X is defined as $A=\{<x,\alpha_A(x)>, x\in X\}$ where $\alpha_A(x)$ is called the membership value of x in A and $0\le\alpha_A(x)\le 1$.

Definition 1.2 A (right) near-algebra Y over a field X is a linear space Y over X on which a multiplication is defined such that (i) Y forms a semi group under multiplication and (ii) multiplication is right distributive over addition and (iii) $(\lambda a)b=\lambda(ab)$ for all $a,b\in Y$ and $\lambda\in X$.

Definition 1.3 A fuzzy subset F of X is called a fuzzy field of X, if it satisfies the following four conditions for all $x,y\in X$:

(i) $\alpha_F(x+y)\ge\alpha_F(x)\wedge\alpha_F(y)$,

(ii) $\alpha_F(-x)\ge\alpha_F(x)$,

(iii) $\alpha_F(xy)\ge\alpha_F(x)\wedge\alpha_F(y)$,

(iv) $\alpha_F(x^{-1})\ge\alpha_F(x)$ for any $x\neq 0$.

Definition 1.4 Let X be a field, F be a fuzzy field of X and Y be a (right) near-algebra over a field X. Let A be the fuzzy subset of Y. Then A is called a fuzzy near-algebra in Y, if the following conditions are satisfied,

(i) $\alpha_A(y_1+y_2)\ge\alpha_A(y_1)\wedge\alpha_A(y_2)$

(ii) $\alpha_A(\lambda y_1)\ge\alpha_F(\lambda)\wedge\alpha_A(y_1)$

(iii) $\alpha_A(y_1y_2)\ge\alpha_A(y_1)\wedge\alpha_A(y_2)$

(iv) $\alpha_F(1)\ge\alpha_A(y_1)$

for all $y_1, y_2 \in Y$ and $\lambda \in X$.

Definition 1.5 An intuitionistic fuzzy set A over X is an object having the form $A = \{< x, \alpha_A(x), \beta_A(x) >, x \in X\}$, where $\alpha_A(x): X \to [0,1]$ and $\beta_A(x): X \to [0,1]$ with the condition $0 \le \alpha_A(x) + \beta_A(x) \le 1$ for all $x \in X$. The numbers $\alpha_A(x)$ and $\beta_A(x)$ denote, respectively, the degree of membership and degree of non membership of the element x in the set A. Obviously when $\beta_A(x) = 1 - \alpha_A(x)$ for every $x \in X$, the set A become a fuzzy set. A intuitionistic fuzzy set $A = \{< x, \alpha_A(x), \beta_A(x) >, x \in X\}$ over X is denoted by $A = (\alpha_A, \beta_A)$.

Definition 1.6 A mapping f of a near-algebra Y_1 onto a near-algebra Y_2 is called a *near-algebra homomorphism*, if it satisfies the following three conditions:

(i) $f(y_1 + y_2) = f(y_1) + f(y_2)$,

(ii) $f(\lambda y_1) = \lambda f(y_1)$,

(iii) $f(y_1 y_2) = f(y_1) f(y_2)$,

for all $y_1, y_2 \in Y, \lambda \in X$.

2. Intuitionistic Fuzzy near-algebra over a Fuzzy field

We now study the concept of intuitionistic fuzzy near-algebra (IFN-algebra) over a fuzzy field and we investigate some properties and theorems related to this new concept.

Definition 2.1 Let X be a field, F be a fuzzy field of X and Y be a (right) near-algebra over a field X. Let $A = (\alpha_A, \beta_A)$ be the intuitionistic fuzzy subset of Y. Then A is called a intuitionistic fuzzy near-algebra in Y over a fuzzy field F, if the following conditions are satisfied,

(i) $\alpha_A(y_1 + y_2) \ge \alpha_A(y_1) \wedge \alpha_A(y_2)$ and $\beta_A(y_1 + y_2) \le \beta_A(y_1) \vee \beta_A(y_2)$

(ii) $\alpha_A(\lambda y_1) \ge \alpha_F(\lambda) \wedge \alpha_A(y_1)$ and $\beta_A(\lambda y_1) \le \alpha_F(\lambda) \vee \beta_A(y_1)$

(iii) $\alpha_A(y_1 y_2) \ge \alpha_A(y_1) \wedge \alpha_A(y_2)$ and $\beta_A(y_1 y_2) \le \beta_A(y_1) \vee \beta_A(y_2)$

(iv) $\alpha_F(1) \ge \alpha_A(y_1)$ and $\beta_F(1) \le \beta_A(y_1)$

for all $y_1, y_2 \in Y$ and $\lambda \in X$. A intuitionistic fuzzy near-algebra A of Y is denoted by (A, Y).

