DIFFERENTIAL SCATTERING CROSS-SECTIONS OF e^+-Mg-TARGET

[1]Mukesh Kumar Sinha and[2]Tarun Kumar Dey
[1]Department of Physics, L.N.J.P.Institute of Technology, Chapra - 841302 (Bihar)
[2]Post Graduate Department of Physics, L.S.College, Muzaffarpur-842001 (Bihar)

Abstract:-The differential cross section (DCS) of e^+-Mg scattering in the intermediate energy range has been calculated by using distorted wave (screened Coulomb waves) method without polarization potential (DCSWP) and with polarization potential (DCSPP). The impact of polarization potential has been analyzed. Thefirst Born approximation has been used. The our results are compared with theoretical results ofKhare,Kumar and Lata.

***Keywords*: - Elastic Scattering, distorted wave method, polarization potential.**

1. INTRODUCTION:-

A lot of theoretical and experimental investigations have been done on e^--Mg scattering but theoretical investigations on e^+-Mg Scattering are few. The partial wave method has been used in analysis of e^+-Mg scattering by Khare, Kumar and Lata(1,2,3). They have used real optical potential and energy dependent Buckingham type polarization potential. It observed that the spin-orbit effect is not appreciable for the energy range 100eV-500eV

We have used the distorted wave method that account for the effect of continuum partially. In this method, we included a part of Coulomb potential in continuum wave function in the initial channel. The nucleus of the target atom has been taken as screened nucleus due to presence of target electrons.

So, incident positron will see an effective charge. The Valence electron of Mg is loosely bound, so the effect of polarization is high.

The polarization potential has been included in our calculation which account for excited states partially. Moreover, the effect of continuum in our method has been taken via distorted continuum wave function.

Infinite no. of channels are open in e^+-Mg scattering in the intermediate energy range. The effect of loss of inelastic flux is not negligible. It has been found that positronium formation cross-section for e^+-Mg scattering is very high at low energy. With increase of energy, capture cross-section decreases very rapidly. As a result, the effect of capture channel on direct channel is not expected to be appreciable. So, in our investigation, positronium formation channel has been neglected.

2. BASIC THEORY

The Hamiltonian $H$for the e^+-Mg atom can be expressed as :

$$H = \frac{-1}{2}\left(\nabla_{r_1}^2 + \nabla_{r_2}^2 + \nabla_{r_3}^2\right) - \frac{Z}{r_1} - \frac{Z}{r_2} + \frac{1}{r_{12}} + V \qquad \text{- - - - - (1)}$$

$$V = \frac{Z}{r_3} - \frac{1}{r_{13}} - \frac{1}{r_{23}} \qquad \text{- - - - -(2)}$$

i.e. V = interaction potential of incident positron with target electrons.

In the centre of mass frame of reference:-$\bar{r}_1$,$\bar{r}_2$and $\bar{r}_3$are position Vectors of target electrons and the incident positron w.r.t screened nucleus respectively.

$\nabla_{r_1}^2$, $\nabla_{r_2}^2$ and $\nabla_{r_3}^2$ are Kinetic energy operators .

Here ,

$$H = H_o + V \qquad \text{- - - - - - (3)}$$

Where, H_o= unperturbed Hamiltonian

The Hamiltonian H is split as :

$$H = H_1 + W \quad \text{------(4)}$$

With

$$H_1 = H_0 + U \quad \text{------(5)}$$

Where, $U = \frac{\delta}{r_3}$ ------(6)

$$W = \frac{Z-\delta}{r_3} - \frac{1}{r_{13}} - \frac{1}{r_{23}} \quad \text{------(7)}$$

And $\delta =$ Screening parameter

The functions ϕ_r, χ_r and φ_r satisfy the following Schrodinger wave equations with HamiltonianH_o, H_1 and H respectively.

$$\left.\begin{aligned} H_o\phi_r &= E_r\phi_r \\ H_1\chi_r &= E_r\chi_r \\ H\psi_r &= E_r\psi_r \end{aligned}\right\} \quad \text{--------(8)}$$

Where, E_r =total energy of the system .

Let the system move from initial bound state $|i>$ with momentum $\bar{K}_i$to the final bond state $|f>$ with momentum$\bar{K}_f$. i.e. target electrons are exited from i[th] state to f[th]state.

Then the$T$-matrix element can be expressed as:

$$T_{i\to f} = \langle \phi_f | V | \psi_i{}^{(+)} \rangle \text{------(9)}$$

Here, $\psi_i{}^{(+)}$is expanded w.r.t.H_1

$$\psi_i{}^{(+)} = \sum_{n=o}^{\infty} \left(G^{(+)}w\right)^n \chi_i{}^{(+)} \quad \text{-----(10)}$$

Where ,$G^{(+)}$ is the Green function for H_1 .

It would not be crude approximation to retain only the first order term in the above expression.

