LUCKY EDGE AND L(2,1)-LABELING OF HUMAN CHAIN GRAPH

K.Anitha[1], B.Selvam[2] and K.Thirusangu[3]

[1]Department of Mathematics, Sri Sai Ram Engineering College, Chennai-44, India

[2,3]Department of Mathematics, S.I.V.E.T. College, Gowrivakkam, Chennai-73, India

Abstract- Motivation of the graph labeling, we determine lucky edge number and L(2,1)- number of the human chain graph in this paper.

Key words- Human chain graph, L(2,1)-labeling, Lucky edge labeling

I. Introduction

Human chain graph was studied by [1]. Lucky edge labeling was studied by [2]. L(2,1)-labeling was studied by [3]. We determine in this paper, L(2,1)- number and lucky edge number of human chain graph.

II. Basic Notions

Definition-1[1]

A human chain graph is constructed from a path $u_1, u_2, \ldots, u_{2n+1}$, n∈N by attaching a cycle of length m (C_m) and Y-tree (Y_{m+1}), m≥3 to each u_{2i} for 1≤i≤n. The vertices of the cycle (C_m) and Y-tree (Y_{m+1}) are $v_1, v_2, \ldots, v_{(m-1)n}$ and $w_1, w_2, \ldots, w_{mn}$ respectively and the vertices of path is $u_1, u_2, \ldots, u_{2n+1}$. The human chain graph is denoted by $HC_{n,m}$.

Definition-2[2]

A graph G admits lucky edge labeling if the edge coloring of G is $l^*(e_i) \neq l^*(e_j)$, the edges e_i and e_j are adjacent. The lucky edge number $\eta_e(G)$ is the least integer of G has a lucky edge labeling from the set {1,2,…,k}.

Definition-3[3]

L(2,1)-labeling of a graph G is a function l:V(G)→N such that $|l(x) - l(y)| \geq 2$ if d(x,y)=1 and $|l(x) - l(y)| \geq 1$ if d(x,y)=2. The L(2,1)-labeling number $\lambda(G)$ is the smallest number k such that l(v)=k.

III. Main Results

Algorithm-1

Procedure. (Lucky edge labeling of $HC_{n,m}$, n≥ 1 and $m \geq 3$)

Input. V←{ $w_1, w_2, \ldots, w_{mn}, u_1, u_2, \ldots, u_{2n+1}, v_1, v_2, \ldots, v_{(m-1)n}$}

E←{ $e_1, e_2, \ldots, e_{2mn+2n}$}

if n≥1

if m≥3

for i = 1 to $\left\lfloor \frac{n+2}{2} \right\rfloor$ do

$u_{4i-3} \leftarrow 5$

end for

for i = 1 to $\left\lfloor\frac{n+1}{2}\right\rfloor$ do

$u_{4i-2} \leftarrow 1$

$u_{4i-1} \leftarrow 4$

$w_{2mi-2m+1} \leftarrow 1$

$v_{2(m-1)i-2m+4j-1} \leftarrow 2$

end for

for i = 1 to n do

$w_{mi} \leftarrow 3$

$v_{(m-1)i} \leftarrow 3$

end for

end if

if m>3

for i = 1 to $\left\lfloor\frac{n+1}{2}\right\rfloor$ do

for j = 1 to $\left\lfloor\frac{m}{4}\right\rfloor$ do

$v_{2(m-1)i-2m+4j} \leftarrow 2$

$w_{2mi-2m+4j-1} \leftarrow 2$

end for

end for

end if

if m>4

for i = 1 to $\left\lfloor\frac{n+1}{2}\right\rfloor$ do

for j = 1 to $\left\lfloor\frac{m-1}{4}\right\rfloor$ do

$w_{2mi-2m+4j} \leftarrow 1$

end for

end for

end if

if m>5

for i = 1 to $\left\lfloor\frac{n+1}{2}\right\rfloor$ do for j = 1 to $\left\lfloor \frac{m-2}{4} \right\rfloor$ do

