

MINIMIZATION OF TRANSPORTATION AND INVENTORY COST IN LOGISTIC PLANNING

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Abstract: In today's competitive environment, supply chain management is a major concern for a company. Two of the key issues in supply chain management and inventory management are transportation and inventory management. To achieve significant savings, companies should integrate these two issues instead of treating them separately. This work considers the problem of selecting the appropriate distribution strategy for delivering a family of products from a set of suppliers to a set of plants so that the total transportation, pipeline inventory, and plant inventory costs are minimized. With reasonable assumptions, a simple model is presented to provide a good solutions that can serve as a guideline for the design and implementation of the distribution network. Due to the plant inventory cost, the problem is formulated as a nonlinear integer programming problem. The problem is difficult to solve because the objective function is highly nonlinear and neither convex nor concave thus a greedy heuristic is proposed to find an initial solution an upper bound. A heuristic and a branch-and-bound algorithm are developed based on the lagrangian relaxation of the nonlinear program and computational experiments are performed. Based on the results the transportation and inventory cost are minimized by considering capacities of forms and distribution strategies

Index Terms: *Supply Chain Management, Inventory Costs, Non Linear Integer Programming, Greedy Heuristic.*

1. INTRODUCTION

TRANSPORTATION MANAGEMENT SYSTEM

A **Transportation Management System (TMS)** is a subset of **supply chain management** concerning transportation operations and may be part of an enterprise resource planning systems.

A TMS usually "sits" between an ERP or legacy order processing and warehouse/distribution module. A typical scenario would include both inbound (procurement) and outbound (shipping) orders to be evaluated by the TMS Planning Module offering the user various suggested routing solutions. These solutions are evaluated by the user for reasonableness and are passed along to the transportation provider analysis module to select the best mode and least cost provider. Once the best provider is selected, the solution typically generates electronic load tendering and track/trace to execute the optimized shipment with the selected carrier, and later to support freight audit and payment (settlement process). Links back to ERP systems (after orders turned into optimal shipments), and sometimes secondarily to wms programs also linked to ERP are also common. Transportation management systems manage four key processes of transportation management:

1. **Planning and decision making** – TMS will define the most efficient transport scheme according to given parameters, which have a lower or higher importance according to the user policy: transport cost, shorter lead-time, fewer stops possible to ensure quality, flows regrouping coefficient, etc.
2. **Transportation execution** – TMS will allow for the execution of the transportation plan such as carrier rate acceptance, carrier dispatching, EDI etc.,
3. **Transport follow-up** – TMS will allow following any physical or administrative operation regarding transportation: traceability of transport event by event (shipping from A, arrival at B, customs clearance, etc.), editing of reception, custom clearance, invoicing and booking documents, sending of transport alerts (delay, accident, non-forecast stops...)
4. **Measurement** - TMS have or need to have a logistics key performance indicator (KPI) reporting function for transport.

2. PRESENT PROBLEM

It is considered the problem of selecting the appropriate distribution strategy for delivering a family of products from a set of suppliers to a set of plants so that the total transportation, pipeline inventory and plant inventory costs are minimized. Latter, it is considered the interaction between plants to meet the gap between actual requirement and forecasted requirement by minimizing the total cost internally between plants

3. METHODOLOGY

It is considered the problem of selecting the appropriate distribution strategy for delivering a family of products from a set of suppliers to a set of plants so that the total transportation, pipeline inventory and plant inventory costs are minimized. Latter, it is considered the interaction between plants to meet the gap between actual requirement and forecasted requirement by minimizing the total cost internally between plants.

3.1 BRIEF DESCRIPTION OF THE PROBLEM

The proposed model is a two stage distribution inventory model with suppliers, cross-dock and plants with limited capacities. It is assumed that products are being distributed from the suppliers to plants in the system. The demand for the products will be forecasted before beginning of every period and will be used as the reference for suppliers to transfer stocks from them to plants in a particular period. All plants and customers in the downstream are identical; i.e., they have identical cost structures and demand distributions. Any demand that is not met is considered as the back order. All cross-docks and plants will be managed by a single organization only. Random demands occur at plants, which were then accommodated from other suppliers for replenishing inventories. Excess demands at plants are completely backlogged. This model is suitable to any kind of industry that follows the two-stage supply chain model with finite number of suppliers and plants.

