Study of thermodynamic fluctuation theory with application to some critical phenomena in black hole

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Abstract:

There have been found some critical behaviors, like scaling power laws, divergence of fluctuations etc. in the black holes, which have certain intricacies in interpreting these notions. Using the thermodynamic fluctuation theory it is seen that divergences at a continuous turning point of the thermodynamic function indicates a change in stability. This gives some the relation between fluctuations and critical phenomena. It is found that the extremal phase of the black hole corresponds to a critical point of a continuous phase transition. The formalism of fluctuation theory can be extended to give a geometric interpretation of some of the usual critical behavior of black holes. Certain algorithms may exist between the critical behaviours and the thermo dynamic fluctuation taking place in the black holes which needs to be explored in this article.

Key words: Thermodynamic- fluctuation , critical behaviors, black holes.

1. Introduction: Those physical quantities which describe a macroscopic body in the state `of equilibrium are almost always close to their mean values. However, there are always certain deviations in the physical quantities from their mean values. This is the natural behavior of the macroscopic system. These deviations are known as thermodynamic fluctuations. The first statistical–mechanical theory of fluctuations was developed by Gibbs [1], But this work of Gibbs was not explored up to Einstein at that time. The Gibb's explanation of fluctuations was based on classical dynamics of particles in the micro-states defined in the phase space of position coordinates and momentum coordinates. Einstein's theory [2] is more general. The comparison of the Gibbs and Einstein approaches is found in Redound- Sukhanov article [3]. If we take the view that the thermodynamic fluctuation theory is to be relatively autonomous, then the simplicity of its internal logical structure is more important. Thus, we propose to build this theory on a fluctuation law which is postulated for an isolated system.

In the formation of matter a very important role is played by the scalar field, basically which has two components. The first component is the gravity field which is produced due to the mass of the matter and the second field is the pressure field. The effects of these two fields are opposite to each other. If initially the field is weak and the pressure is greater, then the pressure overcomes the field and the matter ball expands away leaving nothing behind it. If the gravity field is strong enough to overcome the pressure field then the ball collapsed to form a black hole. But there may be a third case such that the field cannot decide whether to evaporate away or collapse to form a black hole on the other hand it begins to oscillates rapidly performing a large number of oscillations in a finite time. This frame shows the scalar field as a function of position. The gravitational field changes the frequency of the field. This produces characteristics colors to be found in the black hole "Black Holes of nature are the most perfect macroscopic objects that are in the universe. since the general theory of relativity gives only a single unique family of solutions for their descriptions, they are the simplest objects as well"-This is a very beautiful explanation of black holes in the book , "The Mathematical Theory of Black Holes" given by S.Chandrashekar. But it was the revelation of Hawking, that black holes emit radiation, while they were considered only to absorb objects .This phenomena may be explained on the basis of the third case of both the fields forming the matter. It became very clear when Beckenstein showed that the temperature of the radiation can be attributed as the temperature of the black hole itself. While the fact that black holes are described by a set of very minimal number of fundamental quantities like mass, spin and charge, it is very puzzling that it be attributed with a thermo dynamical concept of temperature. We know that the very reason we have thermodynamically description of matter, is due to the macroscopic constituency. This puzzling contrast in the behavior of black holes stays unexplained even today, except for the proposed solutions of String theory. If we attribute equilibrium of thermo dynamical functions to black holes, then it can tempt many researchers to proceed further and study fluctuations and phase transitions related phenomena in them. There may be found critical behaviors, like scaling power laws, divergence of fluctuations etc. in which there are certain intricacies in interpreting these notions in the context of black holes. Also we do not have a complete microscopic picture of black holes as of now. This is the case that there has been attempts to apply methods of renormalization to the entanglement entropy of the black holes, which have been found to contribute in part to the entropy of Hawking radiation.

