

# $T_g^m$ -SPACES

S.SHEERIN FARZANA

Research Scholar Madurai Kamaraj University ,Madurai -21,Tamilnadu,Madurai

K.S.ABDUL GANI

Research Scholar Madurai Kamaraj University ,Madurai -21,Tamilnadu,Madurai.

## 1.1 INTRODUCTION

Levine introduced the notion of  $T_{1/2}$ -spaces which properly lie between  $T_1$ -spaces and  $T_0$ -spaces. Many authors studied properties of  $T_{1/2}$ -spaces: Dunham [9], Arenas et al. [4] etc. In this chapter, we introduce the notions called  $T_g^m$ -spaces,  $gT_g^m$ -spaces and  $\alpha T_g^m$ -spaces and obtain their properties and characterizations.

## 1.2 PRELIMINARIES

Throughout this thesis  $(X, \tau)$  (or  $X$ ) represent topological space on which no separation axioms are assumed unless otherwise mentioned. For a subset  $A$  of a space  $(X, \tau)$ ,  $cl(A)$ ,  $int(A)$  and  $A^c$  denote the closure of  $A$ , the interior of  $A$  and the complement of  $A$  respectively.

We recall the following definitions which are useful in the sequel.

### Definition 1.2.1

A subset  $A$  of a space  $(X, \tau)$  is called:

- (i) semi-open set [11] if  $A \subseteq cl(int(A))$ ;
- (ii) preopen set [13] if  $A \subseteq int(cl(A))$ ;
- (iii)  $\alpha$ -open set [14] if  $A \subseteq int(cl(int(A)))$ ;
- (iv)  $\beta$ -open set [1] (= semi-preopen [3]) if  $A \subseteq cl(int(cl(A)))$ .

The complements of the above mentioned open sets are called their respective closed sets.

The preclosure [15] (resp. semi-closure [6],  $\alpha$ -closure [7], semi-pre-closure [2]) of a subset  $A$  of  $X$ , denoted by  $\text{pcl}(A)$  (resp.  $\text{scl}(A)$ ,  $\alpha \text{cl}(A)$ ,  $\text{spcl}(A)$ ), is defined to be the intersection of all preclosed (resp. semi-closed,  $\alpha$ -closed, semi-preclosed) sets of  $(X, \tau)$  containing  $A$ . It is known that  $\text{pcl}(A)$  (resp.  $\text{scl}(A)$ ,  $\alpha \text{cl}(A)$ ,  $\text{spcl}(A)$ ) is a preclosed (resp. semi-closed,  $\alpha$ -closed, semi-preclosed) set. For any subset  $A$  of an arbitrarily chosen topological space, the semi-interior [6] (resp.  $\alpha$ -interior [7], preinterior [15]) of  $A$ , denoted by  $\text{sint}(A)$  (resp.  $\alpha \text{int}(A)$ ,  $\text{pint}(A)$ ), is defined to be the union of all semi-open (resp.  $\alpha$ -open, preopen) sets of  $(X, \tau)$  contained in  $A$ .

### Definition 1.2.2

A subset  $A$  of a space  $(X, \tau)$  is called:

- (i) a generalized closed (briefly  $g$ -closed) set [10] if  $\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $(X, \tau)$ . The complement of  $g$ -closed set is called  $g$ -open set;
- (ii) a generalized semi-closed (briefly  $gs$ -closed) set [5] if  $\text{scl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $(X, \tau)$ . The complement of  $gs$ -closed set is called  $gs$ -open set;
- (iii) an  $\alpha$ -generalized closed (briefly  $\alpha g$ -closed) set [12] if  $\alpha \text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $(X, \tau)$ . The complement of  $\alpha g$ -closed set is called  $\alpha g$ -open set;
- (iv) a generalized semi-preclosed (briefly  $gsp$ -closed) set [15] if  $\text{spcl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $(X, \tau)$ . The complement of  $gsp$ -closed set is called  $gsp$ -open set;
- (v) a  $\hat{g}$ -closed set [17] (=  $\omega$ -closed [16]) if  $\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is semi-open in  $(X, \tau)$ . The complement of  $\hat{g}$ -closed set is called  $\hat{g}$ -open set;
- (vi) a  $g'''$ -closed set [3] if  $\text{cl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $gs$ -open in  $(X, \tau)$ . The complement of  $g'''$ -closed set is called  $g'''$ -open set;
- (vii) a  $g^*$ -preclosed (briefly  $g^*p$ -closed) set [18] if  $\text{pcl}(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $g$ -open in  $(X, \tau)$ . The complement of  $g^*p$ -closed set is called  $g^*p$ -open set.

