PADAVON GRACEFUL LABELING FOR SOME PATH RELATED GRAPHS

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Abstract
A (p,q) connected graph is padavon graceful graph if there exists an injective map f: E(G) → { 1,1,1,2,2,3,4,5,7,…,2q-1} so that induced map f_+ : V(G) → { 0, 1, …,p+1} defined by $f_+(x)$ = |f(u)-f(v)| where the vertex x is incident with other vertex y and makes all the edges distinct. In this article, the padavon gracefulness of some path related graphs graphs are obtained.

Keywords Padavon sequence, vertex labeling, edge labeling, graceful, padavon graceful

1. Introduction

In this paper we consider only finite, undirected non-trivial graphs *G = (V(G),E(G))* with the vertex set *V(G)* and the edge set *E(G)*.We refer to Gallian for all detailed survey of graph labeling. For standard terminology and notations we follow Harary. Graph labeling is a strong communication between number theory and structure of graphs. The study of graceful graphs and graceful labeling methods was introduced by Rosa .Rosa defined a β-valuation of a graph G with q edges an injection from the vertices of G to the set {0,1, …,q} such that when each edge uv is assigned the label |f(u)-f(v)|, the resulting edges are distinct. β – valuation is a function that produces graceful labeling. However the term graceful labeling was not used until Golomb studied such labeling several years later. The Graph labeling is an assignment of numbers to the vertices or edges or both subject to certain conditions. If the domain of the mapping is the set of vertices(edges) then the labeling is called a vertex labeling(edge labeling).

2. DEFINITIONS.

Definition 2.1

A walk *W* in a graph *G* is an alternating sequence of vertices and edges $v_0, e_1, v_1, e_2, v_2, e_3,\ldots, v_{n-1}, e_n, v_n$ such that $e_i = v_{i-1}v_i$ is an edge of *G*, $1 \le i \le n$. The number of edges in $v_0 - v_n$ walk is the length of the walk. It is also denoted by $v_0\, v_1\, v_2 \ldots v_{n-1}\, v_n$. If $v_0 = v_n$, then *W* is called a closed walk. If $v_0 \neq v_n$, then *W* is called a open walk. If all the edges of *W* are distinct, then it is called a trail.

Definition 2.2

If all the vertices in a walk are distinct, then it is called a path. A path of length *n* is denoted by P_n and it contains *n*+1 vertices. A path in a graph is a sequence of vertices such that from each of its vertices there is an edge to the next vertex in the sequence. The first vertex is called the start vertex and the last vertex is called the end or terminal vertices of the path and the other vertices in the path are internal vertices.

Definition 2.3 The graph obtained by joining a pendent edge at each vertex of a path P_n is called a comb and is denoted by Pn Θ k_1 or P_n^+

Definition 2.4

A generalized star is a tree obtained from a star by extending each edge to a path.

Definition 2.5

The join $G_1 + G_2$ of G_1 and G_2 consists of G_1 U G_2 and all the vertices of G_1 are joined with each vertex of G_2. The graph $P_n + K_1$ is called a fan f_n.

Definition 2.6

$S_{m,\,n}$ denotes a star with *m* spokes in which each spoke is a path of length *n*.

Definition 2.7

A function f of a graph G is called a graceful labeling with m edges, if f is an injection from the vertex set of G to the set {0,1,…,m} such that when each edge uv is assigned the label |f(u)-f(v)| and the resulting labels are distinct. Then the graph G is graceful.

Definition 2.8

The padavon sequence is the sequence of integers P(n) defined by
The initial values P(0) = P(1) = P(2) = 1,
and the recurrence relation P(n) = P(n-2) + P(n-3).

The first few values of P(n) are

1, 1, 1, 2, 2, 3, 4, 5, 7, 9,12, 16, 21, 28, 37, 49, 65, 86, 114, 151, 200, 265, 351, 465, 616,…

Definition 2.9

Let G be a (p, q) graph. An injective function f: $V(G) \rightarrow \{1,2,3, \dots, N \geq q\}$ is said to be a padavon graceful labeling if an induced edge labeling f^* defined by $f^*(uv) = |f(u) - f(v)|$ is a bijection from $E(G)$ onto the set $\{p_1, p_2, p_3, \dots, p_q\}$ where p_i is the i^{th} padavon number. Then G is called a padavon graceful graph if it admits a padavon graceful labeling.

3. Padavon graceful labeling for some path related graphs

In this section, the gracefulness of padavon sequence for some path related graphs P_n, P_n^+ - v, $(n \geq 1)$, $S_{m,n}$ are discussed in detail.