2. Entropy and thermodynamics: Entropy is a measure of disorder of the system. It was first of all introduced by R.Claussius in 1850 to find the amount of heat reversibly exchanged at constant temperature T. It establishes a relation between the thermodynamics and the statistical mechanics. It is the most extensive parameter of thermodynamics. The entropy is a part of the first law of thermodynamics as well as the second law of thermodynamics.

According to the first law of thermodynamics:

$$\delta Q = \delta U + P\,\delta V + \mu\,\delta N \quad \text{--} \quad 1$$

But according to second law of thermo dynamics : $\delta S = \frac{\delta Q}{T}$ ---------------------2

From equations (1) and (2), We can write :

$$T\,\delta S = \delta U + P\,\delta V + \mu\,\delta N$$

$$\delta S = \frac{1}{T}\,[\delta U + P\,\delta V + \mu\,\delta N] \quad \text{---} 3$$

Equation (3) represents the differential form of the entropy. Here we can say that entropy is a function of (U,V,N). So Knowing these quantity the entropy of the system can be determined and hence all the thermodynamic parameters of the ideal gas can be determined. For an isolated system, where there is no exchange of heat, ie $\delta Q = 0$. Therefore, $\delta S = 0$. Hence entropy is maximum. But for irreversible process it can be shown that $\delta S > 0$.Thus in thermodynamics the state of equilibrium may be defined as the state of the maximum entropy.

The entropy of an ideal gas may be written as:

$$S(N,T,P) = NK_B[S_O(T_O,P_O) + Log_e\{(\frac{T}{T0})^{5/2}(\frac{Po}{P})\}] \quad \text{-------------------------------------} 4$$

Where $S_O(T_O,P_O)$ is an arbitrary dimension less function of the state (T_O,P_O) .It comes due to integration .Since $PV = NK_BT$ and $U = \frac{3}{2}NK_BT$. Thus equation (4) gives the complete information about the ideal gas. Futher the entropy of the ideal gas can also be written in the form of f(N,V,U) as given below:

$$S(N,V,U) = NK_B[S_O(N0,V_O,U_O) + Log_e\{(\frac{N0}{N})^{5/2}(\frac{U}{U0})^{3/2}(\frac{V}{V0})\}] \text{ -------------------------5}$$

From this equation all the equations of the ideal gas can be obtained. As for example: $(\frac{\partial S}{\partial U})_{NV} = = \frac{3}{2}NK_BT$.--6

And $(\frac{\partial S}{\partial V})_{NV} = NK_BT$.---7

From statistical mechanics the entropy of the system is given by the natural logarithm of the number of microscopic states Ω which is read as:

$$S = Log_e\Omega \text{ ---8}$$

Here the Boltzmann constant is taken as unity. The microstate Ω will be functionof the macro state i.e. Ω = Ω(U,V,N). When entropy is a function of these variables equation (8) is very important because it provides the basic relation between the macroscopic thermodynamic quantity *entropy and the statistical microscopic physics ie number of microscopic states. From this equation it is apparent that when Ω* =1 then S =0.Thus when there is only one microscopic state, there is no probability of disorder so there will be no entropy.

3. ENTROPY AND Fluctuation: The main problem is to deduce the probability distribution of these deviations. The equation No. (8) may be written as : $\Omega = e^S$ This is the starting point of the thermodynamic fluctuation. It was first of all done by Einstein. However , we preferably denote P as the probability distribution , ie $P \propto e^S$ .The entropy about the fluctuation quantity xup to the second order may be written as $S(x) = S(0) - \frac{1}{2}\beta X^2$, Where $\beta = \frac{\partial 2s}{\partial X2}$, and S(o) = 0 because S has maximum value, therefore,

$\partial S/ \partial X$ is also zero. Therefore P(X) = Ae - $\frac{1}{2}$ β X^2

P(X) dx =(Ae - $\frac{1}{2}$ β X^2) dX --9

The constant A is given by the normalization condition,

$\int_{-\infty}^{+\infty} \mathrm{P(X)\ dx} = 1$---10

In this way the constant is found to be $\sqrt{\frac{\beta}{2\ \pi}}$ By Gaussiun Integration formula. There the probability distribution of the various values of Gaussian function may be written as:

P(X) = $\sqrt{\frac{\beta}{2\pi} e\ -\frac{1}{2}\ \beta\ X^2}$ ---11

This probability distribution is recognized as a Gaussias distribution function.