$v_{2(m-1)i-2m+4j+1} \leftarrow 1$

$w_{2mi-2m+4j+1} \leftarrow 1$

end for

end for

end if

if m>6

for i = 1 to $\left\lfloor \frac{n+1}{2} \right\rfloor$ do

for j = 1 to $\left\lfloor \frac{m-3}{4} \right\rfloor$ do

$v_{2(m-1)i-2m+4j+2} \leftarrow 1$

end for

end for

end if

end if

if n>1

if m≥3

for i = 1 to $\left\lfloor \frac{n}{2} \right\rfloor$ do

$u_{4i} \leftarrow 2$

$w_{2mi-m+1} \leftarrow 2$

end for

for i = 1 to $\left\lfloor \frac{n}{2} \right\rfloor$ do

for j = 1to$\left\lfloor \frac{m+1}{4} \right\rfloor$ do

$v_{2(m-1)i-m+4j-2} \leftarrow 1$

$w_{2mi-m+4j-2} \leftarrow 1$

end for

end for

end if

if m>3

for i = 1 to $\left\lfloor \frac{n}{2} \right\rfloor$ do

for j = 1 to $\left\lfloor \frac{m}{4} \right\rfloor$ do

$v_{2(m-1)i-m+4j-1} \leftarrow 1$

$w_{2mi-m+4j-1} \leftarrow 1$

end for

end for

end if

if m>4

for i = 1 to $\left\lfloor \frac{n}{2} \right\rfloor$ do

for j = 1 to $\left\lfloor \frac{m-1}{4} \right\rfloor$ do

$w_{2mi-m+4j} \leftarrow 2$

end for

end for

end if

if m>5

for i = 1 to $\left\lfloor \frac{n}{2} \right\rfloor$ do

for j = 1 to $\left\lfloor \frac{m-2}{4} \right\rfloor$ do

$v_{2(m-1)i-m+4j1} \leftarrow 2$

$w_{2mi-m+4j+1} \leftarrow 1$

end for

end for

end if

if m>6

for i = 1 to $\left\lfloor \frac{n}{2} \right\rfloor$ do

for j = 1 to $\left\lfloor \frac{m-3}{4} \right\rfloor$ do

$v_{2(m-1)i-m+4j+2} \leftarrow 2$

end for

end for

end if

end if

if m ≡ 1, 2 (mod4) do

for i = 1 to n do

$v_{(m-1)i-1} \leftarrow 3$

end for

end if

end procedure

Theorem-1

For m≥3, n≥1, the lucky edge labeling number of the human chain graph= $\begin{cases} 6 \ if \ n = 1 \\ 7 \ if \ n > 1 \end{cases}$

Proof.

Let $HC_{n,m}$ be a finite, undirected, simple graph with n≥1 and m≥3.