The collection of all  $g^m$ -closed (resp.  $\omega$ -closed,  $\alpha$  g-closed, gsp-closed, gs-closed,  $\alpha$ -closed,  $g^*p$ -closed) sets is denoted by  $G^m C(X)$  (resp.  $\omega C(X)$ ,  $\alpha G C(X)$ ,  $GSP C(X)$ ,  $GS C(X)$ ,  $\alpha C(X)$ ,  $G^*PC(X)$ ).

The collection of all  $g^m$ -open (resp.  $\omega$ -open,  $\alpha$  g-open, gsp-open, gs-open,  $\alpha$ -open,  $g^*p$ -open) sets is denoted by  $G^m O(X)$  (resp.  $\omega O(X)$ ,  $\alpha G O(X)$ ,  $GSP O(X)$ ,  $GS O(X)$ ,  $\alpha O(X)$ ,  $G^*PO(X)$ ).

We denote the power set of X by P(X).

### Definition 1.2.3

A space  $(X, \tau)$  is called:

- (i)  $T_{1/2}$ -space [10] if every  $g$ -closed set is closed.
- (ii)  $T_b$ -space [8] if every gs-closed set is closed.
- (iii)  $\alpha T_b$ -space [7] if every  $\alpha$  g-closed set is closed.
- (iv)  $T_\omega$ -space [16] if every  $\omega$ -closed set is closed.
- (v)  $T_{p^*}$ -space [18] if every  $g^*p$ -closed set is closed.
- (vi)  ${}^*T_p$ -space [18] if every gsp-closed set is  $g^*p$ -closed.
- (vii)  $\alpha T_d$ -space [7] if every  $\alpha$  g-closed set is  $g$ -closed.
- (viii)  $\alpha$ -space [14] if every  $\alpha$ -closed set is closed.

### Definition 1.2.4 [45]

Let  $(X, \tau)$  be a topological space and  $A \subseteq X$ . We define the gs-closure of A (briefly  $gs-cl(A)$ ) to be the intersection of all gs-closed sets containing A.

### Remark 1.2.5 [4]

For a topological space X, the followings hold:

- (i) Every closed set is  $g^m$ -closed but not conversely.
- (ii) Every  $g^m$ -closed set is  $\omega$ -closed but not conversely.
- (iii) Every  $g^m$ -closed set is  $g$ -closed but not conversely.

- (iv) Every  $g'''$ -closed set is  $\alpha$   $g$ -closed but not conversely.
- (v) Every  $g'''$ -closed set is  $gs$ -closed but not conversely.
- (vi) Every  $g'''$ -closed set is  $gsp$ -closed but not conversely.

### Theorem 1.2.6 [ 4 ]

A set  $A$  is  $g'''$ -closed in  $X$  if and only if  $cl(A) - A$  contains no nonempty  $gs$ -closed set.

## 1.3 PROPERTIES OF $T_{g'''}$ -SPACES

We introduce the following definition:

### Definition 1.3.1

A space  $(X, \tau)$  is called a  $T_{g'''}$ -space if every  $g'''$ -closed set in it is closed.

