

# SUBJUGATE STRICT DOMINATION INFUSED IN FASCINATING FUZZY GRAPHS

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**Abstract :** A scholarly flagship exposition of strict domination is put to light incorporating membership values of strict arcs weaved in fuzzy graphs. The strict domination number  $rs$  of complete fuzzy graph and complete bipartite fuzzy graph is identified and the respective bands is procured for strict domination number of fuzzy graphs. In the meantime the relationship between the strict domination number of a fuzzy graph and that of its compliment are put under the scanner.

**Keywords -** Fuzzy graph , Strict arcs, Weight of arcs, Strict domination

## I. INTRODUCTION

The innate concept of fuzzy graphs were launched by Rosenfield [21], there by initiating fuzzy analogue of several graph theoretic concepts like paths, cycles, trees and connectedness and instigated their properties. Bhutani and Rosenfeld have introduced the concept of strict arcs [10],

Enumerous works on fuzzy graphs are also initiated by Akram, Samanta, Dudek, Dawaz, Sunitha Pal and Pramanik [1-8, 14, 18-20, 22-25, 31, 32], In the course of 1850, a study of dominating set in graphs ignited purely as a problem in the game of chess.

Chess fanatics in Europe admind the problem of determining the minimum number of queens that can be placed on a chess board thereby all the squares are either attacked by a queen or engrossed by a queen. The concept of domination in graphs was put forth by Ore and Berge in 1962 and further in depth study by Cockaine and Hedetniemi [12], Domination in fuzzy graphs empowered by effective edges was beguiled by Somasundaram and Somasundaram [27]. In accordance to then a dominating set of fuzzy graph  $H : (W, \sigma, \mu)$  is a set  $J$  of nodes of  $H$  such that every node in  $W - J$  has at least one strict neighbor in  $J$ . Apart from it they reinstated domination number of  $G$  in two ways, one as the minimum number of nodes in  $J$  and the other as the minimum scalar cardinality of any  $J$  (i.e., using the weights of nodes in  $J$ ). In this paper, we reinstate the domination number of fuzzy graph using the weights of strict arcs thereby minimizing the parameter in future..

This fascinating paper energises Section 2 with preliminaries and the classical definition of strict domination number of fuzzy graphs in Section 3. The strict domination number of complete fuzzy graph is redefined complete fuzzy graphs of the weakest arc in fuzzy graph (Proposition 3.1). The biconditional statement is obtained by which the strict domination number of a fuzzy graph  $H$  will be the size of  $H$  (Proposition 3.2). An upper bound for the total sum of the strict domination number embedded is a fuzzy graph and that of its complement (devoid of isolated nodes) is also narrated. The bottom line is a lower bound and an upper bound embarked for the strict domination number of fuzzy graphs is recorded by employing the concept of using minimum arc strength order and maximum strict neighbourhood.

## II. BASIC CONCEPTS

The models of relations are well narrated is the graphs. It is the suitable way of representing data involving relationship between objects. Objects are represented by the way of vertices and relations by edges. The innate presence of a voidness is the narration of objects or is its relationships or is both, it is quite natural relationships or in both, it is quite natural that the fuzzy graph model need to redesigned. A brief summarization pertaining to some basic definitions in fuzzy graphs which are presented in [9,10,13,15-17,21, 26-28],