The problem is divided into two stage optimization model and in the first stage we consider suppliers, cross-dock and plants and interaction between them. Based on the forecasted demand values, plants will try to meet the need from all suppliers. The capacities of all suppliers and plants will also be taken into consideration in optimizing the inventory cost and transportation cost. Once the optimal solution is found then these values will be used in the second stage to further optimize the model.

In the second stage, it is considered the interaction among plants to meet the gap between actual requirement and forecasted projections. Here forecasted projections are done by using moving averages method. The Optimal values from first stage are taken as input to minimize the inventories and back orders cost at each plant by considering the lateral transshipment among plants. For modelling simplicity, we did not consider the lead times between plants at the end of every period, and we assume it as negligible. That is, we assume that this lateral transshipment of stocks among plants will be taken place before starting of next period.

The purpose of this model is to provide an optimal inventory level for the supplier and plant and also minimizing the total cost of the supply chain at each period. In designing the proposed model, the three subsystems need to be analyzed.

- i. The inventory level at each supplier for all periods.
- ii. The inventory level at each plant for all periods.
- iii. Route between entities where the product is transferred from supplier to plant and among plants with back orders for all periods.

Assumptions:

- Product quantities are infinitely split table, i.e., a product can be shipped in any quantity within a vehicle shipment.
- Delivery frequency can be any positive number and is not limited to a set of potential numbers.
- Products are always available for shipping at suppliers, no matter which distribution strategy is chosen.
- Inbound-outbound coordination at the cross-dock is ignored.
- All units of the same flow (a flow is a combination of supplier, plant, and product) are assigned to the same transportation option, i.e., direct or through the same cross-dock.
- Each truck is fully loaded. Only the volume of product is concerned when calculating truck capacity usage. The transportation costs are only determined by the source and destination, regardless of the weight.
- Only one truck type is considered.

3.2 TWO STAGE CONSTRAINED OPTIMIZATION MODEL

The proposed model deals with optimizing the inventory levels of the supply chain entities with consideration of single-echelon supplier to plant and among plants.

3.3 FIRST STAGE OF THE PROBLEM

In the first stage total cost is minimized by considering all the three distribution strategies.

The first stage objective function is given below:

$$MIN TC_{PI} = g_{di}(X) + g_{mi}(X) + g_{cd}(X)$$

➤ $g_{di}(X)$;

Function $g_{di}(X)$ is the total cost of direct delivery. For each supplier-plant pair (s,p), the frequency of shipment is the ratio of quantity delivered from supplier to plant to capacity of truck.

$$fr_{sp}^d = \sum_{i \in I_{sp}} b_i \cdot q_{spi} / c_i$$

Therefore, transportation cost is $fr_{sp}^d \cdot C_{sp}^d$.

The pipeline inventory cost is $\sum_{i \in I_{sp}} t_{sp}^d \cdot h_i \cdot q_{spi}$

The plant inventory cost is $\sum_{i \in I} h_i \cdot \frac{q_{spi}}{2fr_{sp}^d}$. hence, we have

$$g_{di}(X) = \sum_{s \in S} \sum_{p \in P} [fr_{sp}^d \cdot C_{sp}^d + \sum_{i \in I_{sp}} (t_{sp}^d \cdot h_i \cdot q_{spi} / (2fr_{sp}^d))]$$

➤ $g_{mi}(X)$;

Function $g_{mi}(X)$ is the total cost of milk-run delivery. Here goods of one supplier are distributed to available plants in the same trip. Here frequency of shipment is the ratio of total quantity delivered to the plants from a single supplier to capacity of the truck.

$$fr_{sp}^m = \sum_{p \in P} \sum_{i \in I_{sp}} b_i \cdot q_{spi} / C_i$$

Therefore, transportation cost is $fr_{sp}^m \cdot C_{sp}^m$.