### Example 1.3.2

Let  $X = \{a, b, c\}$  with  $\tau = \{\phi, \{b\}, X\}$ . Then  $G'''C(X) = \{\phi, \{a, c\}, X\}$ . Thus  $(X, \tau)$  is a  $T_{g'''}$ -space.

### Example 1.3.3

Let  $X = \{a, b, c\}$  with  $\tau = \{\phi, \{a, c\}, X\}$ . Then  $G'''C(X) = \{\phi, \{b\}, \{a, b\}, \{b, c\}, X\}$ . Thus  $(X, \tau)$  is not a  $T_{g'''}$ -space.

### Proposition 1.3.4

Every  $T_{1/2}$ -space is  $T_{g'''}$ -space but not conversely.

### Proof

Follows from Remark 1.2.5 (iii).

The converse of Proposition 1.3.4 need not be true as seen from the following example.

**Example 1.3.5**

Let  $X$  and  $\tau$  be as in the Example 1.3.2,  $GC(X) = \{\emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$ . Thus  $(X, \tau)$  is not a  $T_{1/2}$ -space.

**Proposition 1.3.6**

Every  $T_\omega$ -space is  $T_{g''}$ -space but not conversely.

**Proof**

Follows from Remark 1.2.5 (ii).

The converse of Proposition 1.3.6 need not be true as seen from the following example.

**Example 1.3.7**

Let  $X = \{a, b, c\}$  with  $\tau = \{\emptyset, \{a\}, \{b, c\}, X\}$ . Then  $\omega C(X) = P(X)$  and  $G''C(X) = \{\emptyset, \{a\}, \{b, c\}, X\}$ . Thus  $(X, \tau)$  is  $T_{g''}$ -space but not a  $T_\omega$ -space.

**Proposition 1.3.8**

Every  $\alpha T_b$ -space is  $T_{g''}$ -space but not conversely.

**Proof**

Follows from Remark 1.2.5 (iv).

The converse of Proposition 1.3.8 need not be true as seen from the following example.

**Example 1.3.9**

Let  $X$  and  $\tau$  be as in the Example 1.3.2. Then  $\alpha GC(X) = \{\emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$ . Thus  $(X, \tau)$  is not a  $\alpha T_b$ -space.

**Proposition 1.3.10**

Every  ${}^*T_p$ -space and  $T_p^*$ -space is  $T_{g''}$ -space but not conversely.

**Proof**

Follows from Remark 1.2.5 (vi) and Definition 1.2.3 (vi) and (v).

The converse of Proposition 1.3.10 need not be true as seen from the following example.

**Example 1.3.11**

Let  $X$  and  $\tau$  be as in the Example 1.3.2. Then  $GSP C(X) = \{\phi, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$  and  $G^*PC(X) = \{\phi, \{a\}, \{c\}, \{a, c\}, \{b, c\}, X\}$ . Thus  $(X, \tau)$  is neither  ${}^*_sT_p$ -space nor  $T_p^*$ -space.

**Proposition 1.3.12**

Every  $T_b$ -space is  $Tg^m$ -space but not conversely.

**Proof**

Follows from Remark 1.2.5 (v).

The converse of Proposition 1.3.12 need not be true as seen from the following example.

**Example 1.3.13**

Let  $X$  and  $\tau$  be as in the Example 1.3.2. Then  $GS C(X) = \{\phi, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$ . Thus  $(X, \tau)$  is not a  $T_b$ -space.

**Remark 1.3.14**

We conclude from the next two examples that  $Tg^m$ -spaces and  $\alpha$ -spaces are independent.

**Example 1.3.15**

Let  $X$  and  $\tau$  be as in the Example 1.3.2. Then  $\alpha C(X) = \{\phi, \{a\}, \{c\}, \{a, c\}, X\}$ . Thus  $(X, \tau)$  is a  $Tg^m$ -space but not an  $\alpha$ -space.

