Implementing Image Compression via Singular Value Decomposition

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Abstract – Image compression is an application of data compression that encodes the original image with few bits. The main goal of such system is to reduce the storage quantity as much as possible, and the decoded image displayed in the monitor can be similar to the original image as much as can be. The research paper focuses on working of SVD (Singular value decomposition) technique of image compression which falls under the lossy category. The paper depicts the working of SVD via considering an input image and obtaining compression at different levels in bits. The obtained results illustrates that the as the level of compression increases the quality of image gets better and the compression is significantly reduced.

Keywords: Image compression, lossy compression, lossless compression, SVD.

I. INTRODUCTION

Image compression is minimizing the size in bytes of a graphics file without degrading the quality of the image to an unacceptable level. The reduction in file size allows more images to be stored in a given amount of disk or memory space. It also reduces the time required for images to be sent over the Internet or downloaded from Web pages. There are several different ways in which image files can be compressed. For Internet use, the two most common compressed graphic image formats are the JPEG format and the GIF format. The JPEG method is more often used for photographs, while the GIF method is commonly used for line art and other images in which geometric shapes are relatively simple [1, 2]. A text file or program can be compressed without the introduction of errors, but only up to a certain extent. This is called lossless compression. Beyond this point, errors are introduced. In text and program files, it is crucial that compression be lossless because a single error can seriously damage the meaning of a text file, or cause a program not to run. In image compression, a small loss in quality is usually not noticeable. There is no "critical point" up to which compression works perfectly, but beyond which it becomes impossible [3, 4]. When there is some tolerance for loss, the compression factor can be greater than it can when there is no loss tolerance. For this reason, graphic images can be compressed more than text files or programs. Proper use of image compression can make a huge difference in the appearance and size of your image files [5].

The goal of image compression is to save storage space and to reduce transmission time for image data. Its aims to achieving a high compression ratio (CR) while preserving good fidelity of decoded images [6, 7]. The techniques used to compress and decompress a single gray level image are expected to be easily modified to encode/decode color image sends image sequences [8]. Recent compression methods can be briefly classified into five categories: (a) Wavelet, (b) JPEG/DCT, (c) VQ, (d) Fractal methods and (e) Genetic algorithm, which are briefly discussed as under.

- Wavelet - Wavelet compression is very popular compression approach in mathematics and digital image processing area because of their ability to effectively represent and analyze data. Image compression algorithms based on Discrete Wavelet Transform (DWT), such as Embedded Zero Wavelet (EZW) is capable of excellent compression performance, both in terms of statistical peak signal to noise ratio (PSNR) and subjective human perception of their constructed image [9, 10].
- JPEG/DCT - The key to the JPEG baseline compression process is a mathematical transformation known as the Discrete Cosine Transform (DCT). The DCT is in a class of mathematical operations that includes the well-known Fast Fourier Transform (FFT), as well as many others [11]. The basic purpose of these operations is to take a signal and transform it from one type of representation to another. For example, an image is a two-dimensional signal that is perceived by the human visual system. The DCT can be used to convert the

signal (spatial information) into numeric data ("frequency" or "spectral" information) so that the image's information exists in a quantitative form that can be manipulated for compression [12].

- VQ – VQ is a powerful method for lossy compression of data such as sounds or images, because their vector representations often occupy only small fractions of their vector spaces [13, 14]. Vector quantization is the procedure of approximating continuous with discrete values; in practice, the input values to the quantization procedure are often also discrete, but with a much finer resolution than that of the output values. The goal of quantization usually is to produce a more compact representation of the data while maintaining its usefulness for a certain purpose. For example, to store color intensities you can quantize floating-point values in the range [0.0, 1.0] to integer values in the range 0-255, representing them with 8 bits, which is considered a sufficient resolution for many applications dealing with color [15].
- Fractal methods - Fractal compression is a lossy compression method for digital images, based on fractals. The method is best suited for textures and natural images, relying on the fact that parts of an image often resemble other parts of the same image [16, 17]. Fractal algorithms convert these parts into mathematical data called "fractal codes" which are used to recreate the encoded image.
- Genetic algorithm - Genetic Algorithms (GAs) are procedures based on the principles of natural selection and natural genetics, that have proved to be very efficient searching for approximations to global optima in large and complex spaces in relatively short time [18]. The basic components of GAs are: genetic operators (mating and mutation), an appropriate representation of the problem that is to be solved, a fitness function, and an initialization procedure [19].