The pipeline inventory cost is $\sum_{i \in I_{sp}} t_{sp}^m \cdot h_i \cdot q_{spi}$

The plant inventory cost is $\sum_{i \in I_{sp}} h_i \cdot \frac{q_{spi}}{2fr_{sp}^m}$. hence, we have

$$g_{mi}(X) =$$

$$\sum_{s \in S} \sum_{p \in P} [(\sum_{p \in P} \sum_{i \in I_{sp}} fr_{sp}^m) \cdot C_{sp}^m + \sum_{s \in S} \sum_{p \in P} (\sum_{i \in I_{sp}} t_{sp}^m \cdot h_i \cdot q_{spi} + \sum_{i \in I_{sp}} h_i \cdot \frac{q_{spi}}{2fr_{sp}^m})]$$

$$= \sum_{s \in S} \sum_{p \in P} [(\sum_{p \in P} \sum_{i \in I_{sp}} fr_{sp}^m) \cdot C_{sp}^m + \sum_{i \in I_{sp}} t_{sp}^m \cdot h_i \cdot q_{spi} + \frac{\sum_{i \in I_{sp}} h_i \cdot q_{spi}}{2 \cdot fr_{sp}^m}]$$

➤ $g_{cd}(X)$;

Function $G_{cd}(X)$ is the cost of shipping flows that travel through cross-dock c. the transportation cost consists of two parts; the inbound-transportation cost and the outbound transportation cost. For $s \in S$, the frequency of inbound shipment is

$fr_{sc}^{ib} = \sum_{p \in P} \sum_{i \in I_{sp}} b_i \cdot q_{spi} / C_i$ And the total inbound transportation cost is $\sum_{s \in S} fr_{sc}^{ib} \cdot C_{sc}^{ib}$. For each $p \in P$, the frequency of outbound shipment is $fr_{cp}^{ob} = \sum_{s \in S} \sum_{i \in I_{sp}} b_i \cdot q_{spi} / C_i$ and the outbound transportation cost is $\sum_{p \in P} fr_{cp}^{ob} \cdot C_{cp}^{ob}$.

Because the transportation time for shipping flow (s,p,i) through cross-dock c is $t_{sc}^{ib} + T_c + t_{cp}^{ob}$, the pipeline inventory cost is $(t_{sc}^{ib} + T_c + t_{cp}^{ob}) \cdot h_i \cdot q_{spi}$, and the plant inventory cost is $h_i / 2fr_{cp}^{ob}$. Hence, we have

$$g_{cd}(X) = \sum_{i \in I_{sp}} fr_{sc}^{ib} \cdot C_{sc}^{ib} + \sum_{p \in P} fr_{cp}^{ob} \cdot C_{cp}^{ob} + \sum_{p \in P} \sum_{(s,i) \in I_p} [t_{sc}^{ib} + T_c + t_{cp}^{ob}] \cdot h_i \cdot q_{spi} + h_i \cdot \frac{q_{spi}}{2fr_{cp}^{ob}} + \text{fixed cost.}$$

Now, the first stage objective function is given below:

$$MIN TC_{pi} = g_{di}(X) + g_{mi}(X) + g_{cd}(X)$$

In the first stage, we have the following constrains:

1. Total quantity transported item wise equals to sum of quantities transported by direct cross-dock and milk-run.

$$Qp_i = \sum_{p \in P_1} \sum_{i \in I_{sp}} (\sum_{s \in S} q_s p_p i_i d_i + q_c p_p i_{icd} + \sum_{s \in S} q_s p_p i_i + m_i)$$

$$Qp_2 = \sum_{p \in P_2} \sum_{i \in I_{sp}} (\sum_{s \in S} q_s p_p i_i d_i + q_c p_p i_{icd} + \sum_{s \in S} q_s p_p i_i + m_i)$$

2. Quantity which is taken as input to cross-dock to both the suppliers of both the items should be \geq quantity of output from cross-dock to both the plants.

$$\sum_{s \in S} \sum_{i \in I_{sp}} (q_s p_p i_i C_i) \geq \sum_{p \in P} \sum_{i \in I_{sp}} (q_s p_p i_i)$$

3. Short-term buffer is always maintained at cross-dock to meet sudden surprises in no stock levels. Stock available at cross-dock is given by

$$\sum_{s \in S} \sum_{i \in I_{sp}} (q_s p_p i_i C_i) - \sum_{p \in P} \sum_{i \in I_{sp}} (q_s p_p i_i) \geq \text{Available stock in cross-dock.}$$

4. Quantity bounds for suppliers and plants for both the items are specified. Lower bound $\leq QS \leq$ upper bound, $\forall s \in S$.