If we assume that W is a weak potential. Then the T-matrix element can be expressed as:

$$T_{i\to f} = \langle \phi_f | V | \chi_i{}^{(+)} \rangle \quad \text{-------(11)}$$

In atomic unit, differential cross section is expressed as :

$$\frac{d\sigma}{d\Omega} = \frac{\mu^2}{4\pi^2}\frac{K_f}{K_i}|T|^2 \quad \text{------(12)}$$

In present case, wave functions have been taken in the following forms:

$$\phi_r = \exp(i\bar{K}.\bar{r}_3)\phi_{nlm}(\bar{r}_1, \bar{r}_2) \quad \text{------(13)}$$

$$\chi_i^{(+)} = \Gamma(1+ia).exp\left(i\bar{K}.\bar{r}_3 - \frac{\pi a}{2}\right){}_1F_1(-ia;1;iK_ir_3 - i\bar{K}_i.\bar{r}_3)\phi_{nlm}(\bar{r}_1, \bar{r}_2)$$

- - - - - (14)

Where ,$a = \frac{\delta}{K}$

Here,$\bar{K}$ is the momentum for the direct channel in which distortion is introduced. The screening parameter δ is evaluated by the method of Junker(4).

This method includes the part of Coulomb potential in incident continuum wave function. The effect of continuum is taken into account partially.

The radical part of atomic wave function (HartreeFock wave function)for Mg-atom is expressed as

$$\phi(r) = \sum_{i=1}^{2} a_i r \exp(-b_i r) + \sum_{i=3}^{4} a_i r^2 \exp(-b_i r) \quad \text{(in ground state)} \qquad \text{- - - - - - (15)}$$

Now ,T - matrix can be expressed as:

$$T_{i\to f} = MF\,[I_o + (I_2 + I_3)] \text{- - - - - - - (16)}$$

Where ,

$$MF = \Gamma(1 + ia)\, exp\,(-\frac{\pi a}{2}) \text{- - - - - - (17)}$$

And

$$I_o = \int \exp(i\bar{q}.\bar{r}_3).{}_1F_1(-ia; 1; iK_i r_3 - i\bar{K}_i.\bar{r}_3)(\frac{Z}{r_3})d\bar{r}_3 \quad \text{- - - - - - - (18)}$$

$$\text{Where },\bar{q} = \bar{K}_i - \bar{K}_f$$

The integrals $I_2$and I_3have the same form and they are expressed in terms of function G:-

$$G = \iint \exp(i\bar{q}.\bar{r}_3)\ {}_1F_1(-ia; 1; .ik_i r_3 - i\bar{K}_i.\bar{r}_3).\exp\left(-\lambda_{ij} r_1\right)\left(\frac{1}{r_{13}}\right) d\bar{r}_1 d\bar{r}_3$$

$$\text{- - - - - - - (19)}$$

$$I_2 = \left[\sum_{i=1}^{2}\sum_{j=1}^{2} N_{ij}\frac{\partial^3 G}{\partial\lambda_{ij}^3} + \sum_{i=1}^{2}\sum_{j=3}^{4} N_{ij}\frac{\partial^4 G}{\partial\lambda_{ij}^4} + \sum_{i=3}^{4}\sum_{j=3}^{4} N_{ij}\frac{\partial^5 G}{\partial\lambda_{ij}^5}\right] \quad \text{- - - - - - (20)}$$

Where ,$N_{ij} = (-\frac{1}{4\pi})a_i a_j$ and $\lambda_{ij} = b_i + b_j$.

All integrals are evaluated by using Nordsieck(5) Method.

In present case, the asymptotic form of the polarization potential is

$V_p = (\frac{-\propto}{2r^4})$, where $r =$ dipole polarisability

The differential cross-sections are calculated with polarization potential (DCSPP) and without polarization potential (DCSWP) , So that effect of polarization potential may be properly analyzed.

Moreover, the first Born approximationhas been calculated for the analysis of $e^+ -$ Mg scattering.

3.RESULTS AND DISCUSSION

The differential cross section calculated by using the distorted wave method without polarization potential (DCSWP) , First Born approximation (DCSB) and distorted wave method with polarization potential (DCSPP) are represented by curve A,B and C respectively.

The differential cross-section calculated by using partial wave method of Khare, Kumar and Lata(1) is represented by the curve D.