II. IMAGE COMPRESSION AND TECHNIQUES

Image compression coding is to store the image into bit-stream as compact as possible and to display the decoded image as exact as possible. The encoder receives the original image file, the image file will be converted into a series of binary data, which is called the bit-stream. The decoder then receives the encoded bit-stream and decodes it to form the decoded image. If the total data quantity of the bit-stream is less than the total data quantity of the original image, then this is called image compression [20, 21]. The full compression flow is shown in Fig. 1. The compression ratio is defined as follows:

$$Cr = n1 / n2$$

where n1 is the data rate of original image and n2 is that of the encoded bit-stream.

In order to evaluate the performance of the image compression coding, it is necessary to define a measurement that can estimate the difference between the original image and the decoded image [22]. Two common used measurements are the Mean Square Error (MSE) and the Peak Signal to Noise Ratio (PSNR). Most image compression systems are designed to minimize the MSE and maximize the PSNR. The Mean Square Error (MSE) and the Peak Signal to Noise Ratio (PSNR) are the two error metrics used to compare image compression quality [23]. The MSE represents the cumulative squared error between the compressed and the original image, whereas PSNR represents a measure of the peak error. The lower the value of MSE, the lower the error.

$$MSE = \frac{1}{m\,n}\sum_{i=0}^{m-1}\sum_{j=0}^{n-1}[I(i,j) - K(i,j)]^2$$

The PSNR is defined as:

$$PSNR = 10 \cdot \log_{10}\left(\frac{MAX_I^2}{MSE}\right) = 20 \cdot \log_{10}\left(\frac{MAX_I}{\sqrt{MSE}}\right)$$

Compression is a term used to quantify the reduction in data-representation size produced by a data compression algorithm. There are two forms of compression are Lossless compression and Lossy Compression [24, 25].

- **Lossless** form of compression algorithms, are those that are non-destructive and reversible. They do not change content of files. Files can be compressed and decompressed and the data in the file remains the same. Examples of lossless algorithms for compression are Flate / deflate compression, Huffman compression, and RLE compression.
- **Lossy** compression algorithms are those that gets rid of some information in order to attain size reduction. This form of compression is irreversible, meaning one can't compress and decompress and have the same amount of information in the file. An example of such algorithm is Singular Value Decomposition.

III. SVD (SINGULAR VALUE DECOMPOSTION)

Every image can be represented in a matrix form. Pixel elements in images can be represented in matrix form with each element denoting the various levels of brightness (or film density) and colors. There are several methods available for compression of still images. SVD (Singular Value Decomposition) is a linear matrix transformation used for compressing images.

Given a Matrix *A* with dimension *m* by *n* the SVD can be defined mathematically as

$$A_{m\times n} = U_{m\times m}\Sigma_{m\times n}V_{n\times n}^{T}$$

Where U and V are orthogonal matrices and the middle matrix is a diagonal matrix. The middle matrix contains the singular values which are basically the square root of the eigenvalues of either the product of the matrix in question (A) and its transpose or the product of the transpose of the matrix in question and the matrix itself. U contains eigenvectors of the product of A and it's transpose in column form and V contains eigenvectors of the product of A transpose and A in row form. So, it can be concluded that SVD is a way of decomposing a matrix into three other matrices with the middle matrix being a diagonal matrix which has the some figures called the singular values.

IV. IMPLEMENTATION AND CONTRIBUTION

This section illustrates the results obtained after implementing SVD using Matlab simulation tool. A colored PNG image having dimensions 351 * 392 of size 169 kb is given as input and the image compression results are obtained at different compression levels denoted by k ranging between 1 to 100. σ_1 refers to compression rate at level 1 and σ_k refers to the compression rate at level k where k is any positive integer greater than 0. Fig. 1 to Fig. 12 below shows different figures obtained at different compression levels. Table 1 depicts the details of the compression reached at different compression levels.

