A Study on Quasi Weak Commutative Gamma Near Rings

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Abstract : -

The concept of gamma near rings (defined by Satyanarayana in 1984) is generalization of both the concepts “gamma ring” and “near ring”. The role of Commutativity plays a vital factor in near ring theory as Quasi weak, weak, weakly sub commutative, pseudo commutative, etc. Here, we introduce a new notion called Quasi weak Commutative gamma near ring, It is proved that every Quasi Weak Commutative gamma near ring is zero symmetric and is gamma pseudo commutative also when it is weak gamma commutative. Homomorphic image will be a right gamma pseudo commutative and also isomorphic to a sub-direct product of sub directly irreducible Quasi weak Commutative gamma near ring. If N is a regular Quasi weak commutative gamma near ring, then N is reduced and has $(*, \text{IFP})$.

Keywords :-

Gamma near ring, weak Commutativity, pseudo Commutativity, Zero-Symmetric, ideal, Semi-prime ideal, completely semi prime ideal, reduced.

1. Introduction

Near rings can be thought of as generalized rings: if in a ring we ignore the commutativity of addition and one distributive law, we get a near ring. Gunter Pilz [3] "Near Rings" is an extensive collection of the work done in the area of near rings.

The concept Γ-ring, a generalization of a ring was introduced by Nobusawa and generalized by Barnes. A generalization of both the concepts near-ring and the gamma-ring, namely Γ-near-ring was introduced by Satyanarayana [1] and later studied by several authors.

1) Let $(M, +)$ be a group (not necessarily Abelian) and Γ be a non-empty set. Then M is said to be a **Γ-near-ring** if there exists a mapping $M \times \Gamma \times M \rightarrow M$ (the image of (a, α, b) is denoted by $a\alpha b$), satisfying the following conditions:

   (i) $(a + b)\alpha c = a\alpha c + b\alpha c$ and

   (ii) $(a\alpha b)\beta c = a\alpha(b\beta c)$ for all $a, b, c \in M$ and $\alpha, \beta \in \Gamma$ [1].

2) M is said to be a **zero-symmetric Γ-near-ring** if $a\alpha o = o$ for all $a \in M$ and $\alpha \in \Gamma$, where o is the additive identity in M [1].

2.Preliminaries

Definition 2.1 [2]

A near ring N is said to be **weak commutative** near ring if $xyz = xzy$ for all $x, y, z \in N$.

Definition 2.2 [2]

A near ring N is said to be **quasi weak commutative** near ring if $xyz = yxz$ for all $x, y, z \in N$.

Definition 2.3 [21]

A near ring N is said to be **pseudo commutative** near ring if $xyz = zyx$ for all $x, y, z \in N$.

Definition 2.4 [3]

A near ring N is said to have property $\boldsymbol{P_4}$ if $ab \in I \Longrightarrow ba \in I$ where I is any ideal of N.

Definition 2.5 [2]

An ideal I of N is called a **prime ideal** if for all ideals A, B of N, AB is subset of $I \Longrightarrow A$ is subset of I or B is subset of I.

Definition 2.6 [2]

I is called a **semi-prime ideal** if for all ideals A of N,A^2 is subset of I implies A is subset of I.

Definition 2.7 [2]

I is called a **completely semi – prime ideal**, if for any $x \in N$, $x^2 \epsilon I \Longrightarrow x \in I$.

Definition 2.8 [2]

A completely prime ideal, if for any $x, y \epsilon N, xy \in I \Longrightarrow x \in I$ or $y \in I$.

Definition 2.9 [2]

A near ring N is said to be reduced if N has no nonzero nilpotent elements.

3.Quasi Weak Commutative Gamma Near ring

Definition 3.1

A gamma near ring is said to be Quasi Weak Commutative if $x\gamma y\gamma z = y\gamma x\gamma z$ for all x, y, z in N and $\gamma \in \Gamma$.

Remark 3.2

Every commutative gamma near ring is Quasi weak, but not the converse.

Theorem 3.3

Every Quasi Weak Commutative Gamma near ring is zero symmetric.

Proof:

Let N be Quasi Weak Commutative gamma near ring.

For every $a \in N$, $a\gamma 0 = a\gamma(0\gamma 0)$

$$= (0\gamma a)\gamma 0$$

$$= 0\gamma 0$$

$$= 0$$

Hence N is Γ-zero symmetric.

Theorem 3.4

Let (N, Γ) be both weak gamma commutative and Quasi weak gamma commutative near ring. Then it is gamma pseudo commutative.

Proof:

For all x, y, z in N and $\gamma \in \Gamma$.

$x\gamma y\gamma z = y\gamma x\gamma z$

$\quad = y\gamma z\gamma x$

$= z\gamma y\gamma x$

This proves N as gamma pseudo commutative.

Theorem 3.5

Homomorphic image of Quasi Weak Commutative gamma near ring is a right pseudo commutative gamma near ring.

