REGULAR GENERALIZED FUZZY b-CLOSED SET IN FUZZY TOPOLOGICAL SPACES

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*Abstract :*In this paper, we introduce a new form of fuzzy generalized b-closed sets namely Regular generalized fuzzy b-closed sets in fuzzy topological spaces and investigate their properties.

***IndexTerms* -** Regular generalized fuzzy b-open sets.

I. INTRODUCTION

After Zadeh [5] and Chang [4] introduced the concept of a fuzzy subset and fuzzy topological space, several concepts of general topology have been extended to fuzzy topology. In this paper we define a new class of generalized closed sets namely, regular generalized fuzzy b-closed sets and investigate its properties.

II. PRELIMNARIES

Throughout this paper X denotes the fuzzy topological spaces (fts.) (X,τ). For a fuzzy set A, the operators fuzzy closure and fuzzy interiors are denoted and defined by $\mathrm{ClA} = \wedge\{B : B \geq A, 1-B \in \tau\}$ and $\mathrm{IntA} = \vee\{B : B \leq A, B \in \tau\}$.

The following concepts are used in the sequel.

2.1 Definition [3]: A fuzzy set A in X is called

(i) Fuzzy b-open set iff $A \leq (\mathrm{IntClA}) \vee (\mathrm{ClIntA})$.

(ii) Fuzzy b-closed set iff $A \geq (\mathrm{IntClA}) \wedge (\mathrm{ClIntA})$.

2.2 Theorem [3]: For a fuzzy set A in X

(i) A is a fuzzy b-open set iff $1-A$ is a fuzzy b-closed set.

(ii) A is a fuzzy b-closed set iff $1-A$ is a fuzzy b- open set.

2.3 Definition [3]: Let A be a fuzzy set in X. Then

(i) $\mathrm{bClA} = \wedge\{B : B \text{ is a fuzzy b-closed set of } X \text{ and } B \geq A\}$.

(ii) $\mathrm{bIntA} = \vee\{C : C \text{ is a fuzzy b-open set of } X \text{ and } A \geq C\}$.

2.4 Lemma [3]: In X, every fuzzy open set is fuzzy b–open.

2.5 Lemma [1]: In X every fuzzy regular open (closed) set is fuzzy open (closed).

2.6 Definition[2]:A fuzzy set A in X is called fuzzy generalized b-closed (fgb-closed) if $\mathrm{bCl}(A) \leq B$, whenever $A \leq B$ and B is fuzzy open.

2.7 Lemma [1,3,4] : In a fuzzy topological space X,

(i) every fuzzy regular open(closed) set is fuzzy open(closed).

(i) every fuzzy open set is fuzzy b–open.

(ii) every fb-closed set is fgb-closed.

III.REGULAR GENERALIZED FUZZY b-CLOSED SETS

In this section we define a new class of fuzzy generalized closed sets called regular generalized fuzzy b-closed sets and study its properties.

3.1 Definition : A fuzzy set A in a X is called a regular generalized fuzzy b-closed (rgfb-closed) set X if $\mathrm{bClA} \leq B$, whenever $A \leq B$ and B is fuzzy regular open set in X.

3.2 Remark : A fuzzy set A in X is called rgfb-open iff 1-A is rgfb-closed in X.

The following theorem shows that the class of regular generalized fuzzy b-closed sets contains the class of fuzzy closed sets and fuzzy b-closed sets.

3.3 Theorem : Every fuzzy closed set in X is regular generalized fuzzy b-closed.

Proof: Let A be a fuzzy closed set in X. Suppose that $A \leq B$ and B is a fuzzy regular -open set in X. Since A is fuzzy closed it is fb-closed,then $Cl(A) = bClA = A \leq B$. Hence it follows that $bClA \leq B$. Therefore A is regular generalized fuzzy b-closed.

3.4 Remark : Every fuzzy b-closed set in X is regular generalized fuzzy b-closed.

The converse of the above theorem is not true which is shown in the following example.

3.5 Example : Let $X = \{a, b\}$ and $\tau = \{0, 1, A\}$, where $A = \{(a,.6),(b,1)\}$, $B = \{(a,.6),(b,0)\}$. $C = \{(a,1),(b,0.4),(c,0)\}$. C is not a fuzzy b-closed set in X, but C is rgfb-closed in X.

3.6 Theorem : Every regular generalized fuzzy b-closed set in X is fgb-closed.
Proof Let A be a regular generalized fuzzy b-closed set in X. Then $bClA \leq B$ whenever $A \leq B$ and B is fuzzy regular open set in X. By lemma 2.7 (i) B is fuzzy open. Then $bClA \leq B$ and B is fuzzy-open in X Hence A is fgb-closed.

3.7 Theorem :A fuzzy set A of a X is called rgfb-open iff $B \leq bInt(A)$, whenever B is fuzzy-regular closed set and $B \leq A$.
Proof: Suppose A is rgfb-open set in X. Then 1-A is rgfb-closed in X. Let B be a fuzzy regular closed set in X such that $B \leq A$. .Then, $1 - A \leq 1 - B$, 1-B is fuggy regular open set in X. Since 1-A is rgfb-closed, $bCl(1 - A) \leq 1 - B$, which implies $1 - bInt(A) \leq 1 - B$. Thus $B \leq bInt(A)$.

Conversely, assume that $B \leq bInt(A)$, wherever $B \leq A$ and B is fuzzy regular closed in X. Then $1 - bInt(A) \leq 1 - B = C$, where C is fuzzy regular open set in X. That is $bCl(a)(A) \leq C$, which implies $1 - A$ is rgfb-closed. Hence A is rgfb-open.

3.8 Theorem : Let A be a fuzzy rgfb-closed set in X and $A \leq B \leq bCl(A)$, then B is rgfb-closed set in X.
Proof: Let C be fuzzy regular open set in X such that $B \leq C$ then $A \leq B$, $A \leq C$, Since A is a rgfb-closed set in X, it follows $bClB \leq C$. Now $bCl(B) \leq bCl(bCl(A)) = bCl(A)$. Thus $bCl(B) \leq C$. Hence B is rgfb-closed set in X.

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V. REFERENCES

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