Free Vibration Analysis of Functionally Graded Materials Using ANSYS

Pavan D
School of Mechanical Engineering
REVA University
Bengaluru, India

Deepak S A
School of Mechanical Engineering
REVA University
Bengaluru, India.

Abstract — Functionally graded materials have received a lot of interest in recent days by their diversified and potential applications in automotive, aerospace and other industries. They have high specific mechanical properties and high temperature capabilities which makes them special over all the exiting advanced materials. This work investigated dynamic analysis of functionally graded plate. The material properties vary continuously from metal to ceramic. The effective material properties of functionally graded materials for the structures are assumed to be temperature independent and graded in the plate thickness direction according to a power law distribution of the volume fractions of the constituents. The present model is developed using ANSYS platform. A higher order quadrilateral shell element is used to discretise this model for dynamic analysis.

Keywords— FGM, dynamic analysis, shell element.

I. Introduction

Laminated composites have received a lot of interest in recent days by diversified and potential applications in automotive and aerospace industry due to their strength to weight, stiffness to weight ratio, low fatigue life and toughness and other higher material properties. These are made from two or more constituent materials which have different chemical or physical properties and produced a material having different behaviour from the individual. These are used in buildings, storage tanks, bridges etc. Each layer is laminated in order to get superior material properties. The individual layer has high strength fibres like graphite, glass or silicon carbide and matrix materials like epoxies, polyamides. By varying the thickness of laminas desired properties (strength, wear resistance, stiffness) can be achieved.

Although these materials have superior properties, their major drawback is the weakness of laminated materials. This is known as delaminating phenomenon which leads to the failure of the composite structure. Residual stresses are present due to difference in thermal expansion of the matrix and fibre. It is well known that at high temperature the adhesive being chemically unstable and fails to hold the lamination. Sometimes due to fibre breakdown it also prematurely fails.

Functionally Graded Material (FGM) is combination of a ceramic and a metal. A material in which its structure and composition both varies gradually over volume in order to get certain specific properties of the material hence can perform certain functions. The properties of material depend on the spatial position in the structure of material. The effect of inter-laminar stress developed at the laminated composite interfaces due to sudden change of material properties reduced by continuous grading of material properties. Generally micro structural heterogeneity or non-uniformity is introduced in functionally graded material. The main purpose is to increase fracture toughness, increase in strength because ceramics only are brittle in nature. Brittleness is a great disadvantage for any structural application. These are manufactured by combining both metals and ceramics for use in high temperature applications. Material properties are varies smoothly and continuously in one or many directions so FGMs are inhomogeneous. FGM serves as a thermal barrier capable of withstanding 2000K surface temperature. Fabrication of FGM can be done by different processing such as layer processing, melt processing, particulate processing etc. FGM has the ability to control shear deformation, corrosion, wear, buckling etc. and also to remove stress concentrations. This can be used safely at high temperature also as furnace liners and thermal shielding element in microelectronics and thermal protection systems for spacecraft, hypersonic and supersonic planes and in combustion chamber also.

II. Literature Review

A. Static and vibration analysis

Talha and Singh [1] investigated the free vibration and static analysis of rectangular FGM plates using higher order shear deformation theory with a special modification in the transverse displacement in conjunction with finite element models. Neves et al. [2] studied the static deformations analysis of functionally graded plates by collocation with radial basis functions, according to a sinusoidal shear deformation formulation for plates. Aragh and Hedayati [3] studied the characteristics of free vibration and static response of a 2-D FGM open cylindrical shell. Formulations are done by 2-D generalized differential quadrature method (GDQM). Ferreira et al. [4] studied static deformations of functionally graded square plates of different aspect ratios using meshless collocation method, the multiquadric radial basis functions and a third-order shear deformation theory. Reddy[5] studied static and dynamic analysis of FGM plates using third-order shear deformation theory. Navier solutions are obtained for a simply supported square plate. Abrate[6] investigated static, buckling and free vibration deflections of FGM plates by using classical plate theory, FSDT model and HSDT model. Zenkour[7] studied the static behaviour of a rectangular FG plate under simply supported condition and subjected to uniform transverse load. Ferreira et al. [8] studied static deformations of simply supported functionally graded plate by using HSDT and multiquadric radial basis functions. Vel and Batra[9] investigated the exact 3-D elasticity solutions of simply supported rectangular FG plates under thermo-mechanical load. The author has assumed power law for material volume fractions. The exact solutions of displacements and stresses are used to find out the accuracy of the solutions. Qian et al.[10] investigated plain strain static thermostatic deformations of simply supported thick rectangular FG elastic plate. Displacement and stress are computed and validated from the 3D exact solutions of the problem. Ramirez et al.[11] studied static analysis of 3D, elastic, anisotropic FG plates. The author has taken simply supported graphite/epoxy material for analysis. Zenkour [12] further studied the static response of FG plates using shear deformation plate theory using power law for grading. Bhangale and Ganesan[13] investigated static analysis of simply supported FG plates which are exponentially graded in the thickness direction. Aghdam et al.[14] studied static analysis foe bending of FG clamped thick plates. The solutions are compared with the solutions of finite element code ANSYS, power law is used for grading the properties in thickness direction. Neves et al.[15] investigated the static deformations of FG square plates using radial basis function. Talha and Singh [16] investigated the static and free vibration analysis using C0 finite element with 13 degrees of freedom per node and formulated by HSDT. Nguyen-Xuan et al.[17] studied the static, free vibration and mechanical/thermal buckling problems of FG plates by Reisnner/Mindlin plate theory.