**Example 1.3.16**

Let  $X$  and  $\tau$  be as in the Example 1.3.3. Then  $\alpha C(X) = \{\phi, \{b\}, X\}$ . Thus  $(X, \tau)$  is an  $\alpha$ -space but not a  $Tg^m$ -space.

**Theorem 1.3.17**

For a space  $(X, \tau)$  the following properties are equivalent:

- (i)  $(X, \tau)$  is a  $T_{g''}$ -space.
- (ii) Every singleton subset of  $(X, \tau)$  is either  $g$ -closed or open.

**Proof**

(i)  $\rightarrow$  (ii). Assume that for some  $x \in X$ , the set  $\{x\}$  is not a  $g$ -closed in  $(X, \tau)$ . Then the only  $g$ -open set containing  $\{x\}^c$  is  $X$  and so  $\{x\}^c$  is  $g''$ -closed in  $(X, \tau)$ . By assumption  $\{x\}^c$  is closed in  $(X, \tau)$  or equivalently  $\{x\}$  is open.

(ii)  $\rightarrow$  (i). Let  $A$  be a  $g''$ -closed subset of  $(X, \tau)$  and let  $x \in \text{cl}(A)$ . By assumption  $\{x\}$  is either  $g$ -closed or open.

Case (a) Suppose that  $\{x\}$  is  $g$ -closed. If  $x \notin A$ , then  $\text{cl}(A) - A$  contains a nonempty  $g$ -closed set  $\{x\}$ , which is a contradiction to Theorem 1.2.6. Therefore  $x \in A$ .

Case (b) Suppose that  $\{x\}$  is open. Since  $x \in \text{cl}(A)$ ,  $\{x\} \cap A \neq \emptyset$  and so  $x \in A$ . Thus in both case,  $x \in A$  and therefore  $\text{cl}(A) \subseteq A$  or equivalently  $A$  is a closed set of  $(X, \tau)$ .

**Definition 1.3.18**

A topological space  $(X, \tau)$  is called generalized semi- $R_0$  (briefly  $gs-R_0$ ) if and only if for each  $g$ -open set  $G$  and  $x \in G$  implies  $gs\text{-cl}(\{x\}) \subset G$ .

**Definition 1.3.19**

A topological space  $(X, \tau)$  is called:

- (i) generalized semi- $T_0$  (briefly  $gs-T_0$ ) if and only if to each pair of distinct points  $x, y$  of  $X$ , there exists a  $g$ -open set containing one but not the other.
- (ii) generalized semi- $T_1$  (briefly  $gs-T_1$ ) if and only if to each pair of distinct points  $x, y$  of  $X$ , there exists a pair of  $g$ -open sets, one containing  $x$  but not  $y$ , and the other containing  $y$  but not  $x$ .

**Theorem 1.3.20**

For a topological space  $X$ , each of the following statement is equivalent:

- (i)  $X$  is a  $gs-T_1$ .
- (ii) Each one point set is  $gs$ -closed set in  $X$ .

**Proof**

(i)  $\Rightarrow$  (ii) Let a space  $X$  be  $gs-T_1$  and  $x \in X$ . Suppose  $gscl(\{x\}) \neq \{x\}$ . Then we can find an element  $y \in gscl(\{x\})$  with  $y \neq x$ . Since  $X$  is  $gs-T_1$ , there exist  $gs$ -open sets  $U$  and  $V$  such that  $x \in U$ ,  $y \notin U$  and  $y \in V$ ,  $x \notin V$ . Now  $x \in V^c$  and  $V^c$  is  $gs$ -closed. Therefore  $gscl(\{x\}) \subseteq V^c$  which implies  $y \in V^c$ , contradiction. Hence  $gscl(\{x\}) = \{x\}$  or  $\{x\}$  is  $gs$ -closed.