Example 2.2 Let $X = Z_3 = \{0,1,2\}_{\oplus_3 \otimes_3}$ and let $F = (x, \alpha_F)$ be a fuzzy field over X defined by,

$$\alpha_F(x_1) = \begin{cases} 0.2 & \text{if } x_1 = 0 \\ 0.1 & \text{otherwise} \end{cases}$$

For any $x_1, x_2 \in X$, we have $x_1 - x_2 \in X$ and for $x_2 \neq 0$, $x_1 x_2^{-1} \in X$. Thus X is a field. Let $Y = \{0, a, b, c\}$ be a set with operations "+" and "·" as follows,

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

•	0	a	b	c
0	0	0	0	0
a	0	b	0	b
b	0	0	0	0
c	0	b	0	b

Also, if a scalar multiplication on Y is defined by

$$\lambda x = \begin{cases} 0 & \text{if } \lambda = 0 \\ x & \text{otherwise} \end{cases}$$

for every $y_1 \in Y, \lambda \in X$. Clearly Y is a near-algebra over the field X. Let $A = (\alpha_A, \beta_A)$ be a intuitionistic fuzzy subset of Y defined by,

$$\alpha_A(y_1) = \begin{cases} 0.5 & \text{if } x = 0 \\ 0.7 & \text{otherwise} \end{cases} \quad \text{and} \quad \beta_A(y_1) = \begin{cases} 0.03 & \text{if } x = 0 \\ 0.02 & \text{otherwise} \end{cases}$$

Let $\lambda, \mu \in X$ and $y_1, y_2 \in Y$, So that $A = (\alpha_A, \beta_A)$ is a intuitionistic fuzzy near-algebra over the fuzzy field F of X.

Theorem 2.3 Let $A = (\alpha_A, \beta_A)$ be a intuitionistic fuzzy near-algebra of Y. Then $\alpha_A(0) \geq \alpha_A(y_1)$ and $\beta_A(0) \leq \beta_A(y_1)$, for all $y_1 \in Y$.

Proof Since $\alpha_A(0) = \alpha_A(1y_1 - 1y_1 \geq [\alpha_A(1) \wedge \alpha_A(y_1)] \wedge [\alpha_A(-1) \wedge \alpha_A(y_1)] \geq \alpha_A(y_1) \wedge \alpha_A(y_1) \geq \alpha_A(y_1)$. Therefore $\alpha_A(0) \geq \alpha_A(y_1)$. Also $\beta_A(0) = \beta_A(1y_1 - 1y_1) \leq [\beta_A(1) \vee \beta_A(y_1)] \vee [\beta_A(-1) \vee \beta_A(y_1)] \leq \beta_A(y_1) \vee \beta_A(y_1) \leq \beta_A(y_1)$. Therefore $\beta_A(0) \leq \beta_A(y_1)$.

Theorem 2.4 Let F be a fuzzy field of the filed X, Y be the near-algebra over X and A is a intuitionistic fuzzy set of Y. Then (A, Y) is a intuitionistic fuzzy near-algebra over a fuzzy field (F, X) if and only if (i) $\alpha_A(\lambda y_1 + \mu y_2) \geq [\alpha_F(\lambda) \wedge \alpha_A(y_1)] \wedge [\alpha_F(\mu) \wedge \alpha_A(y_2)]$ and $\beta_A(\lambda y_1 + \mu y_2) \leq [\alpha_F(\lambda) \vee \beta_A(y_1)] \vee [\alpha_F(\mu) \vee \beta_A(y_2)]$ (ii) $\alpha_A(y_1 y_2) \geq \alpha_A(y_1) \wedge \alpha_A(y_2)$ and $\beta_A(y_1 y_2) \leq \beta_A(y_1) \vee \beta_A(y_2)$ (iii) $\alpha_F(1) \geq \alpha_A(y_1)$ and $\alpha_F(1) \leq \beta_A(y_1)$ for any $y_1, y_2 \in Y$ and $\lambda, \mu \in X$

Proof Suppose that (A, Y) is a intuitionistic fuzzy near-algebra over a fuzzy field (F, X). Then (i) for any $y_1, y_2 \in Y$ and $\lambda, \mu \in X$, we have $\alpha_A(\lambda y_1 + \mu y_2) \geq \alpha_A(\lambda y_1) \wedge \alpha_A(\mu y_2) \geq [\alpha_F(\lambda) \wedge \alpha_A(y_1)] \wedge [\alpha_F(\mu) \wedge \alpha_A(y_2)]$. Clearly (ii) and (iii) holds directly from the definition of a intuitionistic fuzzy near-algebra of Y.