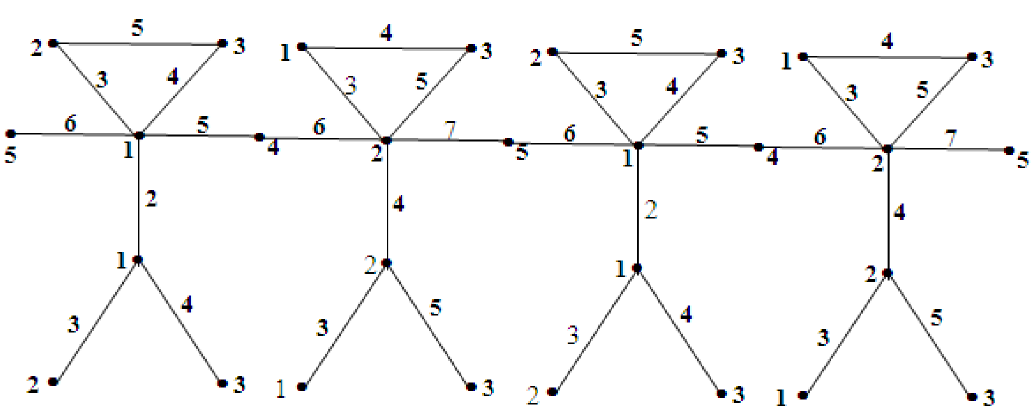
Case(i) if n=1

In $HC_{1,m}$, The neighbors of the vertex u_2 is $\{u_1,u_3,v_1,v_{m-1},w_1\}$. Assign the label $col(u_2)=1$ and $col(u_1)\neq col(u_3)\neq col(v_1)\neq col(v_{m-1})\neq col(w_1)$. Therefore $col(w_1)=1$, $col(v_1)=2$, $col(v_{m-1})=3$, $col(u_1)=4$, $col(u_3)=5$ and the remaining vertices are labeled by algorithm-1. The resulting lucky edge labels are 2,3,4,5,6. Hence $\eta_e(HC_{1,m}) = 6.$

Case(ii) if n>1

Using case(i), label the vertices u_1, u_2, u_3, v_1, v_{m-1}, w_1. The neighbors of the vertex u_4 is $\{u_5,u_3,v_m,v_{2(m-1)},w_{m+1}\}$. Suppose $col(u_4)=1$, the edges u_3u_2 and u_3u_4 are same, which is contradiction to the lucky edge labeling. Therefore $col(u_4)=2$ and $col(w_{m+1})=2$, $col(v_m)=1$, $col(v_{2(m-1)})=3$, $col(u_3)=4$, $col(u_5)=5$, since $col(u_3)\neq col(u_5)\neq col(v_m)\neq col(v_{2(m-1)})\neq col(w_{m+1})$. The resulting lucky edge labels are 2,3,4,5,6,7. Hence $\eta_e(HC_{n,m}) = 7.$

Example-1 (Lucky edge labeling of $HC_{4,3}$)



Algorithm-2

Procedure. (L(2,1) labeling of $HC_{n,m}$, $n\geq 1$ *and* $m \geq 3$)

Input. $V\leftarrow\{ w_1, w_2,\ldots, w_{mn}, u_1, u_2,\ldots, u_{2n+1}, v_1, v_2 ,\ldots, v_{(m-1)n}\}$

$E\leftarrow\{ e_1, e_2, \ldots, e_{2mn+2n}\}$

if n≥1

if m≥3

for i = 1 to $\left\lfloor\frac{n+2}{2}\right\rfloor$ do

$u_{4i-3} \leftarrow 3$

end for

for i = 1 to $\left\lfloor\frac{n+1}{2}\right\rfloor$ do

$u_{4i-2} \leftarrow 1$

$u_{4i-1} \leftarrow 5$

end for

for i = 1 to n do

for j = 1 to $\left\lfloor\frac{m}{3}\right\rfloor$ do

$v_{(m-1)(i-1)+3j-2} \leftarrow 4$

end for

end if

end if

if n>1

if m≥3

for i = 1 to $\left\lfloor\frac{n}{2}\right\rfloor$ do

$u_{4i} \leftarrow 7$

end for

end if

if m>3

if m=0,2 (mod3)

for i = 1 to n do

for j = 1 to $\left\lfloor\frac{m-1}{3}\right\rfloor$ do

$v_{(m-1)(i-1)+3j} \leftarrow 6$

$v_{(m-1)(i-1)+3j-1} \leftarrow 2$

end for

for i = 1 to $\left\lfloor\frac{n+1}{2}\right\rfloor$ do

$v_{2(m-1)i-m+1} \leftarrow 7$

end for

for i = 1 to n do

for j = 1 to $\left\lfloor\frac{m-1}{3}\right\rfloor$ do

$w_{mi-m+3j} \leftarrow 1$

$w_{mi-m+3j-1} \leftarrow 4$

$w_{2mi-2m+3j-2} \leftarrow 6$

end for

end if

for i = 1 to $\left\lfloor\frac{n}{2}\right\rfloor$ do

$v_{2(m-1)i} \leftarrow 1$

end for

for i = 1 to n do

$w_{2mi-m+1} \leftarrow 2$

end for

for i = 1 to n do

for j = 1to $\left\lfloor\frac{m-6}{3}\right\rfloor$ do

$w_{2mi-m+3j+1} \leftarrow 6$

end for

end if

if m=0(mod3)