A fuzzy graph is denoted by  $H : (W, \rho, \varepsilon)$  where  $W$  is a node set,  $\sigma$  and  $\mu$  are mappings defined as  $\rho : W \rightarrow [0,1]$  and  $\varepsilon : W \times W \rightarrow [0,1]$ , where  $\rho$  and  $\varepsilon$  represent the membership values of a vertex and an edge respectively. For any fuzzy graph  $\varepsilon(p, q) \leq \min\{\rho(p), \rho(q)\}$ . We consider fuzzy graph  $H$  with no loops and assume that  $W$  is finite and non-empty,  $\mu$  is reflexive (i.e.,  $\varepsilon(p, p) = \rho(p)$  for all  $p$ ) and symmetric (i.e.,  $\rho(p, q) = \rho(q, p)$  for all  $(p, q)$ ). In all the examples,  $\rho$  is chosen suitably. Also, we denote the underlying crisp graph by  $H^* : (\rho^*, \varepsilon^*)$  where  $\rho^* = \{m \in W / \rho(m) > 0\}$  and  $\varepsilon^* = \{(m, v) \in W \times W / \varepsilon(m, v) > 0\}$ . Throughout we assume that  $\rho^* = W$ . The fuzzy graph  $H : (L, \nu)$  is said to be a partial fuzzy subgraph of  $H : (\rho, \varepsilon)$  if  $\nu \subseteq \varepsilon$  and  $L \subseteq \rho$ . In particular, we call  $H : (L, \nu)$  a fuzzy subgraph of  $H : (\rho, \varepsilon)$  if  $L(m) = \rho(m)$  for all  $m \in \tau^*$  and  $\nu(m, v) = \varepsilon(m, v)$  for all  $(m, v) \in \nu^*$ . A fuzzy graph  $H : (W, \rho, \varepsilon)$  is called trivial if  $|\rho^*| = 1$ . Two nodes  $m$  and  $v$  in a fuzzy graph  $H$  are said to be adjacent if  $\varepsilon(m, v) > 0$ .

A path  $Q$  of length  $n$  is a sequence of distinct nodes  $m_0, m_1, \dots, m_n$  such that  $\mu(m_{i-1}, m_i) > 0$ ,  $i = 1, 2, \dots, n$  and the degree of membership of a weakest arc is defined as its strength. If  $m_0 = m_n$  and  $n \geq 3$ , then  $P$  is called a cycle and  $Q$  is called a fuzzy cycle if it contains more than one weakest arc. The strength of a cycle is the strength of the weakest arc in it. The strength of connectedness between two nodes  $p$  and  $y$  is defined as the maximum of the strength of all paths between  $p$  and  $y$  and is denoted by  $CONN_G(p, y)$ .

A fuzzy graph  $H : (W, \rho, \varepsilon)$  is connected if for every  $p, q$  in  $\rho^*$ ,  $CONN_G(p, q) > 0$ .

An arc  $(m, v)$  of a fuzzy graph is called an effective arc ( $M$ -strict arc) if  $\varepsilon(m, v) = \rho(m) \wedge \rho(v)$ . Then  $m$  and  $v$  are called effective neighbors. The set of all effective neighbors of  $m$  is called effective neighborhood of  $m$  and is denoted by  $EN(m)$ .

A fuzzy graph  $H$  is said to be complete if  $\mu(m, v) = \rho(m) \wedge \rho(v)$  for all  $m, v \in \rho^*$ .

The order  $p$  and size  $q$  of a fuzzy graph  $H : (\sigma, \varepsilon)$  are defined to be  $p = \sum_{p \in V} \rho(p)$  and  $q = \sum_{(p,q) \in V \times V} \varepsilon(p, q)$ .

Let  $G : (W, (\rho, \varepsilon))$  be a fuzzy graph and  $S \subseteq W$ . Then the scalar cardinality of  $S$  is defined to be  $\sum_{v \in S} \rho(v)$  and it is denoted by  $|S|_s$ . Let  $p$  denotes the scalar cardinality of  $W$ , also called the order of  $H$ .

The complement of a fuzzy graph  $H$ , denoted by  $\overline{H}$  is defined to be  $\overline{H} = (W, (\rho, \mu))$  where  $\mu(p, q) = \rho(p) \wedge \rho(q) - \varepsilon(p, q)$  for all  $p, q \in W$  [30].

An arc of a fuzzy graph is called strict if its weight is at least as great as the strength of connectedness of its end nodes when it is deleted. Depending on  $CONN_G(p, q)$  of an arc  $(p, q)$  in a fuzzy graph  $H$ , Mathew and Sunitha [28] defined three different types of arcs. Note that  $CONN_{G-(p,q)}(p, q)$  is the strength of connectedness between  $p$  and  $q$  in the fuzzy graph obtained from  $H$  by deleting the arc  $(p, q)$ . An arc  $(p, q)$  in  $H$  is  $\beta$ -strict if  $\varepsilon(p, q) > CONN_{G-(p,q)}(p, q)$ . An arc  $(p, q)$  in  $H$  is  $\beta$ -strict if  $\varepsilon(p, q) = CONN_{G-(p,q)}(p, q)$ . An arc  $(p, q)$  in  $H$  is  $\delta$ -arc if  $\varepsilon(p, q) < CONN_{G-(p,q)}(p, q)$ .