It reaches a maximum value when X=0 and decreases slowly as X increases. The mean squared fluctuation is defined as :

$\langle X^2 \rangle = \int \mathrm{P(X)\ X^2 dx} = \frac{1}{\beta}$ ---12

So the Gaussian distribution may be written as:

P(X)dx = $\frac{1}{\sqrt{2\pi(X^2}}\ e(\frac{-x^{\ 2}}{2\langle X\rangle^2})$ dx--13

It can be seen that the smaller the value of $\langle X^2 \rangle$, there will be sharper the maximum value of P(X) , which is the characreristics of the Gaussian distribution. Now, let us consider the Gaussian distribution of more than one variable. In this case we will determine simulteneouse deviation for several thermodynamic quantities from their mean values.We define the entropy S as a function of quantities of simultaneous deviations, S (x_1,x_2,x_3,……..x_n). With Taylor expansion method, it can be shown that

$$S-S_0 = -\frac{1}{2}\sum_{ij=1}^{p} \beta ij\ xixj \text{ ---14}$$

Here it is noted that β ij = βji, For simplicity the summation sign is omitted , then the equation may be written as:

$$S-S_0 = -\frac{1}{2}\ \beta ij\ xixj \text{ ---15.}$$

The probability takes the form: $P = A\ \exp.(-\frac{1}{2}\ \beta ij\ xixj\ )$ ------------------------16

A is a normalization constant, whose value is determin

$$\int P(x1, x2 \ldots\ldots xn) \quad dx_1\, dx_2\, dx_3 \text{ ---- } dxn. = 1 \text{ -------------------------------------17}$$

After some manipulations we find, $A = \frac{\sqrt{\beta}}{(2\pi)^{n/2}}$, Therefore the desired form of Gaussian distribution for more than one variable may become:

$$P = \frac{\sqrt{\beta}}{(2\pi)^{n/2}} \quad \exp.(-\frac{1}{2}\ \beta ij\ xixj\ ) \text{ ---18}$$

Where $\beta ij\ -\frac{\partial 2s}{\partial \mathrm{Xi2}\, \partial \mathrm{Xj}}$, $\beta\ = |\beta ij|$ ---19

4. Thermodynamic Fluctuation: Fluctuations of thermo dynamic properties play a crucial role in many physical phenomena and diverse applications. Fluctuations are especially important in nanometer-scale systems where they can lead to large variability of mechanical and functional properties. Fluctuations are unavoidable in molecular dynamics and Monte-Carlo simulations of materials, where the system dimensions rarely exceed a few nanometers. In fact, in many atomistic calculations, equilibrium properties of interest are extracted by analyzing statistical fluctuations of other properties [4-6].Presently, fluctuations are primarily discussed in the statistical–mechanical literature and typically in the context of specific models. At the same time, the thermodynamics community traditionally relies on classical thermo dynamics [7-9] which, by the macroscopic and equilibrium nature of this discipline, disregards fluctuations and operates solely in terms of static properties.

5. THERMODYNAMIC GEOMETRY

It has been found that the thermodynamic fluctuation theory gave us insights into the stability and critical behavior of the system. Now the whole formalism can be put into the language of geometry as was done by Ruppeiner [10]. Here ,we Consider a black hole and its environment or the 'universe'. The total entropy is given as :

$S_{tot} = S_{bh} + S_e$. Where S_{bh} = Entropy of the black hole, Se = Entropy of the environment or the universe.