Fig. 1: Figure at compression level 2

Fig. 2: Figure at compression level 5

Fig. 3: Figure at compression level 10

Fig. 4: Figure at compression level 20

Fig. 5: Figure at compression level 30

Fig. 6: Figure at compression level 40

Fig. 7: Figure at compression level 50

Fig. 8: Figure at compression level 60

Fig. 9: Figure at compression level 70

Fig. 10: Figure at compression level 80

Fig. 11: Figure at compression level 90

Fig. 12: Figure at compression level 100

Table 1: Table shows the obtained compression rate at different levels

Compression level (k)	**Compression rate**
1	64002.0539
2	9874.6462
5	4712.5624
10	1921.6512

20	885.3883
30	553.2889
40	396.615
50	318.363
60	267.025
70	227.0331
80	195.1325
90	172.0933
100	151.1152

Fig. 13 shows the results obtained in Table 1 in the form of pie chart.

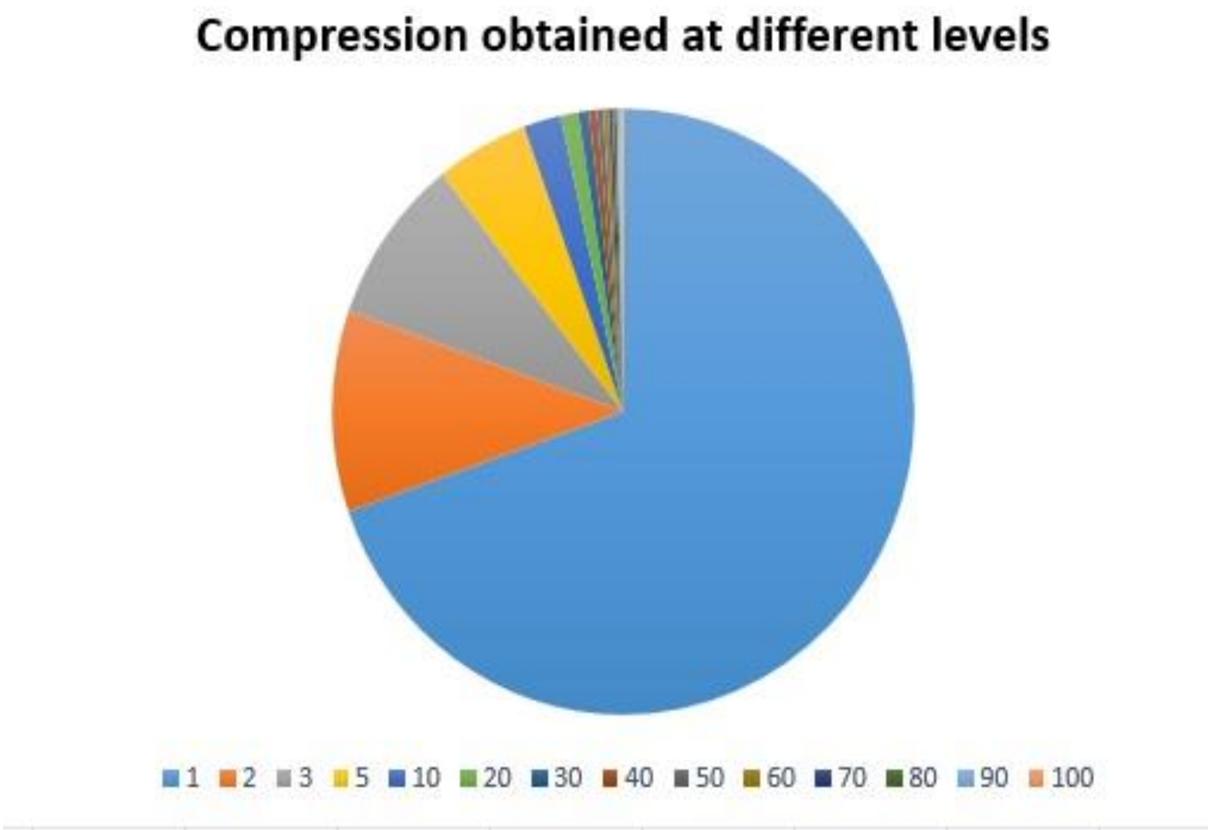

Fig. 13: Figure shows the results obtained in Table 1 in the form of pie chart

V. CONCLUSION

The research work conducted in the paper shows that as the level of compression increases, the quality of image improves along with. The current data compression methods might be far away from the ultimate limits. Interesting issues like obtaining accurate models of images, optimal representations of such models, and rapidly computing such optimal representations are the grand challenges facing the data compression community. Image coding based on models of human perception, scalability, robustness, error resilience, and complexity are a few of the many challenges in image coding to be fully resolved and may affect image data compression performance in the years to come.

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