5. For a particular period of time total demand is divided into trips which are known as frequency of a vehicle during that distribution strategy.

Frequency = quantity transported / capacity of vehicle

6. Frequency bounds for three distribution strategies for both the items are specified.

Lower bound \leq fr-spidi \leq upper bound, $\forall s \in S, p \in P$

Lower bound \leq fr-spimi \leq upper bound, $\forall s \in S, p \in P$

Lower bound \leq fr-scicd \leq upper bound, $\forall s \in S, c \in C$

Lower bound \leq fr-cpicd \leq upper bound, $\forall c \in S, c \in C$

Lower bound ≤ fr-spimixedmi > upper bound, ∀ s ∈ S

3.4 SECOND STAGE OF THE PROBLEM:

In the second stage total cost internally between the plants is minimized.

The second stage objective functions is given below:

$$MIN TC_{p_2} = tot_tr_p_ + tot_inv_p_p + tot_boc_p_p$$

Quantity presents in plant after stage 2 is the difference between quantity present in plant after stage 1 and actual quantity in the plant.

$$q_{p_1} aftstg2mn0 = (act_{p_1i_1} - act_{p_1i_1}) * yk_{p_1i_1}$$

Or

Quantity present in plant after stage 2 in case of back order is difference between actual quantity present in plant and quantity present in plant after stage 1.

$$q_{p_1} aftstg2bo = (act_{p_1i_1} - q_{p_1i_1}) * (1 - yk_{p_1i_1})$$

Here yk_{p_1} and $(1 - yk_{p_1i_1})$ binary variables

Similarly for all plants and items.

- Total inventory cost plant 1 = $(Q_{s_1}i_i - \text{units transferred to } p_2) * \text{unit carrying cost}$.
- Back order cost = net back order cost = backorder-receipts from other plants * rate
- Total transportation cost internally plant to plant = unit transportation cost * quantity from retailer to retailers;
 $tot_tr_p_p = \sum_{i \in I, sp} [(qp_1-p_2i_i * utc_p_2-p_1) + (qp_2-p_2i_i * utc_p_1-p_2)];$
- Total inventory carrying cost = net inventory * unit carrying cost of all plants.
 Net inventory = forecasted inventory at plant - actual projection - quantities transferred to other plants;
 $tot_inv_p_p = \sum_{i \in I, sp} [(qp_1i_i - afttsg_2 - qp_1-p_2i_i) * icc_qp_2-p_2i_i * utc_p_1-p_2];$
- Total back order cost = (state_back order-unit received from other plants) * unit back order
 $tot_boc_p_p = \sum_{i \in I, sp} [(qp_1i_i - afttsg_2 - bo - qp_2-p_1pi_i) * u_bocp_i pi_i + (qp_1i_i - afttsg_2 - bo - qp_1-p_2i_i) * u_bocp_2i_i]$

Now the second stage objective function in given by:

$$MIN TC_{p_2} = tot_tr_p_p + tot_inv_p_p + tot_boc_p_P;$$

In the second stage, we have the following constraints:

1. The sum of the units transferred from a plant to all plants should be less than or equal to the excess inventory of a plant at the end of the stage 1;
 $q_{p_1-p_2i_i} \leq qp_1i_i - afttsg_2; \forall p \in p, i \in I:$
2. The sum of the units received by a plant from all plants should be less than or unequal to the back order of that plant for a particular period 'p';
 $q_{p_1-p_2i_i} = \text{received by palnt 2 to from palnt 1};$
 $q_{p_1-p_2i_i} \leq qp_1i_i - afttsg_2; \forall p \in p, i \in I:$
3. The sum of all transportation costs from all plants to a particular plant should be less than or equal to the transportation cost from any supplier to that plant plus the back ordering cost of that plant;
 $q_{p_1-p_2i_i} * \text{unit transport cost from } p_2 \text{ to } p_1 \leq (q_{p_2i_1} - afttsg_2 - bo) * \text{unit transport cost from } s_1 + (q_{p_2i_1} - afttsg_2 - bo) * \text{unit backorder cost};$
 $\forall p \in p, i \in I:$

