ON PRIME SEMI NEAR RINGS

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Abstract : The notion of a Reverse Derivation as an additive mapping from a ring R into itself satisfying $d(xy) = d(y)x + yd(x)$ for all x, y in R was introduced by Bresar and Vukman [3]. Samman and Alyamani studied the Reverse Derivations on Semi Prime Rings [17]. In this paper we have introduced a Prime Semi Near-ring which admits non-zero Reverse σ-derivation satisfying certain conditions to be a Commutative ring. It is proved that a Prime Semi Near ring S together with d, a non-zero Reverse σ-derivation of S is commutative when $d([x,y]) = [x, d(y)]$ for all x, y in S and proved that for a Prime Semi Near ring S with d a non-zero Reverse σ-derivation of S is commutative when $[x, d(x)] = 0$ for all x in S. Also it is proved that S is commutative if $[d(y)d(x)] = 0$ for all x, y in S. We have also showed that S is commutative if $[x, d(y)]$ belongs to central elements where S is a Prime Semi Near ring and d is a non-zero Reverse σ-derivation of S , it is also proved that if $[d(x), y] = 0$ then we have $d(y) = 0$ or y belongs to central elements, where S is a prime semi near ring and d is a non-zero Reverse σ-derivation of S.

Key words: Reverse derivation, Reverse σ-derivation, commutative in semi near-rings, central elements, Prime near-ring.

1.INTRODUCTION:

In 1987 H.E.Bell and G.Mason [2] initiated the study on commutativity of prime near-rings by using derivations. Later by generalizing some results of Bell and Mason, A.A.M.Kamal in 2001 [5] studied the commutativity of 3-prime near-rings using σ-derivation instead of usual derivation, where σ is an automorphism on the near-ring. [1] Afrah M.Ibraheem used the notion of reverse derivation on prime Γ- near ring *M* to study the commutativity conditions of *M*, when *U* be a non-zero invariant subset of *M*.Throughout the paper *S* will denote a zerosymmetric semi near-ring with multiplicative center Z.

2.PRELIMINARY:

Definition:2.1

A right *near ring* is a set N together with two binary operators '+" and '.' such that

(i) (N,+) is a group (not necessarily abelian)
(ii) (N,.) is a semi group
(iii) For all x, y, x in N, $(x + y)z = xz + yz$ (right distributive law)

Definition:2.2

A near ring *N* is called a *prime near ring* if $xNy = 0$ implies $x = 0$ *or* $y = 0$ for all $x, y \in N$.

Definition:2.3

A nonempty set *S* with two binary operators '+' and '.' is said to be *semi near ring* if (S,+) is a semi group and (S,.) is a semi group satisfying the right distributive law.

Definition:2.4

An additive mapping $d: N \to N$ is called *derivation* if $d(xy) = xd(y) + d(x)y$ or equivalently that $d(xy) = d(x)y + xd(y)\ for\ all\ x, y \in N$.

Definition:2.5

An additive mapping d from a ring R into itself satisfying $d(xy) = d(y)x + yd(x)$ for all $x, y \in R$ is called *reverse derivation.*

Definition:2.6

The symbol $Z(N)$ will represent the multiplicative *center of* N, that is,$Z(N) = \{x \in N | xy = yx\ for\ all\ y \in N\}$.

Notation:2.7

(i) The symbol $[x, y]$ will denote the commutator $xy - yx$ for all $x, y \in N$.
(ii) $[x, yz] = y[x, z] + [x, y]z$ and $[xy, z] = x[y, z] + [x, z]y$ satisfied for all $x, y, z \in N$.

Definition:2.8

A semi near ring *S* is said to have an absorbing zero if $a + 0 = 0 + a = a$ and$a. 0 = 0 = 0. a$ for all $a \in S$.

Definition:2.9

Let *N*be a near ring, and σ is an automorphism on *N*. An additive mapping *d* from *N* into itself is called a *reverse* σ*-derivation* on *N* if satisfying $d(xy) = d(y)x + \sigma(y)d(x)$ for all $x, y \in N$.

