FUNDAMENTALS AND LITERATURE REVIEW OF FOURIER TRANSFORM

[1]Shana Anjum, [2]Prof. Vaishali Sahu

[1]M.Sc Scholar, [2]Assistant Professor
[1&2]Department of Mathematics,
[1&2]Netaji Subhash Chandra Bose College, Sarni, Betul (M.P.), India.

Abstract : The Fourier Transform is a tool that breaks a waveform or a function of signal into an alternate representation, characterized by sine and cosines. The Fourier Transform shows that any waveform can be re-written as the sum of sinusoidal functions. A Fourier series is an expansion of a periodic function f(x) in terms of an infinite sum of sines and cosines. Fourier series make use of the orthogonality relationships of the sine and cosine functions. This paper presents basic of Fourier transform and its review of literature.

***IndexTerms* - Fourier, Signal, Cosines, Transforms.**

I. INTRODUCTION

Any general periodic signal has the automatic property $f(t) = f(\frac{2\pi t}{T})$ where T is the period of the signal. The 2π is "snuck" in because we know that trigonometric functions are good examples of repetition. The complexity of $f(t)$ is irrelevant as long as it repeats itself faithfully. Please keep in mind that 't' for radioastronomy is usually time, but in fact it is an arbitrary variable and so what follows below is applicable provided the variable has functional repetition in some way with a repeat T. Thus spatial repetition is another important variable to which we may apply the theory.
Fourier discovered that such a complex signal could be decomposed into an infinite series made up of cosine and sine terms and a whole bunch of coefficients which can (surprisingly) be readily determined.

$$f(t) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos(\frac{2\pi nt}{T}) + \sum_{n=1}^{\infty} b_n \sin(\frac{2\pi nt}{T})$$

- If you like, we have decomposed the original function $f(t)$ into a series of basis states. For those of you who like to be creative this immediately begs the question of: is this the only decomposition possible? The answer is no.
- The coefficients are "readily" determined by integration.

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos(\frac{2\pi nt}{T})dt$$
$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\sin(\frac{2\pi nt}{T})dt$$

- Introducing complex notation we can simplify all of the above to what you often see in textbooks.

$$f(t) = \sum_{n=-\infty}^{\infty} c_n \exp(-i\frac{2\pi nt}{T})$$

$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\exp(i\frac{2\pi nt}{T})dt$$

- Here $c_0 = \frac{1}{2}a_0$, $c_n = \frac{1}{2}(a_n + ib_n)$, and $c_{-n} = \frac{1}{2}(a_n - ib_n)$.

- The graphical example below indicates how addition of cosine time function terms are Fourier transformed into coefficients. In this case only $c_n = a_n / 2$. Take care the centre line with the big arrow is to mark the axis only – it is **not** part of the coefficient display. Notice also that **two** coefficient lines appear for every frequency. The latter is related to the Nyquist sampling theorem (see below) and is also why the coefficient magnitudes are halved. Notice also the **spacing** of the coefficients to be an integral multiple of $f_0 = 1/T$ with the sign consistent with the input waveform.