Conversely, suppose that the three conditions of the hypothesis hold. Then

(i) $\alpha_A(y_1 + y_2) = \alpha_A(1y_1 + 1y_2)$

$\geq \alpha_A(1y_1) \wedge \alpha_A(1y_2)$

$\geq [\alpha_F(1) \wedge \alpha_A(y_1)] \wedge [\alpha_F(1) \wedge \alpha_A(y_2)]$

$\geq [\alpha_A(y_1) \wedge \alpha_A(y_1)] \wedge [\alpha_A(y_2) \wedge \alpha_A(y_2)]$

$$\geq \alpha_A(y_1) \wedge \alpha_A(y_2)]$$

and $\beta_A(y_1 + y_2) = \beta_A(1y_1 + 1y_2)$

$$\leq [\alpha_F(1) \vee \beta_A(y_1)] \vee [\alpha_F(1) \vee \beta_A(y_2)]$$

$$\leq [\beta_A(y_1) \vee \beta_A(y_1)] \vee [\beta_A(y_2) \vee \beta_A(y_2)]$$

$$\leq \beta_A(y_1) \vee \beta_A(y_2)$$

for every $y_1, y_2 \in Y$ and $\lambda, \mu \in X$. By hypothesis, the remaining two conditions of the definition of a intuitionistic fuzzy near-algebra of Y holds directly. Hence (A,Y) is a intuitionistic fuzzy near-algebra of Y over a fuzzy field F.

Theorem 2.5 Suppose (A,Y) is a intuitionistic fuzzy near-algebra of Y over a fuzzy field F. Then the following conditions holds for any $y_1, y_2 \in Y$ and $\lambda, \mu \in X$

(i) $\alpha_A(y_1 - y_2) \geq \alpha_A(y_1) \wedge \alpha_A(y_2)$ and $\beta_A(y_1 - y_2) \leq \beta_A(y_1) \vee \beta_A(y_2)$

(ii) $\alpha_A(y_1) \leq \alpha_A(y_2)$ implies $\alpha_A(y_1 + y_2) \wedge \alpha_A(y_2) = \alpha_A(y_1)$, $\alpha_A(y_1 y_2) \wedge \alpha_A(y_2) = \alpha_A(y_1)$ and

$\beta_A(y_1) \geq \beta_A(y_2)$ implies $\beta_A(y_1 + y_2) \vee \beta_A(y_2) = \beta_A(y_1)$, $\beta_A(y_1 y_2) \vee \beta_A(y_2) = \beta_A(y_1)$

(iii) $\alpha_A(y_1) \leq \alpha_F(\lambda)$ implies $\alpha_A(\lambda y_1) \wedge \alpha_A(y_1) = \alpha_A(y_1)$ and

$\beta_A(y_1) \geq \alpha_F(\lambda)$ implies $\beta_A(\lambda y_1) \vee \beta_A(y_1) = \beta_A(y_1)$.

Proof (i) (i) $\alpha_A(y_1 - y_2) = \alpha_A(1y_1 - 1y_2)$

$$\geq \alpha_A(1y_1 + (-1)y_2)$$

$$\geq \alpha_A(1y_1) \wedge \alpha_A[(-1)y_2]$$

$$\geq [\alpha_F(1) \wedge \alpha_A(y_1)] \wedge [\alpha_F(-1) \wedge \alpha_A(y_2)]$$

$$\geq [\alpha_A(y_1) \wedge \alpha_A(y_1)] \wedge [\alpha_A(y_2) \wedge \alpha_A(y_2)]$$

$$\geq \alpha_A(y_1) \wedge \alpha_A(y_2)]$$

and $\beta_A(y_1 - y_2) = \beta_A(1y_1 + [(-1)y_2]$

$$\leq [\alpha_F(1) \vee \beta_A(y_1)] \vee [\alpha_F(-1) \vee \beta_A(y_2)]$$

$$\leq [\beta_A(y_1) \vee \beta_A(y_1)] \vee [\beta_A(y_2) \vee \beta_A(y_2)]$$

$$\leq \beta_A(y_1) \vee \beta_A(y_2)$$

(ii) If $\alpha_A(y_1) \leq \alpha_A(y_2)$ and by the definition of intuionistic fuzzy Near algebra, we have

$$\alpha_A(y_1 + y_2) \wedge \alpha_A(y_2) = [\alpha_A(y_1) \wedge \alpha_A(y_2)] \wedge \alpha_A(y_2)$$

$$\geq [\alpha_A(y_1) \wedge \alpha_A(y_2)] \wedge \alpha_A(y_1), \text{if } \alpha_A(y_1) \leq \alpha_A(y_2)$$

$$\geq [\alpha_A(y_1) \wedge \alpha_A(y_1)] \wedge \alpha_A(y_2)$$

$$\geq \alpha_A(y_1) \wedge \alpha_A(y_2)$$

$$\geq \alpha_A(y_1) \wedge \alpha_A(y_1) \text{ if } \alpha_A(y_1) \leq \alpha_A(y_2)$$