3.ON PRIME SEMI NEAR RINGS

Definition:3.1

A Semi near ring *S* is called *prime semi near ring* if $xSy = 0$ implies $x = 0$ or $y = 0$ for all $x, y \in S$.

Definition:3.2

Let S be a semi near ring, and σ is an automorphism on *S*. An additive mapping d from S into itself is called a *reverse* σ*-derivation* on *S* if satisfying $d(xy) = d(y)x + \sigma(y)d(x)$ for all $x, y \in S$.

Lemma:3.3

Let d be an arbitrary additive automorphism of S. Then $d(xy) = \sigma(y)d(x) + d(y)x$ *for all* $x, y \in S$ *if and only if* $d(xy) = d(y)x + \sigma(y)d(x)$ *for all* $x, y \in S$. *Therefore, d is a reverse* σ*-derivation if and only if* $d(xy) = d(y)x + \sigma(y)d(x)$.

Proof:

Suppose, $d(xy) = \sigma(y)d(x)d(y)x$ for all $x, y \in S$.Since $(x + x)y = xy + xy, d\big((x + x)y\big) = d(xy + xy) \Rightarrow d\big((x + x)y\big) = \sigma(y)d(x + x) + d(y)(x + x) = \sigma(y)d(x) + \sigma(y)d(x) + d(y)x + d(y)x \cdots\cdots\cdots (1)$ for all $x, y \in S$.And, $d(xy + xy) = d(xy) + d(xy) = \sigma(y)d(x) + d(y)x + \sigma(y)d(x) + d(y)x \cdots\cdots\cdots (2)$ for all $x, y \in S$.From (1) and (2) we get, $\sigma(y)d(x) + d(y)x = d(y)x + \sigma(y)d(x)$So, $d(xy) = d(y)x + \sigma(y)d(x)$ for all $x, y \in S$.The converse of the proof is similar. Hence the result.

Lemma:3.4

Let S be a prime semi near ring, and d be a non-zero reverse σ-derivation of S. If $d(S) \subset Z(S)$, *then S is a commutative ring.*

Proof:

Let $d(x) \in Z(S)$, for all $x \in S$. Then$d(x)z = zd(x) \cdots\cdots\cdots (1)$Replacing x by xy in (1) we have $d(xy)z = zd(xy) \Rightarrow \big(d(y)x + \sigma(y)d(x)\big)z = z(d(y)x + \sigma(y)d(x))$.Then, $\sigma(y)d(x)z - z\sigma(y)d(x) = -d(y)xz + zd(y)x = -d(y)xz + d(y)zx \cdots\cdots\cdots (2)$ for all $x, y \in S$.Replacing $\sigma(y)$ by $d(x)$ in (2) and using (1) we have $d(x)d(x)z - zd(x)d(x) = d(y)[-xz + zx] \Rightarrow d(y)[z, x] = 0$ for all $x, y, z \in S. \cdots\cdots\cdots (3)$. Replacing z by zy in (3) and using (3) we get, $d(y)z[y, x] = 0\ for\ all\ x, y, z \in S$.Since S is prime, and $d(y) \neq 0$,we have $[y, x] = 0\ for\ all\ x, y \in S$.Therefore S is commutative. Hence Proved.

Lemma:3.5

Let S be a prime semi near ring with center Z, and let d be a non-zero reverse σ-derivation of S, then $d(Z) \subset Z$.

Proof:

For any $z \in Z$ and $x \in S$, we have $d(xz) = d(zx) = d(z)x + \sigma(z)d(x) = \sigma(z)d(x) + d(z)x$(by lemma:3.3) .If we replace $\sigma(z) by\ z$, we get,$d(xz) = zd(x) + d(z)x \cdots\cdots\cdots (1)$ for all $x, z \in S \Rightarrow d(zx) = d(x)z + \sigma(x)d(z) \cdots\cdots\cdots (2)$ for all $x, z \in S$. From (1) and (2) we get,$d(z)x = \sigma(x)d(z)$ and since σ is automorphism, we have $d(z)x = xd(z) for\ all\ x, z \in S$. Therefore,$d(z) \in Z$. Hence the proof.