B. *Dynamic Analysis*

Yang and Shen [18] studied dynamic response of FGM thin plates under initial stress and partially distributed impulsive lateral loads. The author used silicon nitride/ stainless steel rectangular plates, assumed temperature dependent material properties clamped on two opposite edges, used power law for grading and used Modal superposition method for transient response. In 2001 Yang and Shen [19] studied free and forced vibration analysis for the same plate and found functionally graded plate with material properties intermediate to isotropic material do not necessarily have intermediate natural frequency if thermal effects are considered. Liew et al. [20] investigated dynamic stability of symmetrically laminated FGM rectangular plates under uniaxial plane load. Formulation is done by Reddy's third-order shear deformation theory and material is silicon nitride and stainless steel. [21]Kim studied vibration characteristics of rectangular FGM plate under initial stress. Third-order shear deformation plate theory is adopted and Rayleigh-Ritz procedure is applied for getting frequency equation. Lanhe et al. [22] investigated dynamic stability of thick FGM plate under aero-thermo-mechanical loads and used novel numerical solution technique. The equations for dynamic analysis are derived by Hamilton's principle. For different parameters dynamic instability regions are studied. Ansari and Darvizeh [23] investigated vibrational behaviour of functionally graded shells, based on first-order shear deformation shell theory. The grading functions are power law, sigmoid and exponential distribution. Behjat et al. [24] studied dynamic response, static bending of functionally graded piezoelectric material plate (PZT-4/PZT-5H), formulated by using potential energy and Hamilton's principle. Effects of material composition and boundary conditions on dynamic response are also studied. Sladek et al. [25] investigated dynamic analysis of FG plates by MPLG method. For displacement field author used Reissner-Mindlin plate bending theory. Simply supported and clamped boundary conditions are taken in to consideration. Wen et al. [26] studied 3-D analysis of isotropic and orthotropic FG plates with simply supported edge under dynamic loads. The equations formulated is based on state-space approach in Laplace transform domain and solved by RBF method. Grading has done by exponential method as well as volume fraction law. Shariyat [27] studied the vibration and dynamic buckling response of rectangular FG plates under thermo-mechanical loading. A nine noded second-order formulation has done and graphs are studied under temperature dependent material properties.

From the above study it has been seen that very few researcher studied the dynamic analysis of FG plates. Since most of the practical cases deals with transient or dynamic load, its responses has to be analysed with different parameters like volume fraction index. This work analysed dynamic responses of aluminium/zirconia flat panel under step load with different volume fraction index (n=0, 1, 2, ∞).

III. FMG MATERIAL PROPERTIES

The effective material properties of the FGM plate are assumed to be varying continuously along their thickness direction as discussed earlier and are obtained by using a simple power-law distribution or exponential law which counts the volume fraction of each constituent.