for i = 1 to n do

$w_{mi} \leftarrow 3$

$w_{mi-1} \leftarrow 2$

$w_{mi-2} \leftarrow 5$

end for

end if

if m=2(mod3)

for i = 1 to n do

$w_{mi} \leftarrow 5$

$w_{mi-1} \leftarrow 3$

end for

end if

if m=1(mod3)

for i = 1to $\left\lfloor\frac{n+1}{2}\right\rfloor$ do

for j = 1to $\left\lfloor\frac{m-1}{3}\right\rfloor$ do

$v_{2(m-1)i-2(m-1)+3j-1} \leftarrow 2$

$v_{2(m-1)i-2(m-1)+3j} \leftarrow 6$

$w_{2mi-2m+3j-2} \leftarrow 7$

$w_{2mi-2m+3j} \leftarrow 1$

$w_{2mi-2m+3j-1} \leftarrow 4$

end for

end for

for i=1 to $\left\lfloor\frac{n}{2}\right\rfloor$ do

for j=1to $\left\lfloor\frac{m-1}{3}\right\rfloor$ do

$v_{2(m-1)i-(m-1)+3j-1} \leftarrow 6$

$v_{2(m-1)i-(m-1)+3j} \leftarrow 2$

$w_{2mi-2m+3j-2} \leftarrow 1$

$w_{2mi-2m+3j} \leftarrow 7$

$w_{2mi-2m+3j-1} \leftarrow 4$

end for

end for

for i= 1 to n do

$w_{mi} \leftarrow 2$

end for

end if

end if

end procedure

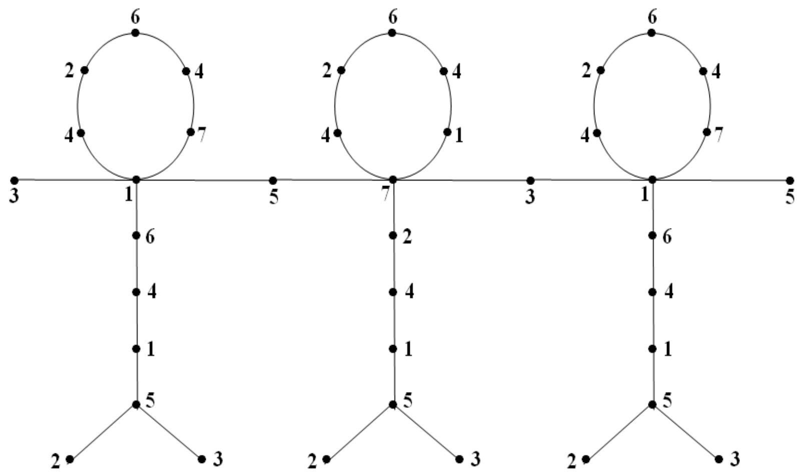
Theorem-2

For m≥3, n≥1, the L(2,1) number of the human chain graph is 7.

Proof.

Let $HC_{n,m}$ be a finite, undirected, simple graph with n≥1 and m≥3. The neighbors of the vertex $u_2 = \{u_1, u_3, v_1, v_{m-1}, w_1\}$. Assign the color c_1 to u_2. The possible colors of the neighbors of u_2 is $\{c_3, c_4, c_5, c_6, c_7\}$ and c_2 as $d(u_2, N(u_2))=1$. We can assign $c(u_1)=c_3$, $c(u_3)=c_5$, $c(v_1)=c_4$, $c(v_{n-1})=c_6$, $c(w_1)=c_7$. Therefore $\lambda(HC_{n,m}) \geq 7$. Using algorithm 2, assign the colors from $\{c_1, c_2, c_3, c_4, c_5, c_6, c_7\}$ to all the vertices of $HC_{n,m}$. Clearly $HC_{n.m}$ graph admits L(2,1) labeling. Hence L(2,1) number of $HC_{n,m}$ is 7.

Example-2 (L(2,1) labeling of $HC_{3,6}$)



IV. Conclusion

We have determined the lucky edge number and L(2,1) number of the human chain graph.

V. References

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