(ii)  $\Rightarrow$  (i) Let  $x, y \in X$  with  $x \neq y$ . Then  $\{x\}$  and  $\{y\}$  are  $gs$ -closed. Therefore  $U = (\{x\})^c$  and  $V = (\{y\})^c$  are  $gs$ -open and  $x \in U$ ,  $y \notin U$  and  $y \in V$ ,  $x \notin V$ . Hence  $X$  is  $gs-T_1$ .

**Theorem 1.3.21**

For a space  $(X, \tau)$  the following properties hold:

- (i) If  $(X, \tau)$  is  $gs-T_1$ , then it is  $Tg'''$ .
- (ii) If  $(X, \tau)$  is  $Tg'''$ , then it is  $gs-T_0$ .

**Proof**

(i) The proof is obvious from Theorem 1.3.20.

(ii) Let  $x$  and  $y$  be two distinct elements of  $X$ . Since the space  $(X, \tau)$  is  $Tg'''$ , we have that  $\{x\}$  is  $gs$ -closed or open. Suppose that  $\{x\}$  is open. Then the singleton  $\{x\}$  is a  $gs$ -open set such that  $x \in \{x\}$  and  $y \notin \{x\}$ . Also, if  $\{x\}$  is  $gs$ -closed, then  $X \setminus \{x\}$  is  $gs$ -open such that  $y \in X \setminus \{x\}$  and  $x \notin X \setminus \{x\}$ . Thus, in the above two cases, there exists a  $gs$ -open set  $U$  of  $X$  such that  $x \in U$  and  $y \notin U$  or  $x \notin U$  and  $y \in U$ .

Thus, the space  $(X, \tau)$  is  $gs-T_0$ .



**Theorem 1.3.22**

For a  $gs-R_0$  topological space  $(X, \tau)$  the following properties are equivalent:

- (i)  $(X, \tau)$  is  $gs-T_0$ .
- (ii)  $(X, \tau)$  is  $T_{g''}$ .
- (iii)  $(X, \tau)$  is  $gs-T_1$ .

**Proof**

It suffices to prove only (i)  $\Rightarrow$  (iii). Let  $x \neq y$  and since  $(X, \tau)$  is  $gs-T_0$ , we may assume that  $x \in U \subseteq X \setminus \{y\}$  for some  $gs$ -open set  $U$ . Then  $x \in X \setminus gs-cl(\{y\})$  and  $X \setminus gs-cl(\{y\})$  is  $gs$ -open. Since  $(X, \tau)$  is  $gs-R_0$ , we have  $gs-cl(\{x\}) \subseteq X \setminus gs-cl(\{y\}) \subseteq X \setminus \{y\}$  and hence  $y \notin gs-cl(\{x\})$ . There exists  $gs$ -open set  $V$  such that  $y \in V \subseteq X \setminus \{x\}$  and  $(X, \tau)$  is  $gs-T_1$ .

**1.4  ${}_gT_{g''}$ -SPACES****Definition 1.4.1**

A space  $(X, \tau)$  is called a  ${}_gT_{g''}$ -space if every  $g$ -closed set in it is  $g''$ -closed.

**Example 1.4.2**

Let  $X$  and  $\tau$  be as in the Example 1.3.3, is a  ${}_gT_{g''}$ -space and the space  $(X, \tau)$  in the Example 1.3.2, is not a  ${}_gT_{g''}$ -space.

**Proposition 1.4.3**

Every  $T_{1/2}$ -space is  ${}_gT_{g''}$ -space but not conversely.

**Proof**

Follows from Remark 1.2.5 (i).

The converse of Proposition 1.4.3 need not be true as seen from the following example.

**Example 1.4.4**

Let  $X$  and  $\tau$  be as in the Example 1.3.3, is a  ${}_gTg''$ -space but not a  $T_{1/2}$ -space.

**Remark 1.4.5**

$Tg''$ -spaces and  ${}_gTg''$ -spaces are independent.

**Example 1.4.6**

The space  $(X, \tau)$  in the Example 1.3.3, is a  ${}_gTg''$ -space but not a  $Tg''$ -space and the space  $(X, \tau)$  in the Example 1.3.2, is a  $Tg''$ -space but not a  ${}_gTg''$ -space.