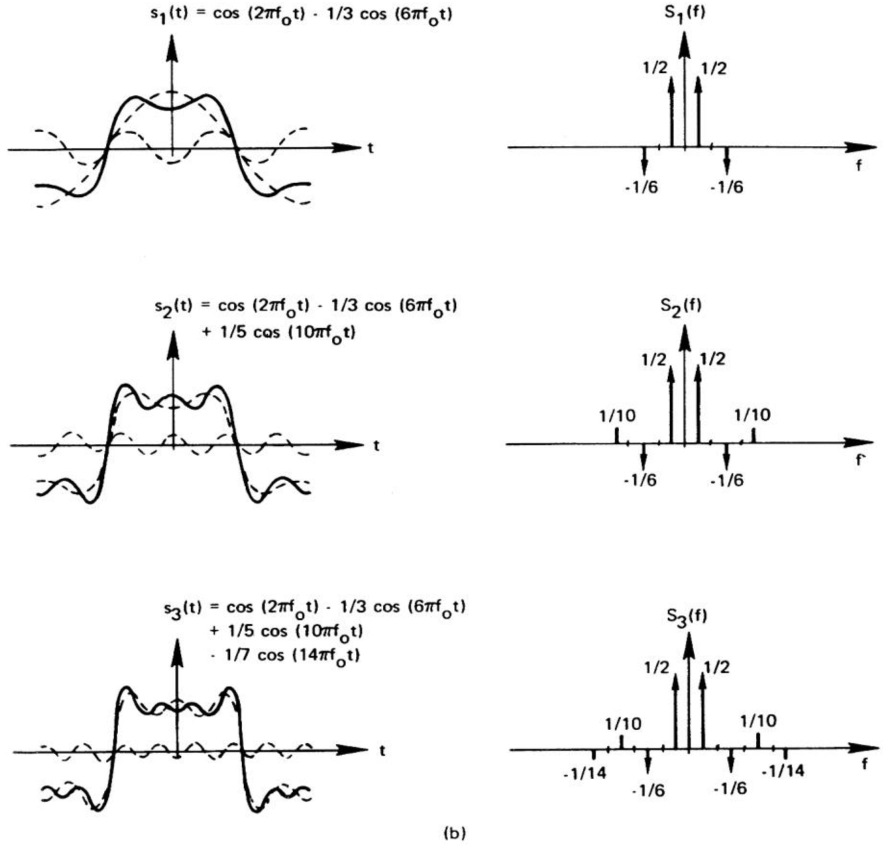

Figure 1: The Fast Fourier Transform

It is important to stress that it is an ***intrinsic*** property that the c_n are discrete. This is sometimes very confusing in text books because they draw them as continuous functions. It ***is*** possible to make them a continuous function by doing a simple trick and imagining that T is enormously large or better still it tends to infinity. Thus the repeat period is infinite.

II. LITERATURE SURVEY

Soo-Chang Pei et al., [1] Traditional Fourier investigation has numerous plans for various sorts of signals. They are Fourier transform (FT), Fourier series (FS), discrete-time Fourier transform (DTFT), and discrete Fourier transform (DFT). The objective of this article is to create two missing plans of fractional Fourier examination strategies. The proposed techniques are fractional Fourier series (FRFS) and discrete-time fractional Fourier transform (DTFRFT), and they are the speculations of Fourier series (FS) and discrete-time Fourier transform (DTFT), separately.

K. M. Snopek, et al., [2] The work indicates relations between two diverse recurrence portrayals of n-dimensional signals (n = 1, 2, 3): the Cayley-Dickson Fourier transform and the Hartley transform. The equations relating the n-D complex Fourier transform and the n-D Hartley transform are displayed. New recipes relating the Quaternion and Octonion Fourier transforms and separately the 2-D and 3-D Hartley transforms are created. The work is delineated with instances of Hartley transforms of 1-D, 2-D and 3-D Gaussian signals.

S. Pei et al., [3] Dependent on discrete Hermite-Gaussian-like functions, a discrete fractional Fourier transform (DFRFT), which gives test approximations of the persistent fractional Fourier transform, was characterized and explored as of late. In this work, it is propose another almost tridiagonal network, which drives with the discrete Fourier transform (DFT) grid. The eigenvectors of the new about tridiagonal framework are demonstrated to be DFT eigenvectors, which are progressively like the nonstop Hermite-Gaussian functions than those created previously. Thorough discourses on the relations between the eigendecomposition of the recently proposed about tridiagonal framework and the DFT network are portrayed. Moreover, by properly consolidating two straightly free grids that both drive with the DFT lattice, it is build up a technique to acquire DFT eigenvectors significantly increasingly like the constant Hermite-Gaussian functions (HGFs). At that point, new forms of DFRFT produce their transform yields nearer to the examples of the nonstop fractional Fourier transform, and their applications are depicted. Related PC trials are performed to show the legitimacy of the works in this paper

W. Hsue et al., [4] Genuine transforms require less unpredictability for calculations and less memory for stockpiles than complex transforms. Be that as it may, discrete fractional Fourier and Hartley transforms are mind boggling transforms. In this work, it is propose reality-safeguarding fractional variants of the discrete Fourier, Hartley, summed up Fourier, and summed up Hartley transforms. The majority of the proposed genuine discrete fractional transforms have the same number of as O(N2) parameters and along these lines are truly adaptable. The proposed genuine discrete fractional transforms have irregular eigenvectors and they have just two unmistakable eigenvalues 1 and - 1. Properties and connections of the proposed genuine discrete fractional transforms are explored. Also, for the genuine ordinary discrete Hartley and summed up discrete Hartley transforms, it is propose their elective reality-saving fractionalizations dependent on corner to corner like networks to further build their adaptability. The proposed genuine transforms have the majority of the required great properties to be discrete fractional transforms. At long last, since the proposed new transforms have irregular yields and numerous parameters, they are on the whole appropriate for information security applications, for example, picture encryption and watermarking.