$=\alpha_A(y_1)$

Therefore $\alpha_A(y_1+y_2)\wedge\alpha_A(y_2)=\alpha_A(y_1)$

Also, $\alpha_A(y_1y_2)\wedge\alpha_A(y_2)=[\alpha_A(y_1)\wedge\alpha_A(y_2)]\wedge\alpha_A(y_2)$

$$\alpha_A(y_1y_2)\wedge\alpha_A(y_2)=[\alpha_A(y_1)\wedge\alpha_A(y_2)\wedge\alpha_A(y_2)$$

$$\geq[\alpha_A(y_1)\wedge\alpha_A(y_2)]\wedge\alpha_A(y_1),\text{if } \alpha_A(y_1)\leq\alpha_A(y_2)$$

$$\geq[\alpha_A(y_1)\wedge\alpha_A(y_1)]\wedge\alpha_A(y_2)$$

$$\geq\alpha_A(y_1)\wedge\alpha_A(y_2)$$

$$\geq\alpha_A(y_1)\wedge\alpha_A(y_1)\text{ if } \alpha_A(y_1)\leq\alpha_A(y_2)$$

$$=\alpha_A(y_1)$$

Similarly we prove $\beta_A(y_1)\geq\beta_A(y_2)$ implies $\beta_A(y_1+y_2)\vee\beta_A(y_2)=\beta_A(y_1)$, $\beta_A(y_1y_2)\vee\beta_A(y_2)=\beta_A(y_1)$.

(iii) If $\alpha_A(y_1)\leq\alpha_F(\lambda)$ and by the definition of by the definition of intuionistic fuzzy Near algebra, we have

$$\alpha_A(\lambda y_1)\wedge\alpha_A(y_1)\geq[\alpha_F(\lambda)\wedge\alpha_A(y_1)]\wedge\alpha_A(y_1)$$

$$\geq\alpha_F(\lambda)\wedge[\alpha_A(y_1)]\wedge\alpha_A(y_1)],\text{if } \alpha_A(y_1)\leq\alpha_F(\lambda)$$

$$\geq\alpha_A(y_1)\wedge\alpha_A(y_1)$$

$$=\alpha_A(y_1)$$

Therefore $\alpha_A(\lambda y_1)\wedge\alpha_A(y_1)=\alpha_A(y_1)$

Similarly we can prove $\beta_A(y_1)\geq\alpha_F(\lambda)$ implies $\beta_A(\lambda y_1)\vee\beta_A(y_1)=\beta_A(y_1)$.

Theorem 2.6 If A and B are two intuitionistic fuzzy near-algebras of Y over a fuzzy field F, then $A+B$ and λA are also intuitionistic fuzzy near-algebra of Y over a fuzzy field F.

Proof Since $\alpha_A(y_1+y_2)\geq\alpha_A(y_1)\wedge\alpha_A(y_2)$, then

(i) $$\alpha_{A+B}(y_1+y_2)\geq\alpha_A(y_1+y_2)\wedge\alpha_B(y_1+y_2)$$

$$\geq[\alpha_A(y_1)\wedge\alpha_A(y_2)]\wedge[\alpha_B(y_1)\wedge\alpha_B(y_2)]$$

$$\geq[\alpha_A(y_1)\wedge\alpha_B(y_1)]\wedge[\alpha_A(y_2)\wedge\alpha_B(y_2)]$$

$$\geq\alpha_{A+B}(y_1)\wedge\alpha_{A+B}(y_2)$$

Similarly, $\beta_{A+B}(y_1+y_2)\leq\beta_{A+B}(y_1)\vee\beta_{A+B}(y_2)$

(ii) $$\alpha_{A+B}(\lambda y_1)\geq\alpha_A(\lambda y_1)\wedge\alpha_B(\lambda y_1)$$

$$\geq[\alpha_F(\lambda)\wedge\alpha_A(y_1)]\wedge[\alpha_F(\lambda)\wedge\alpha_B(y_1)]$$

$$\geq\alpha_F(\lambda)\wedge[\alpha_A(y_1)\wedge\alpha_B(y_1)]$$

$$\geq\alpha_F(\lambda)\wedge\alpha_{A+B}(y_1)$$

Similarly, $\beta_{A+B}(\lambda y_1) \le \alpha_F(\lambda) \vee \beta_{A+B}(y_1)$