OUTPUT DATA FOR STAGE 1

QUANTITY AND FREQUENCY VALUES FOR THE THREE DISTRIBUTION STRATEGIES:

Table 6.1 Quantities and frequency values for three distribution strategies.

		Q (units)	Fr (trips)
Direct Delivery	Quantity delivered from supplier 1 to plant 1 for item 1 ($s_1p_1i_1 di$)	8000	100
	Quantity delivered from supplier 1 to plant 2 for item 1 ($s_1p_2i_1 di$)	4000	50
	Quantity delivered from supplier 2 to plant 1 for item 1 ($s_2p_1i_1 di$)	4000	50
	Quantity delivered from supplier 2 to plant 2 for item 1 ($s_2p_2i_1 di$)	4000	50
	Quantity delivered from supplier 1 to plant 1 for item 2 ($s_1p_1i_2 di$)	9964	94
	Quantity delivered from supplier 1 to plant 2 for item 2 ($s_1p_2i_2 di$)	5300	50
	Quantity delivered from supplier 2 to plant 1 for item 2 ($s_2p_1i_2 di$)	5300	50
	Quantity delivered from supplier 2 to plant 2 for item 2 ($s_2p_2i_2 di$)	9646	91
	Quantity delivered from supplier 1 to plant 1 for items 1 ($s_1p_1i_1 mihomo$)	56	4

Milk-run delivery	Quantity delivered from supplier 1 to plant 2 for items 1 ($s_1p_2i_1mihomo$)	264	4
	Quantity delivered from supplier 2 to plant 1 for items 1 ($s_2p_1i_1mi$)	80	2
	Quantity delivered from supplier 2 to plant 2 for items 1 ($s_2p_2i_1mi$)	80	2
	Quantity delivered from supplier 1 to plant 1 for items 2 ($s_1p_1i_2mihomo$)	42	4
	Quantity delivered from supplier 1 to plant 2 for items 2 ($s_1p_2i_2mihomo$)	382	4
	Quantity delivered from supplier 2 to plant 1 for items 2 ($s_2p_1i_2mi$)	106	2
	Quantity delivered from supplier 2 to plant 2 for items 2 ($s_2p_2i_2mi$)	106	2
	Quantity delivered from supplier 1 to plant 1 for items 1 ($s_1p_1i_1mihetro$)	21	4
	Quantity delivered from supplier 1 to plant 1 for items 2 ($s_1p_1i_2mihetro$)	28	4
	Quantity delivered from supplier 1 to plant 2 for items 2 ($s_1p_2i_2mihomo$)	396	4
	Quantity delivered from supplier 2 to plant 2 for items 2 ($s_2p_2i_2mihomo$)	299	4
Cross-dock delivery			
	Quantity delivered from supplier 1 to cross-dock for item1 ($s_1c_1i_1cd$)	7360	92
	Quantity delivered from supplier 2 to cross-dock for item1 ($s_2c_1i_1cd$)	11840	148
	Quantity delivered from supplier 1 to cross-dock for item 2 ($s_1c_1i_2cd$)	3922	37
	Quantity delivered from supplier 2 to cross-dock for item 2 ($s_2c_1i_2cd$)	4876	46
	Quantity delivered from cross-dock to plant1 for item 1 ($c_1p_1i_1cd$)	17600	220
	Quantity delivered from cross-dock to plant2 for item 1 ($c_1p_2i_1cd$)	160	2
	Quantity delivered from cross-dock to plant1 for item 2 ($c_1p_1i_2cd$)	7314	69
Quantity delivered from cross-dock to plant2 for item 2 ($c_1p_2i_2cd$)	106	1	