A. M. Grigoryan et al., [5] This work displays a novel idea of the reversible integer discrete Fourier transform (RiDFT) of request 2r, r > 2, when the transform is part by the combined portrayal into a base arrangement of short transforms, i.e., transforms of requests 2k, k < r. By methods for the combined transform the signal is spoken to as a lot of short signals which convey the otherworldly data of the signal at explicit and disjoint arrangements of frequencies. The combined transform-based fast Fourier transform (FFT) includes a couple of tasks of augmentation that can be approximated by integer transforms. Instances of 1-point transforms with one control bit are portrayed. Control bits enable us to rearrange such approximations. Two control bits are required to play out the 8-point RiDFT, and 12 (or even 8) bits for the 16-point RiDFT of genuine sources of info. The proposed forward and reverse RiDFTs are fast, and the computational unpredictability of these transforms is relative with the multifaceted nature of the FFT. The 8-point immediate and opposite RiDFTs are depicted in detail.

J. G. Vargas-Rubio et al., [6] Existing adaptations of the discrete fractional Fourier transform (DFRFT) depend on the discrete Fourier transform (DFT). These methodologies need a full premise of DFT eigenvectors that fill in as discrete forms of Hermite-Gauss functions. In this letter, it is characterize a DFRFT dependent on a focused variant of the DFT (CDFRFT) utilizing eigenvectors got from the Gru/spl uml/nbaum tridiagonal commutor that fill in as fantastic discrete approximations to the Hermite-Gauss functions. it is build up a fast and productive approach to process the multiangle adaptation of the CDFRFT for a discrete arrangement of edges utilizing the FFT calculation. it is then demonstrate that the related trill recurrence portrayal is a helpful investigation device for multicomponent peep signals.

O. K. Ersoy, et al., [7] Major consistent time, discrete-time, and discrete Fourier-related transforms just as Fourier-related series are talked about both with genuine and complex bits. The mind boggling Fourier transforms, Fourier series, cosine, sine, Hartley, Mellin, Laplace transforms, and z-transforms are secured on a relative premise. Speculations of the Fourier transform bit lead to various novel transforms, specifically, exceptional discrete cosine, discrete sine, and genuine discrete Fourier transforms, which have effectively discovered use in various applications. The fast calculations for the genuine discrete Fourier transform give a bound together way to deal with the ideal fast calculation of all discrete Fourier-related transforms. The brief timeframe Fourier-related transforms are examined for applications including nonstationary signals. The one-dimensional transforms talked about are additionally reached out to the two-dimensional transforms.<>

Soo-Chang Pei et al., [8] The integer transform, (for example, the Walsh transform) is the discrete transform that every one of the passages of the transform framework are integer. It is a lot simpler to execute on the grounds that the genuine number duplication tasks can be maintained a strategic distance from, yet the exhibition is typically more regrettable. Then again, the noninteger transform, for example, the DFT and DCT, has a decent exhibition, however genuine number augmentation is required. W infer the integer transforms practically equivalent to some mainstream noninteger transforms. These integer transforms hold a large portion of the exhibition nature of the first transform, yet the execution is a lot less complex. Particularly, for the two-dimensional (2-D) square transform in picture/video, the sparing can be gigantic utilizing integer activities. In 1989, Cham had determined the integer cosine transform. Here, it is will determine the integer sine, Hartley, and Fourier transforms. it is likewise acquaint the general technique with get the integer transform from some noninteger transform. Additionally, the integer transform determined by Cham still requires genuine number augmentation for the backwards transform. it is alter the integer transform presented by Cham and present the total integer transform. It requires no genuine number duplication activity, regardless of what the forward or converse transform. The integer transform it is determine would be more effective than the first transform. For instance, for the 8-point DFT and IDFT, there are in all out four genuine numbers and eight fixed-point augmentation activities required, however for the forward and converse 8-point total integer Fourier transforms, there are absolutely 20 fixed-point duplication tasks required. Nonetheless, for the integer transform, the usage is less complex, and a large number of the properties of the first transform are kept.