(iii) $\alpha_{A+B}(y_1 y_2) \ge \alpha_A(y_1 y_2) \wedge \alpha_B(y_1 y_2)$

$$\ge [\alpha_A(y_1) \wedge \alpha_A(y_2)] \wedge [\alpha_B(y_1) \wedge \alpha_B(y_2)]$$

$$\ge [\alpha_A(y_1) \wedge \alpha_B(y_1)] \wedge [\alpha_A(y_2) \wedge \alpha_B(y_2)]$$

$$\ge \alpha_{A+B}(y_1) \wedge \alpha_{A+B}(y_2)$$

Similarly, $\beta_{A+B}(y_1 y_2) \le \beta_{A+B}(y_1) \vee \beta_{A+B}(y_2)$

(iv) Since $\alpha_F(1) \ge \alpha_A(y_1)$ and $\alpha_F(1) \ge \alpha_B(y_1)$

$$\alpha_F(1) \ge \alpha_A(y_1) \wedge \alpha_B(y_1) = \alpha_{A+B}(y_1)$$

Similarly, $\alpha_F(1) \le \beta_{A+B}(y_1)$

Therefore $A+B$ is a intuitionistic fuzzy near-algebra of Y over a fuzzy field F.

Now, to show that λA are also intuitionistic fuzzy near-algebra of Y over a fuzzy field F.

(i) $\alpha_{\lambda A}(y_1 + y_2) \ge \alpha_{\lambda A}(y_1) \wedge \alpha_{\lambda A}(y_2)$

$$\ge [\alpha_\lambda(y_1) \wedge \alpha_A(y_1)] \wedge [\alpha_\lambda(y_2) \wedge \alpha_A(y_2)]$$

$$\ge [\alpha_\lambda(y_1) \wedge \alpha_\lambda(y_2)] \wedge [\alpha_A(y_1) \wedge \alpha_A(y_2)]$$

$$\ge \alpha_\lambda(y_1 + y_2) \wedge \alpha_A(y_1 + y_2)$$

Similarly, $\beta_{\lambda A}(y_1 + y_2) \le \beta_\lambda(y_1 + y_2) \vee \beta_A(y_1 + y_2)$

(ii) $\alpha_{\lambda A}(\lambda y_1) \ge \alpha_\lambda(\lambda y_1) \wedge \alpha_A(\lambda y_1)$

$$\ge [\alpha_F(\lambda) \wedge \alpha_\lambda(y_1)] \wedge [\alpha_F(\lambda) \wedge \alpha_A(y_1)]$$

$$\ge [\alpha_F(\lambda) \wedge \alpha_F(\lambda)] \wedge [\alpha_\lambda(y_1) \wedge \alpha_A(y_1)]$$

$$\ge \alpha_F(\lambda) \wedge \alpha_{\lambda A}(y_1)$$

Similarly, $\beta_{\lambda A}(\lambda y_1) \le \alpha_F(\lambda) \vee \beta_{\lambda A}(y_1)$

(iii) $\alpha_{\lambda A}(y_1 y_2) \ge \alpha_\lambda(y_1 y_2) \wedge \alpha_A(y_1 y_2)$

$$\ge [\alpha_\lambda(y_1) \wedge \alpha_\lambda(y_2)] \wedge [\alpha_A(y_1) \wedge \alpha_A(y_2)]$$

$$\ge [\alpha_\lambda(y_1) \wedge \alpha_A(y_1)] \wedge [\alpha_\lambda(y_2) \wedge \alpha_B(y_2)]$$

$$\ge \alpha_{\lambda A}(y_1) \wedge \alpha_{\lambda A}(y_2)$$

Similarly, $\beta_{\lambda A}(y_1 y_2) \le \beta_{\lambda A}(y_1) \vee \beta_{\lambda A}(y_2)$

(iv) Since $\alpha_F(1) \ge \alpha_A(y_1)$ and $\alpha_F(1) \ge \alpha_B(y_1)$

$$\alpha_F(1) \ge \alpha_\lambda(y_1) \wedge \alpha_A(y_1) \ge \alpha_{\lambda A}(y_1)$$

Similarly, $\alpha_F(1) \le \beta_{\lambda A}(y_1)$

Therefore λA are also intuitionistic fuzzy near-algebra of Y over a fuzzy field F.

Theorem 2.7 Intersection of family of intuitionistic fuzzy near-algebras is a intuitionistic fuzzy near-algebra.