Thus an arc  $(p, q)$  is a strict arc if it is either  $\beta$ -strict or  $\beta$ -strict. A path  $P$  is called strict path if  $P$  contains only strict arcs. If  $\mu(m, v) > 0$ , then  $m$  and  $v$  are called neighbors. The set of all neighbors of  $m$  is denoted by  $N(m)$ . Also  $v$  is called strict neighbor of  $m$  if arc  $(m, v)$  is strict. The set of all strict neighbors of  $m$  is called the strict neighborhood of  $m$  and is denoted by  $N_s(m)$ . The closed strict neighborhood  $N_s(m)$  is defined as  $N_s(m) = N_s(m) \cup \{m\}$ .

The strict degree of a node  $v \in W$  is defined as the sum of membership values of all strict arcs incident at  $v$ . It is denoted by  $d_s(v)$ . That is  $d_s(v) = \sum_{m \in N_s(v)} \varepsilon(m, v)$ . The minimum strict degree of  $H$  is  $\delta_s(H) = \wedge \{d_s(v) \mid v \in W\}$  and maximum strict degree of  $H$  is  $\Delta_s(H) = \vee \{d_s(v) \mid v \in W\}$ .

The strict neighborhood degree of a node  $v$  is defined as  $d_{SN}(v) = \sum_{m \in N_s(v)} \sigma(m)$ . The minimum strict neighborhood degree of  $H$  is  $\delta_{SN}(H) = \wedge \{d_{SN}(v) \mid v \in W\}$  and the maximum strict neighborhood degree of  $H$  is  $\Delta_{SN}(H) = \vee \{d_{SN}(v) \mid v \in W\}$ . A fuzzy graph  $H$  is said to be bipartite [27] if the vertex set  $W$  can be partitioned into two non empty sets  $W_1$  and  $W_2$  such that  $\mu(v_1, v_2) = 0$  if  $v_1, v_2 \in W_1$  or  $v_1, v_2 \in W_2$ . Further if  $\varepsilon(m, v) = \rho(m) \wedge \rho(v)$  for all  $m \in W_1$  and  $v \in W_2$ , then  $H$  is called a complete bipartite graph and is denoted by  $K_{\rho_1, \rho_2}$  where  $\sigma_1$  and  $\sigma_2$  are respectively the restrictions of  $\rho$  to  $W_1$  and  $W_2$ .

A node  $m$  is said to be isolated if  $p(m, v) = 0$  for all  $v \neq m$ .

### III. STRICT DOMINATION IN FUZZY GRAPHS

The concept of domination in graphs was put forth by Ore and Berge in 1962 and further studied by Cockayne and Hedetniemi [12].

We refer to [11] for the terminology of domination in crisp graphs.

For a vertex  $v$  of a graph  $H : (W, E)$ , recall that a neighbor of  $v$  is a vertex adjacent to  $v$  in  $H$ . Also the neighborhood  $N(v)$  of  $v$  is the set of neighbors of  $v$ . The closed neighborhood  $N[v]$  is defined as  $N[v] = N(v) \cup \{v\}$ . A vertex  $v$  in a graph  $H$  is said to dominate itself and each of its neighbors, that is,  $v$  dominates the vertices in  $N[v]$ . A set  $S$  of vertices of  $H$  is a dominating set of  $H$  if every vertex of  $W(H) - S$  is adjacent to some vertex in  $S$ . A minimum dominating set in a graph  $H$  is a dominating set of minimum cardinality. The cardinality of a minimum dominating set is called the domination number of  $H$  and is denoted by  $\gamma(H)$ .

These ideas are extended to fuzzy graphs using strict arcs as follows.

Nagoor gani and Chandrasekaran [16] put forth the concept of domination using strict arcs. These concepts have motivated researchers to reformulate some of the concepts in domination more effectively. The studies in [16] is our main motivation and we have modified the definition of domination number of a fuzzy graph. This modification is required due to the fact that the parameter ‘domination number’ defined by Nagoor gani and Chandrasekharan is inclined more towards graphs than to fuzzy graphs. Using the new definition of domination number we have reduced the value of old domination number and extracted classic results in a fuzzy graph.