W. E. Pelton, et al., [9] The work depicts another fast Fourier transform (FFT) calculation and its execution. The calculation is better than customary FFT calculations, requiring less calculations and having diminished dormancy. A full arrangement of exact traditional coefficients is determined at each example entry. A 8-point processor was mimicked in Matlab. A 64-point ASIC reproduction in 0.18/spl mu/m CMOS gave consequences of 16 mW, region = 1 mm/sup 2/and test rate = 2.5 MS/sec when timed at 330 MHz.

A. E. Gera et al., [10] In this work, the connection between the uneven Z-transform and the uneven discrete-time Fourier transform is considered. It fills in as the partner to a past one inferred for consistent functions. The advantage is in its application to unit step and periodic functions.

III. Fourier series and Fourier transform

A. Fourier Series

If we have a reasonably well behaved, continuous, periodic function x(t), then we can approximate x(t) as the weighted sum of simple sinusoids

.g. $x(t) \approx a_o/2 + \sum_{n} (a_n \cos(2\pi ft) + b_n \sin(2\pi ft))$

if we choose the "weights" a_n and b_n properly. A simple "least squares" procedure yields the well known formulae for calculating a_n and b_n, i.e.

$$a_n = (2/T) \int_{-T/2}^{T/2} x(t) \cos(2\pi ft)\, dt$$

$$b_n = (2/T) \int_{-T/2}^{T/2} x(t) \sin(2\pi ft)\, dt$$

where f = n/T

We can use Euler's relation to rewrite the sinusoidal version of Fourier series in terms of exponentials, e.g.

$x(t) = \Sigma\, \alpha_n e^{i2\pi ft}$ nT/2

where $\alpha_n = (1/T) \int_{-T/2} x(t)\, e^{-i2\pi ft}\, dt$

In the limit of the period T approach infinity, we get the a fit to a non-periodic function x(t) that is the

B. Fourier Transform

$$\mathbf{X}(f) = \int x(t)\, e^{-i2\pi ft}\, dt$$

which converts a function of time into a function of frequency
(or a function of space into a function of wavenumber)

x(t) is then represented by X(f)

The new function has the same information as x(t) but from a different perspective. If this is done properly, then one can always recover the orginal function x(t) by an inverse transform, i.e.

$$x(t) = \int_{-\infty}^{\infty} X(f)\, e^{+i2\pi ft}\, dt$$

The discrete (digital) form of the transform (DFT) for a time series of length N is:

$$X(j\Delta f) = X_j = \sum_{k=0}^{N-1} x_k\, e^{-i2\pi kj/N}$$

and

$$x(k\Delta t) = x_k = 1/N \sum_{j=0}^{N-1} X_j\, e^{i2\pi kj/N}$$

where $\Delta f = 1/N\, \Delta t$

Note the relationship with simply sinuisoids, i.e.

$$e^{-i2\pi kj/N} = \cos(2\pi kj/N) - i \sin(2\pi kj/N)$$

Also note that these equations allow one to compute either X or x for values of j and i respectively that are outside the range of 0 and N-1. These values correspond to the periodic replicas of X and x that are implicit in the discreter formulation.

Now, let's make a simple substitution, letting

$$z = e^{-i2\pi j/N}$$

then the equation for the DFT becomes

$$X(j\Delta t) = X_j = \sum_{k=0}^{N-1} x_k\, e^{-i2\pi kj/N} = \sum_{k=0}^{N-1} x_k\, z^k = X(z).$$

This latter relation is called the Z transform.

C. Digital Fourier Transform

In the real world of experimental physics we do not have the luxury of infinite time nor can we necessarily describe analytically our function $f(t)$. It is normally derived from the experimental equipment attached to something (eg., receiver voltage from an antenna). Thus we need something practical to do an FT.
The above has three simple consequences:

1) We need to choose a total sample time T recognizing 1/T will then be the frequency spacing or resolution we can get out of the transform. Thus if T=1.5s we obtain our frequency coefficients at a spacing of 0.67 Hz.

2) During T, we need to sample our waveform N times to produce a sampling vector representing our continuous time domain. Thus

$$f(t) \leftrightarrow \{x_n\} \text{ with } 0 < n < N-1$$

IV. CONCLUSION

In the literature, there are few review studies with published articles. Most of these studies are related to the explanation of DFT or FFT transforms. The presented paper is the first review study included both detail examination of published articles related to power quality in recent years and classification of these studies. The result of the reviewed published papers on FFT shows that Fourier transform can be used for many issues related to detection and restoration. In addition, it can be used in image processing tasks, filtration, disturbance detection, modulation and demodulation.

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