Proof Let $\{A_i = (\alpha_i, \beta_i)\}_{i\in\Lambda}$ be a family of intuitionistic fuzzy near-algebras of Y over fuzzy field F of X. Let $\alpha_A(x) = \bigcap_{i\in\Lambda} \alpha_i(x) = \inf_{i\in\Lambda} \alpha_i(x) = \wedge_{i\in\Lambda} \alpha_i(x)$. for any $y_1, y_2 \in Y, \lambda, \mu \in X$, we have

(i) $$\alpha_A(\lambda y_1 + \mu y_2) = \inf_{i\in\Lambda} \alpha_{A_i}(\lambda y_1 + \mu y_2)$$

$$\geq \inf_{i\in\Lambda}[\alpha_{A_i}(\lambda y_1) \wedge \alpha_{A_i}(\mu y_2)]$$
$$\geq \inf_{i\in\Lambda}[[\alpha_F(\lambda) \wedge \alpha_{A_i}(y_1)] \wedge [\alpha_F(\mu) \wedge \alpha_{A_i}(y_2)]]$$
$$\geq \inf_{i\in\Lambda}[[\alpha_F(\lambda) \wedge \alpha_F(\mu)] \wedge [\alpha_{A_i}(y_1) \wedge \alpha_{A_i}(y_2)]]$$
$$\geq \inf_{i\in\Lambda}[\alpha_F(\lambda\mu) \wedge \alpha_{A_i}(y_1 y_2)]$$
$$\geq [\inf_{i\in\Lambda} \alpha_F(\lambda\mu)] \wedge [\inf_{i\in\Lambda} \alpha_{A_i}(y_1 y_2)]$$
$$\geq \alpha_F(\lambda\mu)] \wedge \alpha_A(y_1 y_2)]$$

Similarly, $\beta_A(\lambda y_1 + \mu y_2) \leq \beta_F(\lambda\mu) \vee \beta_A(y_1 y_2)$

(ii) $$\alpha_A(y_1 y_2) \geq \inf_{i\in\Lambda}[\alpha_{A_i}(y_1 y_2)]$$

$$\geq \inf_{i\in\Lambda}[\alpha_{A_i}(y_1) \wedge \alpha_{A_i}(y_2)]$$
$$\geq [\inf_{i\in\Lambda} \alpha_{A_i}(y_1)] \wedge [\inf_{i\in\Lambda} \alpha_{A_i}(y_2)]$$
$$\geq \alpha_A(y_1) \wedge \alpha_A(y_2)]$$

(iii) Since each A_i ia intuitionistic fuzzy near-algebra, we have

$$\alpha_F(1) \geq \alpha_{A_i}(y) \geq \inf_{i\in\Lambda} \alpha_{A_i}(y) = \alpha_A(y) \text{ and } \beta_F(1) \geq \beta_{A_i}(y) \geq \sup_{i\in\Lambda} \beta_{A_i}(y) = \beta_A(y).$$

Therefore intersection of family of intuitionistic fuzzy near-algebras is a intuitionistic fuzzy near-algebra.

Theorem 2.8 If $\{A_i = (\alpha_i, \beta_i)\}_{i\in\Lambda}$ be a family of intuitionistic fuzzy near-algebras of Y over fuzzy field F of X, the so is $\vee_{i\in\Lambda} A_i$.

Proof Let $\{A_i = (\alpha_i, \beta_i)\}_{i\in\Lambda}$ be a family of intuitionistic fuzzy near-algebras of Y over fuzzy field F of X. Let $\alpha_A(x) = \bigcap_{i\in\Lambda} \alpha_i(x) = \inf_{i\in\Lambda} \alpha_i(x)$. for any $y_1, y_2 \in Y, \lambda \in X$, we have

(i) $$\vee_{i\in\Lambda} \alpha_{A_i}(y_1 + y_2) = \sup_{i\in\Lambda}[\alpha_{A_i}(y_1 + y_2)]$$

$$\geq \sup_{i\in\Lambda}[\alpha_{A_i}(y_1) \wedge \alpha_{A_i}(y_2)]$$
$$\geq [\sup_{i\in\Lambda} \alpha_{A_i}(y_1)] \wedge [\sup_{i\in\Lambda} \alpha_{A_i}(y_2)]$$
$$\geq [\vee_{i\in\Lambda} \alpha_{A_i}(y_1)] \wedge [\vee_{i\in\Lambda} \alpha_{A_i}(y_2)]$$

Similarly, $\vee_{i\in\Lambda} \beta_{A_i}(y_1 + y_2) = \sup_{i\in\Lambda} \beta_{A_i}(y_1 + y_2)$

(ii) $$\vee_{i\in\Lambda} \alpha_{A_i}(\lambda y_1) = \sup_{i\in\Lambda}[\alpha_{A_i}(\lambda y_1)]$$

$$\geq \sup_{i\in\Lambda}[\alpha_F(\lambda) \wedge \alpha_{A_i}(y_1)]$$
$$\geq [\sup_{i\in\Lambda} \alpha_F(\lambda)] \wedge [\sup_{i\in\Lambda} \alpha_{A_i}(y_1)]$$
$$\geq [\vee_{i\in\Lambda} \alpha_F(\lambda)] \wedge [\vee_{i\in\Lambda} \alpha_{A_i}(y_1)]$$