Quantity delivered from supplier 1 to cross –dock for item 1 ($k = s_1c_1i_1cd$) 7360

Quantity delivered from supplier 2 to cross –dock for item 1 ($m = s_2c_1i_1cd$) 11840

Quantity transported from cross –dock to both the plants (p1, p2) for item 1

$n = c_1p_1i_1cd + c_1p_2i_1cd$

17760

QUANTITY DISTRIBUTED FROM BOTH THE SUPPLIERS TO BOTH THE PLANTS FOR BOTH THE ITEMS BY THE THREE DISTRIBUTION STRATEGIES:

Table 6.2 Quantity distributed from suppliers to plants by three Distribution strategies

	Direct	Cross –dock	Milk-run
Quantity delivered from supplier 1 to plant 1 for item 1 ($s_1p_1i_1$)	8000	1923	77
Quantity delivered from supplier 1 to plant 1 for item 2 ($s_1p_1i_2$)	9964	0	70
Quantity delivered from supplier 1 to plant 2 for item 1 ($s_1p_2i_1$)	4000	5437	563
Quantity delivered from supplier 1 to plant 2 for item 2 ($s_1p_2i_2$)	5300	3922	778
Quantity delivered from supplier 2 to plant 1 for item 1 ($s_2p_1i_1$)	4000	5920	80
Quantity delivered from supplier 2 to plant 1 for item 2 ($s_2p_1i_2$)	5300	4628	106
Quantity delivered from supplier 2 to plant 2 for item 1 ($s_2p_2i_1$)	4000	5920	80
Quantity delivered from supplier 2 to plant 2 for item 2 ($s_2p_2i_2$)	9646	248	106

TOTAL QUANTITY PRESENT IN BOTH THE PLANTS AFTER STAGE1

Table 6.3 Total Quantity Present In Both Plants After Stage 1

	Q (units)
Quantity delivered from supplier 1 to plant 1 for item 1 ($s_1p_1i_1$)	10000
Quantity delivered from supplier 1 to plant 1 for item 2 ($s_1p_1i_2$)	10000
Quantity delivered from supplier 1 to plant 2 for item 1 ($s_1p_2i_1$)	10000
Quantity delivered from supplier 1 to plant 2 for item 2 ($s_2p_2i_2$)	10034
Quantity delivered from supplier 2 to plant 1 for item 1 ($s_2p_1i_1$)	10000
Quantity delivered from supplier 2 to plant 1 for item 2 ($s_2p_1i_2$)	10000
Quantity delivered from supplier 2 to plant 2 for item 1 ($s_2p_2i_1$)	10000

TOTAL QUANTITY DELIVERED FROM BOTH THE SUPPLIERS DURING STAGE1

Table 6.4 Total Quantity Delivered From Both the Suppliers during Stage1

	Q (units)
Quantity delivered from supplier 1 for item 1 (s_1i_1)	20000
Quantity delivered from supplier 2 for item 1 (s_2i_1)	20000
Quantity delivered from supplier 1 for item 2 (s_1i_2)	20034
Quantity delivered from supplier 2 for item 2 (s_2i_2)	20034

TOTAL QUANTITY DELIVERED AND TRANSPORTED FROM CROSS –DOCK FOR ITEM 1:

Table 6.5 Total Quantity Delivered and Transported From Cross-Dock for Item 1

	Q (units)
Quantity input –quantity output for cross-dock for item 1	1440

We get the total transportation cost during the three deliveries as given below:

Total transportation cost in case of direct delivery: $g_{di}(X) = 2838288/-$

Total transportation cost in case of milk-run delivery: $g_{mi}(X) = 183788.7/-$

Total transportation cost in case of cross-dock delivery: $g_{cd}(X) = 1484723/-$

Now, the first stage objective function is given below:

$$\text{MIN } TC_{p1}(X) = g_{di}(X) + g_{mi}(X) + g_{cd}(X)$$

$$\text{MIN } TC_{p1} = 4506599/-$$

STAGE 2:

The second stage objective function is:

$$TC_{p2} = \text{tot_tr_p_p} + \text{tot_inv_p_p} + \text{tot_boc_p_p};$$

Quantity present in plant after stage 2 is the difference between quantity present in plant after stage 1 and actual quantity in the plant.