Theorem 3.1

For any positive integer n, P_n^+ - v is a padavon graceful graph where v is a pendent vertex of P_n^+ attached with either a pendent vertex of P_n or the neighbor of a pendent vertex of P_n

Proof:

Let $\{u_0, u_1, u_2, u_3, \dots, u_n\}$ be the vertices of the path P_n of P_n^+.
Let $G = P_n^+$ - v, where v is either v_0 or v_1.
Let $V(G) = \{u_i : 0 \leq i \leq n\} \cup \{v_i : 0 \leq i \leq n\} \backslash \{v = v_0 \text{ or } v_1\}$
$E(G) = \{u_i u_{i+1} : 0 \leq i \leq n-1\} \cup \{u_i v : 0 \leq i \leq n\} \backslash \{u_1 v_1\}$ (or) $\{u_0 v_0\}$
Then $|V(G)| = 2n+1$ and $|E(G)| = 2n$.
Define f: $V(G) \rightarrow \{0, 1,2,3, \dots, N\}$

$$\text{Let } E_1 = \bigcup_{i=0}^{n-1} \{|f(u_i u_{i+1})|\}$$
$$= \bigcup_{i=0}^{n-1} \{|f(u_i) - f(u_{i+1})|\}$$
$$E_2 = \bigcup_{i=0}^{n} \{f(u_i v_i)\} - \{f(u_0 v_0)\} \text{ (or)}$$
$$= \bigcup_{i=0}^{n} \{f(u_i v_i)\} - \{f(u_1 v_1)\}$$

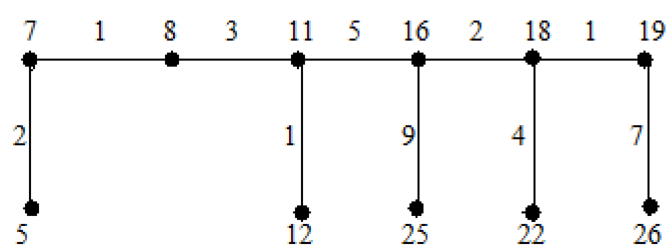
Accordingly as $v = v_0$ (or) $v = v_1$

$$f^*(E(G)) = E_1 \cup E_2$$
$$= \{p_1, p_2, p_3, \dots, p_{2n}\}$$

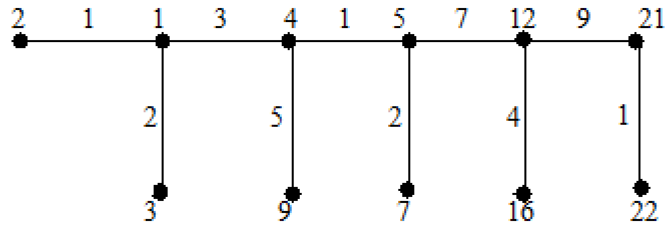
So the edges of G receives the distinct labels and f is a padavon graceful labeling of P_n^+ - v
Hence P_n^+ - v, (n $\geq$ 1 and v is either v_v (or)v_1) is a padavon graceful graph.

Example 3.2

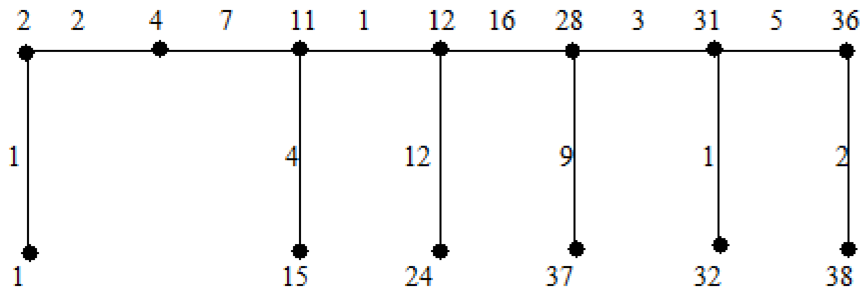
The following are some of the examples of padavon graceful labeling for some path related graphs.



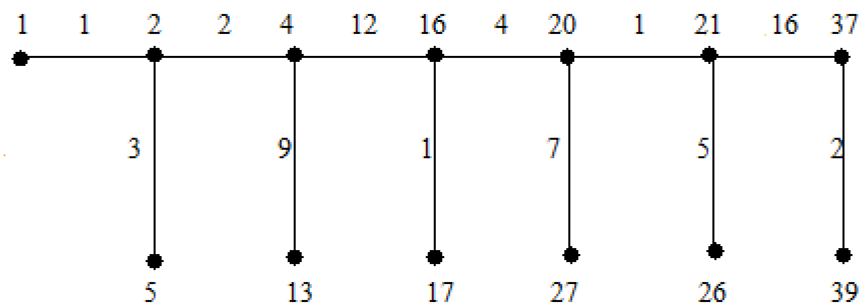
P_5^+ - v_1



$P_5^+ - v_0$



$P_6^+ - v_1$



$P_6^+ - v_0$

Theorem 3.3

$S_{m,n}$ is a padavon graceful graph

Proof:

Let $G = S_{m,n}$

Let $V(G) = \{u_0\} \cup \{u_j^i : 1 \leq i \leq m \text{ and } 1 \leq j \leq n\}$ and

$E(G) = \{u_j^i\, u_{j+1}^i : 1 \leq i \leq m \text{ and } 0 \leq j \leq n-1\}$

Then $|V(G)| = mn+1$ and $|E(G)| = mn$

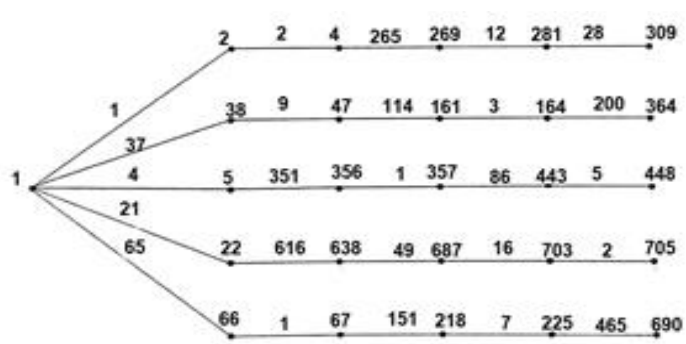
Define f: $V(G) \rightarrow \{0, 1, 2, 3, \ldots, N\}$ by $f(u_0) = p_1$

The following example reveals that it is possible to assign suitable numbers for the values so as the edges receive distinct padavon numbers as it is shown in the following examples.

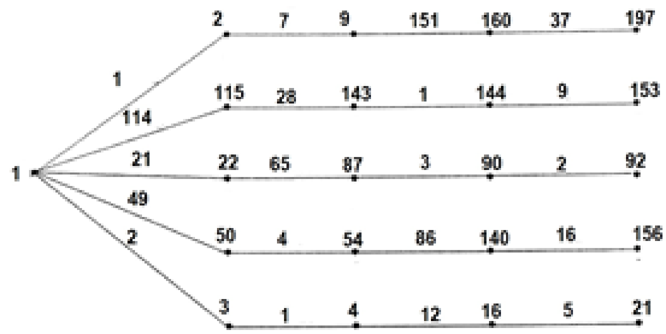
Therefore f is a padavon graceful labeling of G.

Hence $S_{m,n}$ is a padavon graceful graph.

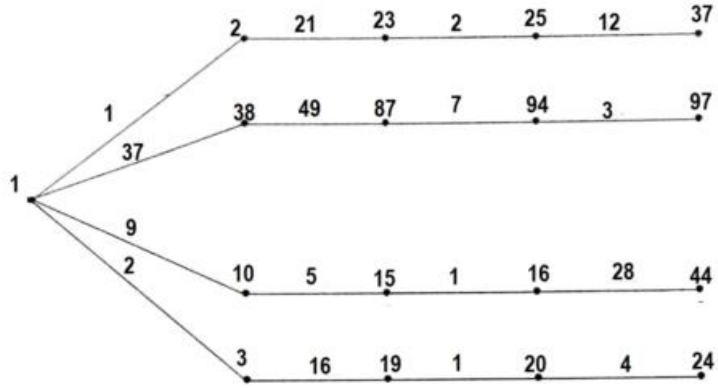
Example 3.4.



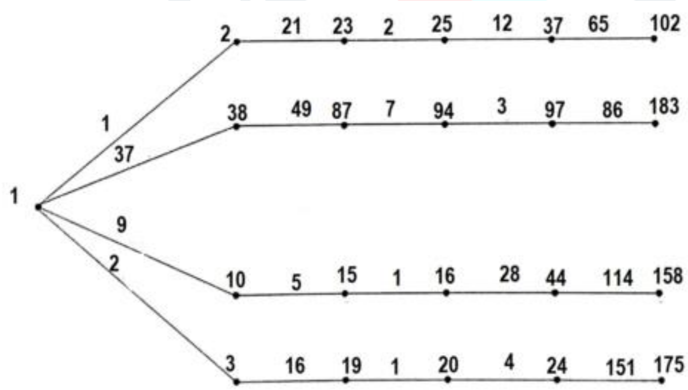
$S_{5,5}$



$S_{5,4}$



$S_{4,4}$



$S_{4,5}$

4. Padavon graceful labeling for some special graphs

In this section it is shown that f_m @ P_n is padavon graceful graph.

Definition 4.1

The graph $G = f_m$ @ P_n consists of a fan f_m and a path P_n of length n which is attached with the maximum degree of the vertex of f_m.

Theorem 4.2

f_m @ P_n is a padavon graceful graph for any positive integer m and n.