The fluctuation in the parameter α may be defined as : $F_\alpha = \frac{\partial Sbh}{\partial X\alpha}$ Where X_α = is a function of (M,J,Q). Now expanding the total entropy to the second order in fluctuations gives:

The linear terms vanish due to conservation laws and for a very large environment the second quadratic term is negligible compared to the first. Therefore the above equation can now be written as:

$$\frac{\Delta\, Stotal}{KB} = -\frac{1}{2}\; g_{\mu\upsilon}(\Delta X^{\mu})\,(\Delta X^{\upsilon}) \quad \text{---20}$$

Where the symmetric matrix g is given by

$$g^{\alpha\beta} \propto \frac{\partial^{2S}}{\partial X^{\alpha}\partial X^{\beta}} \quad \text{---21}$$

From this we can develop a formalism of Thermodynamic Riemannian geometry by defining the line element as:

$$\Delta l^2 = -\,2\Delta Stot\,/\,Kb = g^{\mu\nu}\Delta X^{\mu}\Delta X^{\nu} \quad \text{---22}$$

This unitless positive definite line element can be given a physical interpretation: Farther apart the stateless probable are the fluctuations between the states [11].

Once a matrix is defined, we can go ahead and calculate the curvature of the given geometry. The thermodynamic curvature obtained can be given various interpretations as the range of interaction, the correlation volume etc in various contexts. Refer [10] and references therein for detailed expositions. For most of the ordinary thermo dynamical systems the curvature is negative. But it is very interesting that it is positive for the Kerr-Newman black hole. Even more curiously it is also positive for a Fermi gas. Therefore attempts may be made to establish a correspondence between a two dimensional Fermi gas and a Kerr Newman black hole.

6. Discussion:

When black Holes are characterized by just 3 parameters of mass, charge and angular momentum, they are comparable to elementary particles having just mass, charge, spin. But when low energy systems are being complexly constituted by such elementary particles, they have shown amazing richness and contrast in their equilibrium thermo dynamical and critical behaviours. Generally, black holes being 'simple' objects, generally show such behavior similar to these complex systems. In studying the phase transitions of black holes, we encountered some intricacies in the interpretation of critical behaviours and fluctuations. The use of equilibrium fluctuation theory provided some insight information's into these matters that reveal the critical behaviours of the black holes. It might be possible that further investigations into the field can lead to some information about the hawking radiation related problems. there might be remote or close links between the critical behaviours of the black holes and the microscopic theory of gravity.

References:

[1] J.W.Gibbs, The Collected Works of J.W.Gibbs, vol.2, YaleUniversityPress, New Haven, 1948,pp.1–207.

[2] A.Einstein,Ann.Phys.,Lpz.33(1910)1275–1298.

[3]. Y.D.Rudoi and A.D.Sukhanov,Phys.Usp.43(2000)1169–1199.

[4] D. Frenkel, B. Smit, Understanding Molecular Simulation: from Algorithms to Applications, second ed., Academic, San Diego,2002.

[5] D.P. Landau, K. Binder, A Guide to Monte Carlo Simulations in Statistical Physics, third ed., Cambridge University Press, Cambridge,2009.

[6] J.J.Hoyt,Z.T.Trautt,M.Upmanyu,Math.Comput.Simul.80(2010)1382–1392.

[7] J.W.Gibbs, The Collected Works of J.W.Gibbs, vol.1, YaleUniversity Press ,NewHaven, 1948,pp.55–349.

[8] A.Münster ,Classical Thermodynamics,Wiley-Interscience,London-NewYork- Sydney-Toronto,1970.

[9] H.B.Callen, Thermo dynamics and an Introduciton to Thermo statistics, seconded., Wiley,NewYork,1985.

[10] Rupperiner G , Phys. Rev. D,78 ,024016 (2008).

[11].Hickman,Y.Mishin,Atomistic modeling of grain boundary pre-melting in alloy systems, (2015)