Similarly, $\vee_{i\in\Lambda} \beta_{A_i}(\lambda y_1) = [\vee_{i\in\Lambda} \beta_F(\lambda)] \vee [\vee_{i\in\Lambda} \beta_{A_i}(y_1)]$

(iii) $\vee_{i\in\Lambda} \alpha_{A_i}(y_1 y_2) = \sup_{i\in\Lambda}[\alpha_{A_i}(y_1 y_2)]$

$$\geq \sup_{i\in\Lambda}[\alpha_{A_i}(y_1) \wedge \alpha_{A_i}(y_2)]$$
$$\geq [\sup_{i\in\Lambda} \alpha_{A_i}(y_1)] \wedge [\sup_{i\in\Lambda} \alpha_{A_i}(y_2)]$$
$$= [\vee_{i\in\Lambda} \alpha_{A_i}(y_1)] \wedge [\vee_{i\in\Lambda} \alpha_{A_i}(y_2)]$$

Similarly, $\vee_{i\in\Lambda} \beta_{A_i}(y_1 y_2) = [\vee_{i\in\Lambda} \alpha_{A_i}(y_1)] \vee [\vee_{i\in\Lambda} \alpha_{A_i}(y_2)]$

(iv) Since each A_i ia intuitionistic fuzzy near-algebra, we have

$$\alpha_F(1) \geq \sup_{i\in\Lambda} \alpha_{A_i}(y_1) = \vee_{i\in\Lambda} \alpha_{A_i}(y_1) \text{ and } \beta_F(1) \geq \sup_{i\in\Lambda} \beta_{A_i}(y_1) = \vee_{i\in\Lambda} \beta_{A_i}(y_1)$$

Therefore $\vee_{i\in\Lambda} A_i$ intuitionistic fuzzy near-algebra of Y over fuzzy field .

Theorem 2.9 Let Y and Z be two near-algebras over a field X. Let $f: Y \to Z$ be an onto near-algebra homomorphism. If $A = (\alpha, \beta)$ and $B = (\alpha, \beta)$ are two intuitionistic fuzzy near-algebras of Z and Y over fuzzy field F of X, then $f^{-1}(A)$ and $f(B)$ are two intuitionistic fuzzy near-algebras in Y and Z over the fuzzy field $F = (x, \alpha)$.

Proof For any $y_1, y_2 \in Y, \lambda, \mu \in X$, we have

(i) $\alpha_{f^{-1}(A)}(\lambda y_1 + \mu y_2) = \alpha_A[f(\lambda y_1 + \mu y_2)]$

$$= \alpha_A[\lambda f(y_1) + \mu f(y_2)]$$
$$\geq \alpha_A(\lambda f(y_1)) \wedge \alpha_A(\mu f(y_2))]$$
$$\geq [\alpha_F(\lambda) \wedge \alpha_A(f(y_1))] \wedge [\alpha_F(\mu) \wedge \alpha_A(f(y_2))]$$
$$\geq [\alpha_F(\lambda) \wedge f^{-1}(\alpha_A)(y_1)] \wedge [\alpha_F(\mu) \wedge f^{-1}(\alpha_A)(y_2)]$$
$$\geq \alpha_F(\lambda\mu) \wedge f^{-1}(\alpha_A(y_1 y_2))$$

Similarly, $\beta_{f^{-1}(A)}(\lambda y_1 + \mu y_2) \leq \beta_F(\lambda\mu) \vee f^{-1}(\beta_A(y_1 y_2))$

(ii) $\alpha_{f^{-1}(A)}(y_1 y_2) \geq \alpha_A(f(y_1 y_2))$

$$\geq \alpha_A(f(y_1) f(y_2))$$
$$\geq \alpha_A(f(y_1)) \wedge \alpha_A(f(y_2))$$
$$\geq f^{-1}(\alpha_A)(y_1) \wedge f^{-1}(\alpha_A)(y_2)$$

(iii) Since $A = (\alpha, \beta)$ ia intuitionistic fuzzy near-algebra, we have

$$\alpha_F(1) \geq \alpha_{f^{-1}(A)}(y_1) = \alpha_A(f(y_1)) = f^{-1}(\alpha_A)(y_1) \text{ and } \beta_F(1) \leq \beta_{f^{-1}(A)}(y_1) = \beta_A(f(y_1)) = f^{-1}(\beta_A)(y_1).$$

Therefore $f^{-1}(A)$ is a intuitionistic fuzzy near-algebra of Y over a fuzzy field F. Similarly, we can prove $f(B)$ is intuitionistic fuzzy near-algebras in Z over the fuzzy field F.