Lemma:3.6

Let d be a non-zero reverse σ-derivation of a prime semi near ring S, and $x \in S$. *If* $xd(S) = 0$ *or* $d(S)x = 0$, *then* $x = 0$.

Proof:

Let us assume that, $xd(s) = 0 \cdots\cdots\cdots (1)$ for all $s \in S$. Replacing s by ms in (1), we have $xd(ms) = 0 \Rightarrow xd(s)m + x\sigma(s)d(m) = 0 \cdots\cdots\cdots (2)$ for all $x, m, s \in S$. By using (1) in (2) we get, $x\sigma(s)d(m) = 0$. Also since σ is automorphism, we have $xSd(m) = 0\ for\ all\ x, m \in S$. And, since S is prime and $d(s) \neq 0$, we have $x = 0$.Similarly, we can prove the case when $d(s)x = 0$ for all $s \in S$. Hence proved.

Theorem:3.7

For a prime semi near ring S, let d be a non-zero reverse σ- derivation of S, such that $[x, d(x)] = 0$, *for all* $x \in S$, *then S is commutative.*

Proof:

Let $[x, d(x)] = 0 \cdots\cdots\cdots (1)$ for all $x \in S$. Replacing $d(x)$ by $yd(x)$ in (1) and using (1) again, $[x, yd(x)] = 0$ we get $[x, y]d(x) = 0 \cdots\cdots\cdots (2)$ for all $x, y \in S$. Replace y by zy in (2) and using (2), we get $[x, zy]d(x) = 0$ we get $[x, z]yd(x) = 0$ for all $x, y, z \in S$. Since S is prime, we have either $[x, z] = 0\ or\ d(x) = 0$. Since $d(x) \neq 0, for\ all\ x \in S$, then $[x, z] = 0$ it follows that $x \in Z(S)$ for each fixed $x \in S$ and by lemma:3.3, we get $d(x) \in Z(S)$, that is $d(S) \subset Z(S)$. By lemma:3.4, we get S is commutative.

Theorem:3.8

Let S be a prime semi near ring, and d be a non-zero reverse σ-derivation of S. If $[d(y), d(x)] = 0\ for\ all\ x, y \in S$, *then S is commutative.*

Proof:

Given that $[d(y), d(x)] = 0 \cdots\cdots\cdots (1)$ for all $x, y \in S$. Replacing y by yx in (1) we get $[d(x)y + \sigma(x)d(y), d(x)] = 0$. By using (1) again, we get,$d(x)[y, d(x)] + [\sigma(x), d(x)]d(y) = 0 \cdots\cdots\cdots (2) for\ all\ x, y \in S$. Replacing y by zy, where $z \in Z(S)$ in (2) we get,$d(x)z[y, d(x)] + d(x)[z, d(x)]y + [\sigma(x), d(x)]d(y)z + [\sigma(x), d(x)]\sigma(y)d(z) = 0 \cdots\cdots\cdots (3) for\ all\ x, y, z \in S$. Since σ is automorphism and by using (2) in (3) we get, $[\sigma(x), d(x)]yd(z) = 0\ for\ all\ x, y, z \in S$. Since S is prime, we have either $[\sigma(x), d(x)] = 0$ or $d(x) = 0$. Since $d(z) \neq 0$, we have $[\sigma(x), d(x)] = 0 \cdots\cdots\cdots (4)\ for\ all\ x \in S$. Replacing $\sigma(x)$ by x in (4) and by using Theorem:3.7 we get, S is commutative.