A. *Exponential law*

Exponential law of grading FGM states that for a FGM structure of uniform thickness 'h', the material properties 'P(z)' at any point located at 'z' distance from the mid-plane surface is given by:

$$P(z) = P_t e^{\left(-\lambda\left(1-\frac{2z}{h}\right)\right)} \text{, where, } \quad \lambda = \frac{1}{2}\ln\left(\frac{P_t}{P_b}\right)$$

P(z)denotes material property like Young's modulus of elasticity (E), shear modulus of elasticity (G), Poisson's ratio (), material density ( ) of the FGM structure. and are the material properties at the top (z=+h/2) and bottom (z=-h/2) surfaces. is the material grading indexes which depend on the design requirements.

B. *Power law*

The power-law distribution of a panel considered from the mid-plane reference plane can be written as

$$V_f = \left(\frac{z}{h} + \frac{1}{2}\right)^n$$

Where, n is the power-law index, 0 ≤ N ≤ ∞ . The variations of volume fraction of the ceramic and metal phase through the non-dimensional thickness coordinate are plotted in Figure 2 and 3 for five different values of power-law indices (n = 0.2, 0.5, 1, 2 and 10). The functionally graded material with two constituents and their properties such as, Young's modulus E and the mass density ρ have been obtained using the following steps.

$$E = (E_c - E_m)\left(\frac{z}{h} + \frac{1}{2}\right)^n + E_m$$

$$\rho = (\rho_c - \rho_m)\left(\frac{z}{h} + \frac{1}{2}\right)^n + \rho_m$$

$$\upsilon = (\upsilon_c - \upsilon_m)\left(\frac{z}{h} + \frac{1}{2}\right)^n + \upsilon_m$$

In the present work, the power- law distribution is used for the continuous gradation of material properties in thickness direction.

The effective material properties are calculated based on the shown Eqns. when z = - h/2, E= Em , ρ = ρm and = similarly, when z = + h/2; E= Ec , ρ = ρc and = i.e., the material properties vary continuously from metal at the bottom surface to ceramic at the top surface. The Poisson's ratio ν is assumed to be constant throughout the thickness of the shell panel. The properties of the FGM constituents at room temperature (27^0C) are used for the analysis and presented in Table 1. The different material properties are used to analyse the responses for throughout the study.

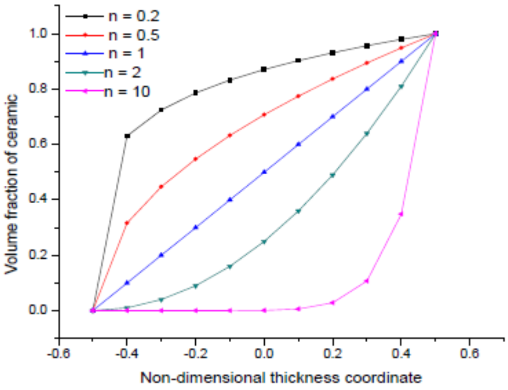

Fig. 1: Variations of volume fraction of ceramic through non-dimensional thickness coordinate

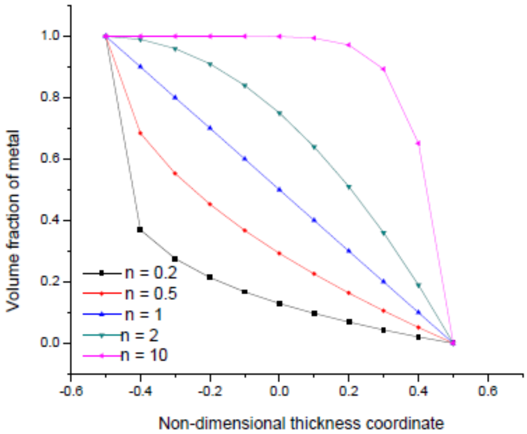

Fig. 2: Variations of volume fraction of metal through non-dimensional thickness coordinate

Table1: Properties of the FGM plates constituents

Materials	*Properties*		
	Young's Modulus E (GPa)	*Poisson's Ratio ϑ*	*Density (Kg/m^3) ρ*
Aluminium (Al)	*70*	*0.3*	*2707*
Alumina (Al_2O_3)	*380*	*0.3*	*3800*
Zirconia (ZrO_2)	*151*	*0.3*	*3000*
Silicon Nitride (Si_3N_4)	*348.43*	*0.28*	*2370*
Steel (SUS304)	*201.04*	*0.28*	*8166*

C. FE Modelling of FGM plates

FGM plates with different length to thickness ratio (a/h) , aspect ratio (a/b) are analysed in this experiment. The analysis is performed in commercially available software (ANSYS). The loading conditions are assumed to be static. The element chosen for this analysis is SHELL281, which is a layered version of the 8-node structural shell model. This is suitable for analysing thin to moderately-thick shell structures. This shell element has six degrees of freedom at each node namely three translations and three rotation in the nodal x, y and z directions respectively. The FGM plate is modelled in ANSYS.