**Theorem 1.4.7**

If  $(X, \tau)$  is a  ${}_gTg''$ -space, then every singleton subset of  $(X, \tau)$  is either  $g$ -closed or  $g''$ -open.

**Proof**

Assume that for some  $x \in X$ , the set  $\{x\}$  is not a  $g$ -closed in  $(X, \tau)$ . Then  $\{x\}$  is not a closed set, since every closed set is a  $g$ -closed set. So  $\{x\}^c$  is not open and the only open set containing  $\{x\}^c$  is  $X$  itself. Therefore  $\{x\}^c$  is trivially a  $g$ -closed set and by assumption,  $\{x\}^c$  is an  $g''$ -closed set or equivalently  $\{x\}$  is  $g''$ -open.

The converse of Theorem 1.4.7 need not be true as seen from the following example.

**Example 1.4.8**

Let  $X$  and  $\tau$  be as in the Example 1.3.2. The sets  $\{a\}$  and  $\{c\}$  are  $g$ -closed in  $(X, \tau)$  and the set  $\{b\}$  is  $g''$ -open. But the space  $(X, \tau)$  is not a  ${}_gTg''$ -space.

**Theorem 1.4.9**

A space  $(X, \tau)$  is  $T_{1/2}$  if and only if it is both  $Tg''$  and  ${}_gTg''$ .

**Proof**

Necessity. Follows from Propositions 1.3.4 and 1.4.3.

Sufficiency. Assume that  $(X, \tau)$  is both  $T_g^m$  and  ${}_gT_g^m$ . Let  $A$  be a  $g$ -closed set of  $(X, \tau)$ . Then  $A$  is  $g^m$ -closed, since  $(X, \tau)$  is a  ${}_gT_g^m$ . Again since  $(X, \tau)$  is a  $T_g^m$ ,  $A$  is a closed set in  $(X, \tau)$  and so  $(X, \tau)$  is a  $T_{1/2}$ .

**1.5  $\alpha T_g^m$ -SPACES****Definition 1.5.1**

A space  $(X, \tau)$  is called a  $\alpha T_g^m$ -space if every  $\alpha g$ -closed set in it is  $g^m$ -closed.

**Example 1.5.2**

Let  $X$  and  $\tau$  be as in the Example 1.3.3, is a  $\alpha T_g^m$ -space and the space  $(X, \tau)$  in the Example 1.3.2, is not a  $\alpha T_g^m$ -space.

**Proposition 1.5.3**

Every  $\alpha T_b$ -space is  $\alpha T_g^m$ -space but not conversely.

**Proof**

Follows from Remark 1.2.5 (i).

The converse of Proposition 1.5.3 need not be true as seen from the following example.

**Example 1.5.4**

Let  $X$  and  $\tau$  be as in the Example 1.3.3, is a  $\alpha T_g^m$ -space but not a  $\alpha T_b$ -space.

**Proposition 1.5.5**

Every  $\alpha T_g^m$ -space is a  $\alpha T_d$ -space but not conversely.

**Proof**

Let  $(X, \tau)$  be an  $\alpha T g'''$ -space and let  $A$  be an  $\alpha g$ -closed set of  $(X, \tau)$ . Then  $A$  is a  $g'''$ -closed subset of  $(X, \tau)$  and by Remark 1.2.5 (iii),  $A$  is  $g$ -closed. Therefore  $(X, \tau)$  is an  $\alpha T_d$ -space.

The converse of Proposition 1.5.5 need not be true as seen from the following example.

**Example 1.5.6**

Let  $X$  and  $\tau$  be as in the Example 1.3.3, is a  $\alpha T_d$ -space but not a  $\alpha T g'''$ -space.