Proof:

Let $G = f_m$ @ P_n

Let $v_1, v_2, v_3, \ldots, v_m, v_{m+1}$ and u_0 be the vertices of a fan f_m and $u_0, u_1, u_2, u_3, \ldots, u_n$ be the vertices of a path P_n

Here $E(G) = \{v_i v_{i+1} : 1 \leq i \leq m\} \cup \{u_0 v_i : 1 \leq i \leq m+1\} \cup \{u_i u_{i+1} : 0 \leq i \leq n-1\}$

Then $|V(G)| = m + n + 2$ and $|E(G)| = 2m + n + 1$

Define $f: V(G) \rightarrow \{p_1, p_2, p_3, \ldots p_{2m+n+1}\}$ such that

$f^*(e_i)$ is the i^{th} padavon number for all $i = 1, 2, 3, \ldots, 2m+n+1$

Let $E(G) = \{f^*(v_i v_{i+1}) : 1 \leq i \leq m\}$

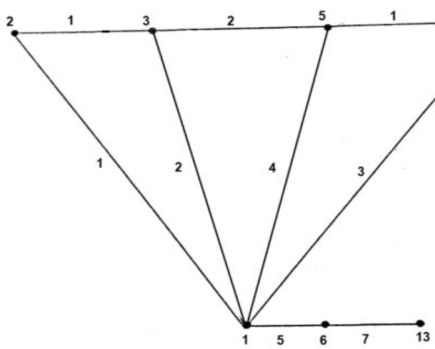
$= \{|f(v_i) - f(v_{i+1})| : 1 \leq i \leq m\}$

Here the edges of f_m @ P_n are labeled in such a manner that they receive the distinct labels.

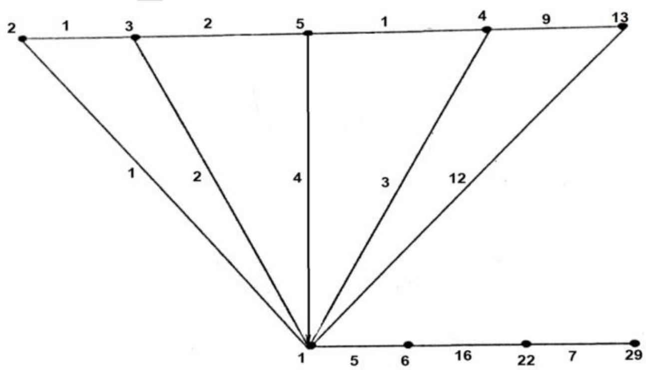
Hence f_m @ P_n is a padavon graceful graph when for any positive integer m.

Example 4.3

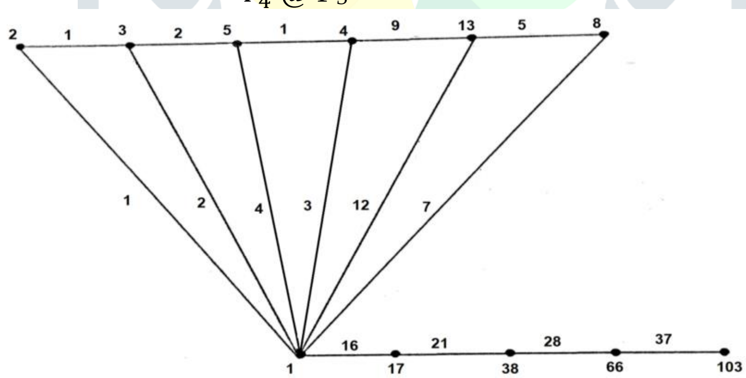
The following graphs shows f_m @ P_n, when $m = n$, $m > n$ and $m < n$ are padavon graceful labeling



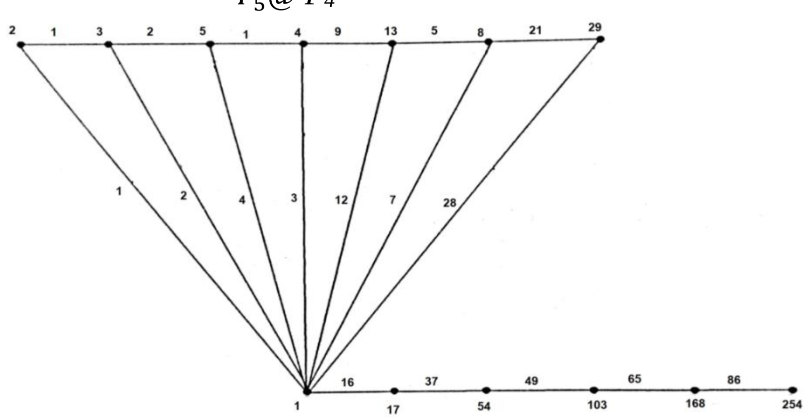
F_3 @ P_2



F_4 @ P_3



F_5@ P_4



F_6 @ P_5

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