Proof:

Let N be given Quasi Weak Commutative gamma near ring. Let $f: N \longrightarrow M$ be endomorphism of gamma near rings. For all x, y, z in N ; $\gamma \in \Gamma$ $f(x)\gamma f(y)\gamma f(z) = f(x\gamma y\gamma z) = f(y\gamma x\gamma z) = f(y)\gamma f(x)\gamma f(z)$. Hence M is a Quasi Weak Commutative gamma near ring.

Theorem 3.6

Every Quasi Weak Commutative gamma near ring is isomorphic to a subdirect product of subdirectly irreducible Quasi Weak Commutative gamma near rings.

Theorem 3.7

Any Weak Commutative gamma near ring with left identity is a Quasi Weak Commutative gamma near ring.

Proof:

For any a, b, c in N and $\gamma \in \Gamma$, $a\gamma b\gamma c = e\gamma(a\gamma b\gamma c)$

$= (e\gamma a\gamma b)\gamma c$

$= (e\gamma b\gamma a)\gamma c$

$= b\gamma a\gamma c$

Hence N is a Quasi Weak Commutative gamma near ring.

Theorem 3.8

Any Quasi Weak Commutative gamma near ring with right identity is weak gamma commutative.

Proof:

Let $a, b, c \in N$; $\gamma \in \Gamma$; $e \in N$ be right identity.

Then, $a\gamma b\gamma c = (a\gamma b\gamma c)\gamma e$

$= a\gamma(b\gamma c\gamma e)$

$= a\gamma(c\gamma b\gamma e)$

$= (a\gamma c\gamma b)\gamma e$

$= a\gamma c\gamma b$

Hence N is gamma weak commutative.

Definition 3.9

A gamma near ring is said to have property $\boldsymbol{P_4}$ if $a\gamma b \in I \Longrightarrow b\gamma a \in I$ where I is any ideal of N.

Theorem 3.10

Let N be a regular Quasi Weak Commutative gamma near ring. Then (i) Every ideal of N is completely semi prime. (ii) N has property P_4.

Proof:

(i) Let I be an ideal of N. Let $a\gamma a \in I$.

$a = a\gamma b\gamma a$ (Since N is gamma regular)

$= b\gamma a\gamma a$

$\in N\Gamma I$

$\subset I$

Hence, every ideal is completely semi prime

(ii) Let $a\gamma b \in I$.

Then $(b\gamma a)^2 = (b\gamma a)(b\gamma a)$

$= b\gamma(a\gamma b)\gamma a$

$\in N\Gamma I\Gamma N$

$\subseteq I$

Therefore by (i), $b\gamma a \in I$. Hence , property P_4 satisfies.

Theorem 3.11

Any Quasi weak commutative gamma near ring N with left identity is gamma commutative.

Proof:

Let $a, b \in N$ and $e \in N$ be the identity.

Then for all $\gamma \in \Gamma$ $a\gamma b = a\gamma b\gamma e$

$= b\gamma a\gamma e$ (Since N is Quasi weak Commutative gamma near ring)

$= b\gamma a$

Hence N is gamma commutative.

Theorem 3.12

Let N be a regular quasi weak commutative gamma near ring. Then N is reduced.

Proof:

Since N is gamma regular, for every $a \in N$, there exists $b \in N$ such that

$a = a\gamma b\gamma a = b\gamma a\gamma a$ .....(1) (Since N is Quasi weak commutative gamma near ring) for all $\gamma \in \Gamma$.

If $a\gamma a = 0$ then by (1) $a = b\gamma a\gamma a$

$= b\gamma(a\gamma a)$

$= b\gamma 0$

$= 0$

This completes the proof.

Definition 3.13

We say that a gamma near ring N has $(*, \text{IFP})$ if

i) N has IFP.
ii) for $a, b \in N$ and $\gamma \in \Gamma$, $a\gamma b = 0 \Longrightarrow b\gamma a = 0$.

Theorem 3.14

Let N be a regular Quasi weak commutative gamma near ring. Then N has $(*, \text{IFP})$.

Proof:

Since N is gamma regular, for every $a \in N$, there exists $b \in N$ such that $a = a\gamma b\gamma a$ for all $\gamma \in \Gamma$.

Now $a = a\gamma b\gamma a$

$= b\gamma a\gamma a$ (Since N is Quasi weak commutative gamma near ring)

Let $a\gamma b = 0$

Then $(b\gamma a)^2 = (b\gamma a)\gamma(b\gamma a)$

$= b\gamma(a\gamma b)\gamma a$

$= b\gamma(0\gamma a)$

$= b\gamma 0$

$= 0$

so by theorem 3.12 $b\gamma a = 0$.

Now for any $n \in N$,

$(a\gamma n\gamma b)^2 = (a\gamma n\gamma b)\gamma(a\gamma n\gamma b)$

$= a\gamma n\gamma(b\gamma a)\gamma n\gamma b$

$= (a\gamma n)0(n\gamma b)$

$= 0$

Again by theorem 3.12, $a\gamma n\gamma b = 0$.

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