**Theorem 1.5.7**

If  $(X, \tau)$  is a  $\alpha T g'''$ -space, then every singleton subset of  $(X, \tau)$  is either  $\alpha g$ -closed or  $g'''$ -open.

**Proof**

Similar to Theorem 1.4.7.

The converse of Theorem 1.5.7 need not be true as seen from the following example.

**Example 1.5.8**

Let  $X$  and  $\tau$  be as in the Example 1.3.2. The sets  $\{a\}$  and  $\{c\}$  are  $\alpha g$ -closed in  $(X, \tau)$  and the set  $\{b\}$  is  $g'''$ -open. But the space  $(X, \tau)$  is not a  $\alpha T g'''$ -space.

## REFERENCES

- [1] Abd El-Monsef, M. E., El-Deeb, S. N. and Mahmoud, R. A.:  $\beta$ -open sets and  $\beta$ -continuous mappings, Bull. Fac. Sci. Assiut Univ., 12 (1983), 77-90.
- [2] Andrijevic, D.: Semi-preopen sets, Mat. Vesnik., 38 (1986), 24-32.
- [3] Antony Rex Rodrigo., Ravi, O., Jeyashri, S. and Vijayalakshmi, K.:  $g'''$ -closed sets in topology, International Journal of Advances in Pure and Applied Mathematics, 2(1) (2012), 31-47.
- [4] Arenas, F. G., Dontchev, J. and Ganster, M.: On  $\lambda$ -sets and dual of generalized continuity, Questions Answers Gen. Topology, 15 (1997), 3-13.

- [5] Arya, S. P. and Nour, T.: Characterizations of  $s$ -normal spaces, Indian J. Pure Appl. Math., 21(8) (1990), 717-719.
- [6] Crossley, S. G. and Hildebrand, S. K.: Semi-closure, Texas J. Sci., 22 (1971), 99-112.
- [7] Devi, R., Balachandran, K. and Maki, H.: Generalized  $\alpha$ -closed maps and  $\alpha$ -generalized closed maps, Indian J. Pure Appl. Math., 29 (1998), 37-49.
- [8] Devi, R., Balachandran, K. and Maki, H.: Semi-generalized closed maps and generalized semi-closed maps, Mem. Fac. Kochi Univ. Ser. A. Math., 14 (1993), 41-54.
- [9] Dunham, W.:  $T_{1/2}$ -spaces, Kyungpook Math. J., 17 (1977), 161-169.
- [10] Levine, N.: Generalized closed sets in topology, Rend. Circ. Math. Palermo, 19(2) (1970), 89-96.
- [11] Levine, N.: Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly, 70 (1963), 36-41.
- [12] Maki, H., Devi, R. and Balachandran, K.: Associated topologies of generalized  $\alpha$ -closed sets and  $\alpha$ -generalized closed sets, Mem. Fac. Sci. Kochi. Univ. Ser. A. Math., 15 (1994), 51-63.
- [13] Mashhour, A. S., Abd El-Monsef, M. E. and El-Deeb, S. N.: On precontinuous and weak pre continuous mappings, Proc. Math. and Phys. Soc. Egypt, 53 (1982), 47-53.
- [14] Njastad, O.: On some classes of nearly open sets, Pacific J. Math., 15 (1965), 961-970.
- [15] Noiri, T., Maki, H. and Umehara, J.: Generalized preclosed functions, Mem. Fac. Sci. Kochi Univ. Math., 19 (1998), 13-20.
- [16] Sheik John, M.: A study on generalizations of closed sets and continuous maps in topological and bitopological spaces, Ph.D Thesis, Bharathiar University, Coimbatore, September 2002.
- [17] Veera Kumar, M. K. R. S.:  $g^*$ -preclosed sets, Acta Ciencia Indica, Vol. XXVIII, (1) (2002), 51-60.
- [18] Veera Kumar, M. K. R. S.:  $g^\#$  semi-closed in topological spaces, Indian J. Math., 44(1) (2002), 73-87.