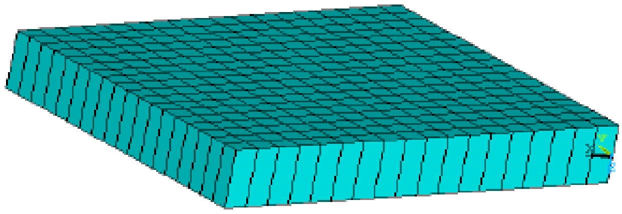

Fig3: FGM Plate modelled in ANSYS using shell281 element

IV. DYNAMIC ANALYSIS

A. FG model (ANSYS)

Rectangular simply supported Aluminium/Zirconia FG flat panel has been developed in ANSYS platform. Time dependant step load has been taken for transient dynamic analysis. Step type loading has been taken in to consideration. From time 0 to 0.001s force is zero and from 0.001 to 0.002s force is 10kN. APDL code has been developed in ANSYS for analysing the above panel.

B. Numerical Results

The analysis is carried out for different volume fraction indices (n=0, 1, 2, ∞). Dynamic behaviour of FG flat panel can be seen in figures 4-7. By time step of 0.0001s analysis has been performed and displacement has been plotted. An enlarged view of dynamic response has been shown in figure 8 for ceramic flat panel.

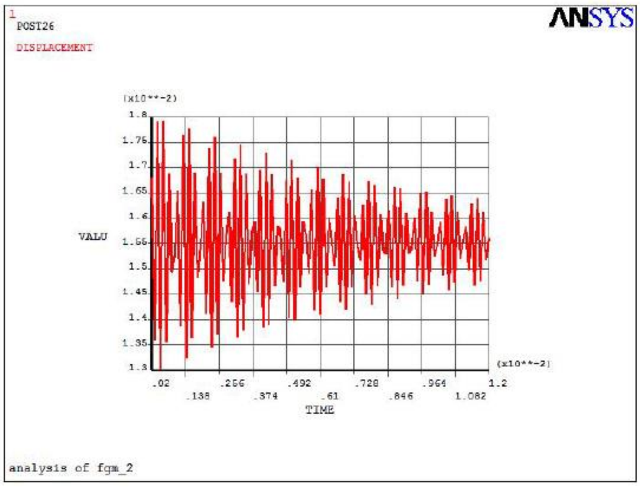

Fig. 4: Deflection of midpoint of simply supported FG flat panel with n=0

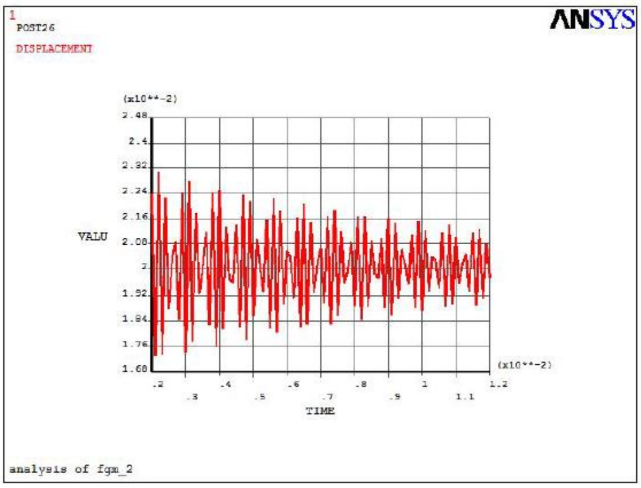

Fig. 5: Deflection of midpoint of simply supported FG flat panel with n=1

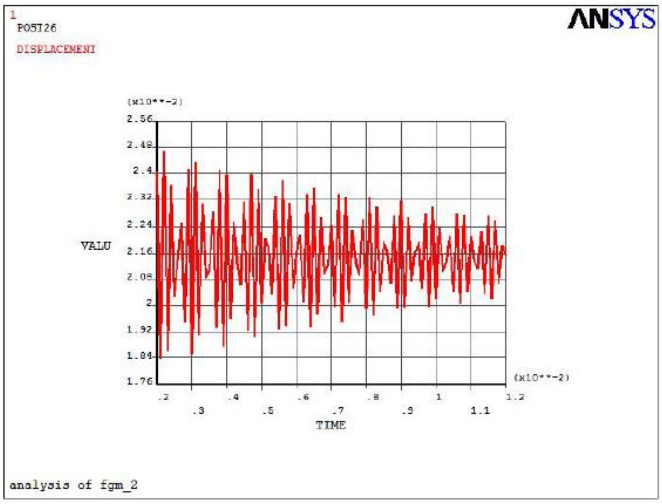

Fig. 6: Deflection of midpoint of simply supported FG flat panel with n=2

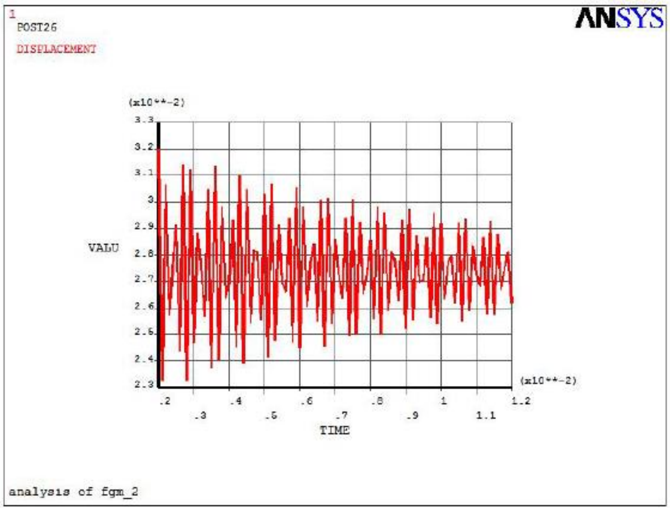

Fig. 7: Deflection of midpoint of simply supported FG flat panel with n=∞

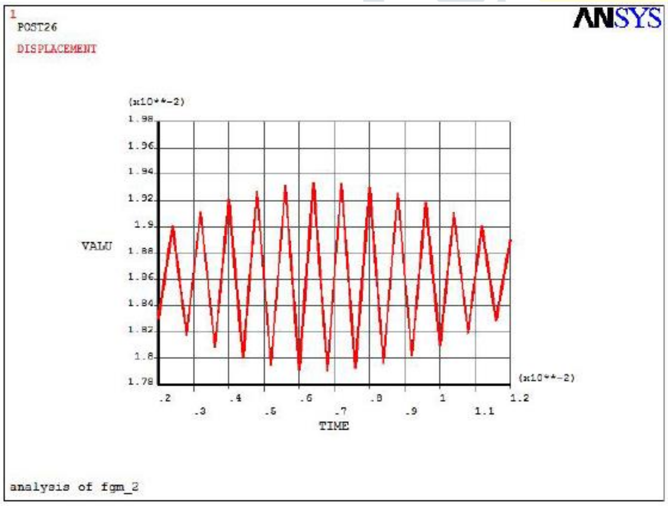

Fig. 8: Deflection of midpoint of simply supported FG flat panel with n=0 in time interval of 0.0001 to 0.012s

V. CONCLUSION

In this study, static and dynamic responses of FGM plates are analysed. The effective material properties of functionally graded materials for the plate structures are assumed to vary continuously through the plate thickness and are graded in the plate thickness direction according to a volume fraction power law distribution. Various boundary conditions have been considered to check the efficacy of ANSYS model. Convergence tests and comparison studies have been carried out with the commercially available software (ANSYS). An eight noded layered shell element (SHELL281) is used throughout the problem. The obtained results have illustrated a good agreement with those available in the literature for different volume fraction indices, thickness ratios, aspect ratios and different support conditions. The following points revealed the concluded remarks for thin to thick FGM plates are:

- For all the boundary conditions, the non-dimensional central deflection increases as the volume fraction index increases.
- For all the boundary conditions, the non-dimensional central deflection increases as the aspect ratio increases.
- For all the boundary conditions, the non-dimensional central deflection increases as the thickness ratio increases.
- For simply supported boundary condition vibration amplitude increases as the volume fraction index increases.

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