Theorem:3.9

Let S be a prime semi near ring, and d be a non-zero reverse σ-derivation of S. If $[x, d(y)] \in Z(S)$ *, for all* $x, y \in S$*, then S is commutative.*

Proof:

Assume that $[x, d(y)] \in Z(N)$ for all $x, y \in S$. Hence for all $s \in S, \big[[x, d(y)], s\big] = 0 \cdots (1)$. Replacing x by $xd(y)$ in (1), and using (1) again, we get$[x, d(y)][d(y), s] = 0 \cdots\cdots (2)$ for all $x, y, s \in S$. Replacing x by sx in (2), and using (2) again, we get$[s, d(y)]x[d(y), s] = 0 \cdots\cdots\cdots (3)$ for all $x, y, s \in S$. Since S is prime, we have either $[s, d(y)] = 0 \cdots\cdots (4)\ for\ all\ s, y \in S\ or\ [d(y), s] = 0 \cdots\cdots (5) for\ all\ y, s \in S$. If we replace $d(y)$ by $md(y)$ in (4) and (5) we get, $[s, md(y)] = m[s, d(y)] + [n, m]d(y) = 0 \Rightarrow [s, m]d(y) = 0$. Also, $[md(y), s] = 0 \Rightarrow m[d(y), s] + [m, s]d(y) = 0 \Rightarrow [m, s]d(y) = 0$ for all $s, m, y \in S$. By using lemma:3.6 in two cases we have $[s, m] = 0$ and $[m, s] = 0$ for all $s, m \in N$. Therefore, S is commutative.

Theorem:3.10

Let S be a prime semi near ring, d be a non-zero reverse σ-derivation of S, and $y \in S$*. If* $[d(x), y] = 0$ *then* $d(y) = 0$ *or* $y \in Z(S)$*.*

Proof:

Let $[x, d(x)] = 0 \cdots\cdots\cdots (1)$ for all $x \in S$. Replacing $d(x)$ by $yd(x)$ in (1) and using (1) again, $[x, yd(x)] = 0 \Rightarrow [x, y]d(x) = 0 \cdots\cdots\cdots (2)$ for all $x, y \in S$. Replace y by zy in (2) and using (2), we get $[x, zy]d(x) = 0$ we get $[x, z]yd(x) = 0$ for all $x, y, z \in S$. Since S is prime, we have either $[x, z] = 0$ or $d(x) = 0$. Since $d(x) \neq 0$, for all $x \in S$, then we have $[x, z] = 0$, it follows that $x \in Z(S)$ for each fixed $x \in S$, and by lemma:3.5, we get $d(x) \in Z(S)$, that is $d(S) \subset Z(S)$. Then by lemma :3.4 , we get S is commutative.

Theroem:3.11

.Let S be a prime semi near ring, and d be a non-zero reverse σ-derivation of S, such that $d([x, y]) = [x, d(y)]\ for\ all\ x, y \in S$*, then S is commutative*

Proof:

Given that $d([x, y]) = [x, d(y)] \cdots\cdots (1)$ for all $x, y \in S$. Replacing y by yx in (1) and using (1), we get, $[x, d(x)]y + [x, \sigma(x)]d(y) = 0 \cdots\cdots (2)$ for all $x, y \in S$. If we replace $\sigma(x)$ by x in (2) $[x, d(x)]y = 0 \cdots\cdots\cdots (3)$for all $x, y \in S$. Replacing y by $yd(x)$ in (3), we get, $[x, d(x)]yd(x) = 0$. Since S is prime, $d(x) \neq 0$ we have, $[x, d(x)] = 0$ for all $x \in S$. Then by Theorem:3.7 we get S is commutative. Hence proved.

4. CONCLUSION:

We have studied in this paper, the commutativity of S, where S is a semi near ring and S has a non-zero reverse σ-derivation d, where d is an additive mapping from S onto itself satisfying $d(xy) = d(y)x + \sigma(y)d(x)$ for all $x, y \in S$ and σ is an automorphism on a semi near ring S. Also we have introduced some conditions on d, to get the commutativity on S, when S is a prime semi near ring.

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