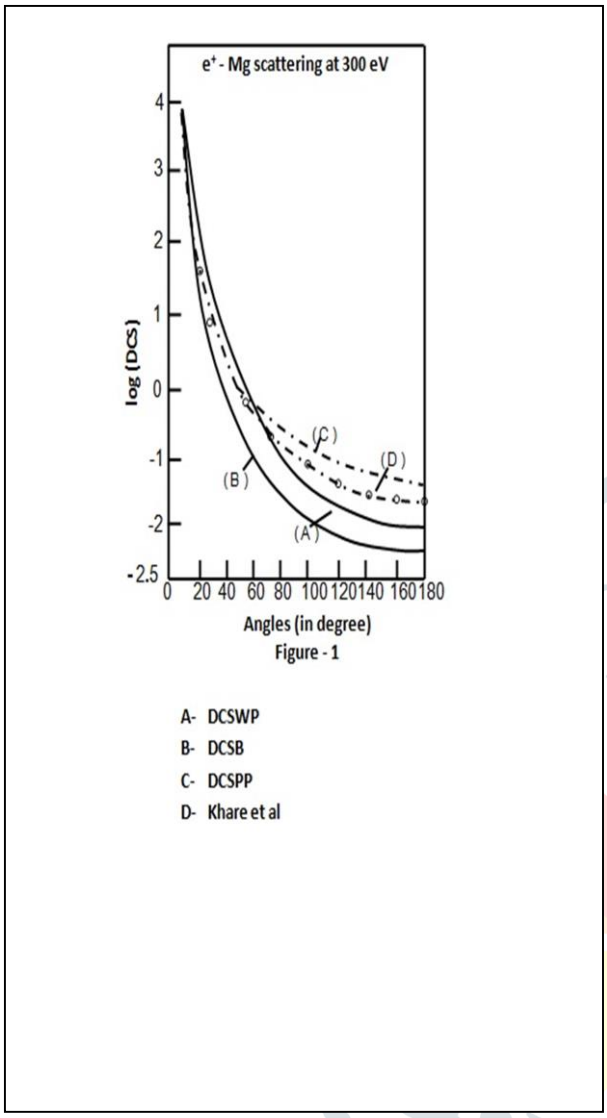

Figure - 1

A- DCSWP
B- DCSB
C- DCSPP
D- Khare et al

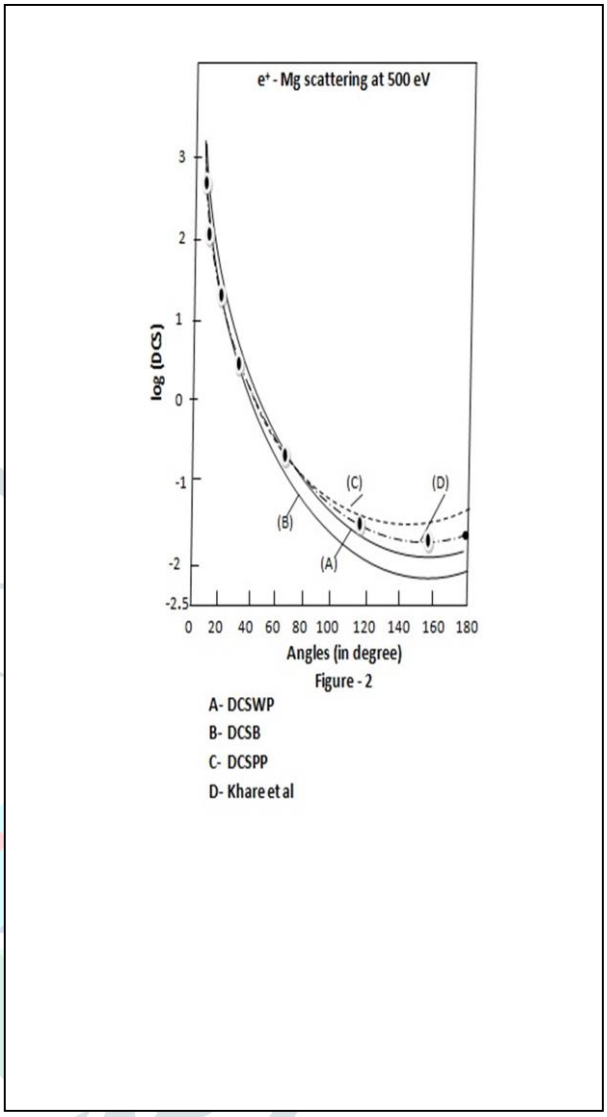

Figure - 2

A- DCSWP
B- DCSB
C- DCSPP
D- Khare et al

At 300 eV, curve B lies below curve A. With increase of energy, closeness between curve A and B has increased.This is expected result.

The relative reduction in DCSPP w.r.t. DCSWP is maximum at the angle of scattering nearly10°. Thus effect of polarization is maximum in forward direction of DCS.

The curve B lies below the curve A and C .Thus DCSPP and DCSWP results are higher than the first Born results.

At 500 eV, closeness between curves A (i.e. DCSWP results) and B (i.e. first Born results) has improved.

The relative reduction in DCSPP w.r.t DCSWP is maximum at angle of scattering nearly 5°. Thus effect of polarization is maximum in forward direction of DCS.

At 300eV and 500 eV, curveC is closer to curve D.

Thus , there is good agreement between DCSPP results and DCS results of Khare, Kumar, and Lata(1).

These facts established the validity of our method in analysis of e^+-Mg scattering in intermediate and high energy ranges.

References

[1]. S.P. Khare, Ashok Kumar and Kusum Lata, J. phys. B: At. Mol. Phys. 16:4419(1983)
[2]. G.F. Gribakin and W.A.King ,Canadian J.Phys.1996,74(7-8):449-459.
[3]. Luis A.Poveda,DeniseAssafrao and Jose R.Mohallem.Eur.phys.J.D(2016) 70: 152
[4]. B.R Junker, phys. Rev. A 2:1552 (1975)
[5]. A. Nordsieck, phys. Rev. 93: 785 (1954)