Demonstrates Nagoor gani a node  $v$  in a fuzzy graph  $H$  is said to strictly dominate itself and each of its strict neighbors, that is,  $v$  strictly dominates the nodes in  $N_s[v]$ . A set  $J$  of nodes of  $H$  is a strict dominating set of if every node of  $W(H) - J$  is a strict neighbor of some node in  $J$ . They defined a minimum strict dominating set in a fuzzy graph  $H$  as a strict dominating set with minimum number of nodes [16].

Also in [17], Nagoor gani defined a minimum strict dominating set as a strict dominating set of minimum scalar cardinality. The scalar cardinality of a minimum strict dominating set is called the strict domination number of  $H$ .

The concept of strict domination in fuzzy graphs has applications to several fields. Strict domination arises in fuzzy location problems in networks. In such applications, the membership values of strict arcs in fuzzy graph give more maximum results for strict domination number than using membership values of nodes. Hence we have modified the definition of strict domination number using membership values of strict arcs and extracted some interesting results using the new definition.

**Definition 3.1** The weight of a strict dominating set  $J$  is defined as  $W(J) = \sum_{u \in D} \mu(m, v)$ , where  $\mu(m, v)$  is the minimum of the membership values (weights) of the strict arcs incident on  $u$ . The strict domination number of a fuzzy graph  $H$  is defined as the minimum weight of strict dominating sets of  $H$  and it is denoted by  $\gamma_s(H)$  or simply  $\gamma_s$ . A minimum strict dominating set in a fuzzy graph  $H$  is a strict dominating set of minimum weight.

Let  $\gamma_s(H)$  or  $\gamma_s$  denote the strict domination number of the complement of a fuzzy graph  $H$

**Remark 3.1** Note that for any undirected fuzzy graph for any  $p, q \in W$ , if  $(p, q)$  is a strict arc, then  $(q, p)$  is also a strict arc. That is if  $p$  strictly dominates  $q$ , then  $q$  strictly dominates  $p$  and hence strict domination is a symmetric relation on  $W$ .

**Remark 3.2** If all the nodes are isolated, then  $W$  is the only strict dominating set of  $H$  of order  $p$  and  $\gamma_s = 0$ . That is  $N_s(m) = \phi$  for each  $m \in W$ .

**Example 3.1** Consider the fuzzy graph given in Fig. 1.

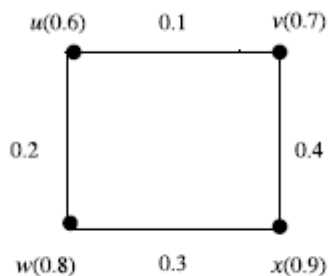


Fig. 1 Illustration of strong domination in fuzzy graphs

In this fuzzy graph, strict arcs are  $(u, w)$ ,  $(w, x)$  and  $(v, x)$ . The minimum strict dominating sets are  $J_1 = \{u, v\}$  and  $J_2 = \{w, x\}$ , where

$$W(J_1) = 0.2 + 0.3 = 0.5 \text{ and } W(J_2) = 0.2 + 0.3 = 0.5.$$

Hence

$$\gamma_s = 0.5.$$

**Proposition 3.1** If  $H : (W, \rho, \epsilon)$  is a complete fuzzy graph, then  $\gamma_s(H) = \Lambda\{\mu(m, v) \mid m, v \in \rho^*\}$ .

**Proof** Since  $H$  is a complete fuzzy graph, all arcs are strict [29] and each node is adjacent to all other nodes. Hence  $J = \{m\}$  is a strict dominating set for each  $u \in \rho^*$ . Hence the result follows.

**Proposition 3.2** Let  $H : (W, \rho, \epsilon)$  be a non trivial fuzzy graph of size  $q$ . Then  $\gamma_s(H) = q$  if and only if all arcs are strict and each node is either an isolated node or has a unique strict neighbor.

**Proof** If all arcs are strict and each node is either an isolated node or has a unique strict neighbor, then the minimum strict dominating set of  $H$  is a set  $D$  containing nodes each of which is either an isolated node or an end node of each unique strict arc. Hence weight of  $D$  is exactly

$$W(D) = \sum_{u \in D} \rho(u, v) = q.$$

Hence  $\gamma_s = q$ .