Theorem 2.10 Let Y be a near-algebra. Then the fuzzy subset $A = (\alpha, \beta)$ of Y is intuitionistic fuzzy near-algebra over a fuzzy field of F if and only if A^C is a intuitionistic fuzzy near-algebra of Y over the fuzzy field of F.

Proof Let $A=(\alpha,\beta)$ be a intuitionistic fuzzy near-algebra of Y. Then for any $y_1, y_2 \in Y$, we have

(i) $$\alpha_{A^C}(y_1+y_2)=1-\alpha_A(y_1+y_2)$$
$$\geq 1-[\alpha_A(y_1)\wedge\alpha_A(y_2)]$$
$$=(1-\alpha_A(y_1))\wedge(1-\alpha_A(y_2))$$
$$=\alpha_{A^C}(y_1)\wedge\alpha_{A^C}(y_2)$$

Similarly, $\beta_{A^C}(y_1+y_2)=\beta_{A^C}(y_1)\vee\beta_{A^C}(y_2)$

(ii) $$\alpha_{A^C}(y_1y_2)=1-\alpha_A(y_1y_2)$$
$$\geq 1-[\alpha_A(y_1)\wedge\alpha_A(y_2)]$$
$$=(1-\alpha_A(y_1))\wedge(1-\alpha_A(y_2))$$
$$=\alpha_{A^C}(y_1)\wedge\alpha_{A^C}(y_2)$$

Similarly, $\beta_{A^C}(y_1y_2)=\beta_{A^C}(y_1)\vee\beta_{A^C}(y_2)$

(iii) $$\alpha_{A^C}(\lambda y_1)=1-\alpha_A(\lambda y_1)$$
$$\geq 1-[\alpha_F(\lambda)\wedge\alpha_A(y_1)]$$
$$=(1-\alpha_F(\lambda))\wedge(1-\alpha_A(y_1))$$
$$=\alpha_{F^C}(\lambda)\wedge\alpha_{A^C}(y_1)$$

Similarly, $\beta_{A^C}(\lambda y_1)=\alpha_{F^C}(\lambda)\vee\beta_{A^C}(y_1)$

(iv) $\alpha_{F^C}(1)\geq 1-\alpha_F(1)\geq 1-\alpha_A(y_1)=\alpha_{F^C}(y_1)$ and $\beta_{F^C}(1)\leq 1-\beta_F(1)=1-\beta_A(y_1)=\beta_{A^C}(y_1)$.

Thus A^C is a intuitionistic fuzzy near-algebra of Y over the fuzzy field of F.

Conversely, Suppose A^C is a intuitionistic fuzzy near-algebra of Y over the fuzzy field of F. Then

(i) $$\alpha_A(y_1+y_2)=1-\alpha_{A^C}(y_1+y_2)$$
$$\geq 1-[\alpha_{A^C}(y_1)\wedge\alpha_{A^C}(y_2)]$$
$$=(1-\alpha_{A^C}(y_1))\wedge(1-\alpha_{A^C}(y_2))$$
$$=\alpha_A(y_1)\wedge\alpha_A(y_2)$$

Similarly, $\beta_A(y_1+y_2)=\beta_A(y_1)\vee\beta_A(y_2)$

(ii) $$\alpha_A(y_1y_2)=1-\alpha_{A^C}(y_1y_2)$$
$$\geq 1-[\alpha_{A^C}(y_1)\wedge\alpha_{A^C}(y_2)]$$
$$=(1-\alpha_{A^C}(y_1))\wedge(1-\alpha_{A^C}(y_2))$$
$$=\alpha_A(y_1)\wedge\alpha_A(y_2)$$

Similarly, $\beta_A(y_1y_2)=\beta_A(y_1)\vee\beta_A(y_2)$

(iii) $$\alpha_A(\lambda y_1)=1-\alpha_{A^C}(\lambda y_1)$$
$$\geq 1-[\alpha_{F^C}(\lambda)\wedge\alpha_{A^C}(y_1)]$$

$$= (1-\alpha_{F^C}(\lambda)) \wedge (1-\alpha_{A^C}(y_1))$$

$$= \alpha_F(\lambda) \wedge \alpha_A(y_1)$$

Similarly, $\beta_A(\lambda y_1) = \alpha_F(\lambda) \vee \beta_A(y_1)$

(iv) $\alpha_F(1) \geq 1-\alpha_{F^C}(1) \geq 1-\alpha_{A^C}(y_1) = \alpha_F(y_1)$ and $\beta_F(1) \leq 1-\beta_{F^C}(1) = 1-\beta_{A^C}(y_1) = \beta_A(y_1)$.

Therefore $A = (\alpha, \beta)$ of Y is intuitionistic fuzzy near-algebra over a fuzzy field of F.

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