Conversely, suppose that  $\gamma_s = q$ . To prove that all arcs are strict and each node is either an isolated node or has a unique strict neighbor. If possible let  $(m, v)$  be an arc of  $H$  which is not strict. Then the weight of this arc is not counted for getting  $\gamma_s$ . Hence  $\gamma_s < q$ , a contradiction. Hence all arcs are strict.

Let  $p$  be any node of  $H$ . If  $p$  is an isolated node, then clearly  $p$  is contained in the minimum strict dominating set. If possible suppose  $p$  has two strict neighbors say  $v$  and  $w$ . Then exactly one of the weights of the arcs  $(p, v)$  and  $(p, w)$  contribute to the weight of the minimum strict dominating set. Hence  $\gamma_s < q$ , a contradiction. Hence each node has a unique strict neighbor.

In particular,  $\gamma_s(H) = 0$  when  $H$  is a complete fuzzy graph.

**Remark 3.3** In Proposition 3.2,  $H$  is exactly  $S \cup N$  where  $S$  is a set of isolated nodes, may be empty and  $N$  is a union of  $K_2$  s.

**Proposition 3.3**  $\gamma_s K_{\rho_1, \rho_2} = \mu(m, v)$  or  $2\epsilon(m, v)$ , where  $\epsilon\{m, v\}$  is the weight of a weakest arc in  $K_{\rho_1, \rho_2}$

**Proof** In  $K_{\rho_1, \rho_2}$  all arcs are strict. Also each node in  $W_1$  is adjacent with all nodes in  $W_2$ . Hence in  $K_{\rho_1, \rho_2}$ , the strict dominating sets are  $W_1, W_2$  and any set containing 2 nodes, one in  $W_1$  and other in  $W_2$ . If  $W_1$  or  $W_2$  contains only one element say  $m$ , then  $D = \{m\}$  is the minimum strict dominating set in  $H$ . Hence  $\gamma_s K_{\rho_1, \rho_2} = \mu(m, v)$  where  $\mu(m, v)$  is the minimum weight of the arcs adjacent on  $m$ . If each  $W_1$  and  $W_2$  contains more than one element, then the end nodes say  $\{m, v\}$  of any weakest arc  $(w, v)$  in  $K_{\rho_1, \rho_2}$  form the minimum strict dominating set.

Hence  $\gamma_s K_{\rho_1, \rho_2} = \epsilon(m, v) + \epsilon(m, v) = 2\epsilon(m, v)$ . So the proposition is proved.

**Remark 3.4** In any fuzzy graph  $H : (q, \rho, \epsilon)$ ,  $\gamma_s < \epsilon$  always holds, since  $\mu(p, q) < \rho(p) \wedge \rho(q)$  for all  $p, q \in \rho^*$ . [ $p$  is the scalar cardinality of  $H$ , which is got by using the node weights and  $\gamma_s$  is the weight of the minimum strict dominating set, which is got by using the arc weights].

For the strict domination number  $\gamma_s$ , the following theorem gives a Nordhaus- Gaddum type result.

**Theorem 3.1** For any fuzzy graph  $H : (W, \rho, \epsilon)$ ,  $\gamma_s + \overline{\gamma}_s < 2p$ .

**Proof** Since  $\gamma_s < p$  and  $\overline{\gamma}_s < p$  by Remark 3.4, we have  $\gamma_s + \overline{\gamma}_s < p + p = 2p$ .

**Definition 3.2 [16]** A strict dominating set  $J$  is called a minimal strict dominating set if no proper subset of  $J$  is a dominating set.

**Example 3.2** In Fig.1 of Example 3.1,  $J = \{u, v\}$  is a minimal strict dominating set.

**Theorem 3.2 [16]** Every non trivial connected fuzzy graph  $H$  has a strict dominating set  $J$  whose complement  $W - J$  is also a strict dominating set.

**Theorem 3.3 [16]** Let  $H$  be a fuzzy graph without isolated nodes. If  $J$  is a minimal strict dominating set, then  $W - J$  is a strict dominating set.

**Theorem 3.4** For any fuzzy graph  $H : (W, \rho, \rho)$  without isolated nodes  $\gamma_s < p/2$ .

**Proof** Let  $D$  be a minimal strict dominating set of  $H$ . Then by Theorem 3.3,  $W - D$  is a strict dominating set of  $H$ . Then  $\gamma_s \leq W(D)$  and  $\gamma_s < W(W - D)$ .

Therefore  $2\gamma_s \leq W(D) + W(W - D) < p$  which implies  $\gamma_s < p/2$ . Hence the proof.

**Corollary 3.1** Let  $H$  be a fuzzy graph such that both  $H$  and  $\overline{H}$  have no isolated nodes. Then  $\gamma_s + \overline{\gamma_s} < p$ . Further equality holds if and only if  $\gamma_s = \overline{\gamma_s} = p/2$ .

Proof By Theorem 3.4,  $\gamma_s < p/2$ ,  $\overline{\gamma_s} \leq p/2$

$$\Rightarrow \gamma_s + \overline{\gamma_s} \leq p/2 + p/2 = p,$$

$$\text{that is } \gamma_s + \overline{\gamma_s} < p.$$

If  $\gamma_s = \overline{\gamma_s} = p/2$ , then obviously  $\gamma_s + \overline{\gamma_s} = p$ . Conversely, suppose  $\gamma_s + \overline{\gamma_s} = p$ . Then by Theorem 3.4, we have  $\overline{\gamma_s} < p/2$ ,  $\overline{\gamma_s} \leq p/2$ . If either  $\gamma_s < p/2$  or  $\overline{\gamma_s} < p/2$ , then  $\gamma_s + \overline{\gamma_s} < p$ , which is a contradiction. Hence the only possibility is that  $\gamma_s = \overline{\gamma_s} = p/2$ .

**Remark 3.5** Note that Theorem 3.4 and Corollary 3.1 are true when we use the definition of domination number in [13].

**Theorem 3.5** In any fuzzy graph  $H : (W, \rho, \varepsilon)$ ,  $\gamma_s = p/2$  if and only if the following conditions hold.

- 1) All nodes have the same weight.
- 2) All arcs are M- strict arcs.
- 3) For every minimum strict dominating set  $J$  of  $H$ ,  $|J| = n/2$ , where  $n$  is the number of nodes of  $H$  and  $n$  is even.

Proof If all the conditions 1), 2), 3) hold, then obviously  $\gamma_s = p/2$ .

Conversely, suppose  $\gamma_s = p/2$ . If some nodes say  $m$  and  $v$  have different weights, then the arc weight corresponding to these nodes is  $\mu(m, v) \leq \rho(m) \wedge \rho(v)$ .

If  $\mu(m, v) < \rho(m) \wedge \rho(v)$ , then obviously  $\gamma_s < p/2$ , a contradiction.

If  $\sigma(m, v) = \sigma(m) \wedge \rho(v)$ , then  $(m, v)$  becomes an M- strict arc.

If  $|D| < n/2$ , then clearly  $\gamma_s < p/2$ , a contradiction.

Hence all the conditions are sufficient.

The following theorem gives a lower bound and an upper bound for the strict domination number of a connected fuzzy graph.

**Theorem 3.6** For any connected fuzzy graph  $H : (W, \rho, \mu)$  of order  $p$ ,

$$\min_{m, v \in W} \mu(m, v) < \gamma_s \leq p - \Delta_{SN}(H).$$

Proof The first part is trivial. For the second part, let  $m$  be a node of  $H$  such that  $d_{SN}(m) = \Delta_{SN}(H)$ . Then  $W - N_s(m)$  is a strict dominating set. Therefore  $\gamma_s < W - N_s(m) < p - \Delta_{SN}(H)$ . Therefore  $\gamma_s \leq p - \Delta_{SN}(H)$ .

**Remark 3.6** The above inequality cannot be improved further. For example, for the complete fuzzy graph  $H$ ,  $\min_{u, v \in V} \varepsilon(m, v) = \gamma_s = p - \Delta_{SN}(H)$ .

**Remark 3.7** Clearly,  $\Delta_s(H) < \Delta_{SN}(H)$ . (since  $\mu(p, q) \leq \rho(p) \wedge \rho(q)$ ) and hence  $\gamma_s \leq p - \Delta_s(H)$ .

#### IV. CONCLUSION

The concept of domination in graph is very rich both in theoretical developments and applications. More than thirty domination parameters have been investigated by different authors, and in this paper the concept of strict domination number has been modified for fuzzy graphs using the membership values of strict arcs.

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