Or

Quantity present in plant after stage2 in case of back order is difference between actual quantity present in plant and quantity present in plant after stage1.

$$q_{p_1i_1} \text{aftstg2} = (q_{p_1i_1} - \text{act}_{p_1i_1}) * yk_{p_1i_1}$$

$$q_{p_1} \text{aftstg2bo} = (\text{act}_{p_1i_1} - q_{p_1i_1}) * (1 - yk_{p_1i_1})$$

$$q_{p_2i_1} \text{aftstg2} = (q_{p_2i_1} - \text{act}_{p_2i_1}) * yk_{p_2i_1}$$

$$q_{p_2} \text{aftstg2bo} = (\text{act}_{p_2i_1} - q_{p_2i_1}) * (1 - yk_{p_2i_1})$$

$$q_{p_1i_2} \text{aftstg2} = (q_{p_1i_2} - \text{act}_{p_1i_2}) * yk_{p_1i_2}$$

$$q_{p_1} \text{aftstg2bo} = (\text{act}_{p_1i_2} - q_{p_1i_2}) * (1 - yk_{p_1i_2})$$

$$q_{p_2i_2} \text{aftstg2} = (q_{p_2i_2} - \text{act}_{p_2i_2}) * yk_{p_2i_2}$$

$$q_{p_2} \text{aftstg2bo} = (\text{act}_{p_2i_2} - q_{p_2i_2}) * (1 - yk_{p_2i_2})$$

Here $yk_{p_1}, 1 - yk_{p_1i_1}, yk_{p_2i_1}, 1 - yk_{p_2i_1}, yk_{p_1i_2}, 1 - yk_{p_1i_2}, yk_{p_2i_2}, 1 - yk_{p_2i_2}$ are binary variables.

➤ tot_tr_p_p is total transportation cost internally between plants.

$$\text{tot_tr_p_p} = \sum_{i \in I_{sp}} [(q_{p_1-p_2i_2} * \text{utc}_{p_2-p_1}) + (q_{p_2-p_1i_1} * \text{utc}_{p_1-p_2})];$$

$$= (q_{p_1-p_2i_2} * \text{utc}_{p_2-p_1}) + (q_{p_1-p_2i_2} * \text{utc}_{p_2-p_1}) + (q_{p_1-p_2i_2} * \text{utc}_{p_1-p_2}) + (q_{p_1-p_2i_2} * \text{utc}_{p_1-p_2});$$

➤ tot_inv_p_p is total inventory cost internally between plants.

$$\text{tot_inv_p_p} = \sum_{i \in I_{sp}} [(q_{p_1i_1} - \text{aftstg}_2 - q_{p_1-p_2i_1}) * \text{icc}_{p_1i_1} + (q_{p_2i_1} - \text{aftstg}_2 - q_{p_2-p_1i_1}) * \text{icc}_{p_2i_1}]$$

$$= 0.5 * ((q_{p_1i_1} - \text{aftstg}_2 - q_{p_1-p_2i_1}) * \text{icc}_{p_1i_1} + (q_{p_1i_2} - \text{aftstg}_2 - q_{p_1-p_2i_2}) * \text{icc}_{p_1i_2} + (q_{p_2i_1} - \text{aftstg}_2 - q_{p_1-p_2i_1}) * \text{icc}_{p_2i_1}) + (q_{p_2i_2} - \text{aftstg}_2 - q_{p_1-p_2i_2}) * \text{icc}_{p_2i_2});$$

- the sum of the units transferred from a plant to all plants should be less than or equal to the excess inventory of a plant at the end of the stage 1;

$$q_{p_1-p_2i_1} \leq q_{p_1i_1} - \text{aftstg}_2$$

$$q_{p_1-p_2i_2} \leq q_{p_1i_2} - \text{aftstg}_2$$

$$q_{p_2-p_1i_1} \leq q_{p_2i_1} - \text{aftstg}_2$$

$$qp_2-p_1i_2 \leq qp_2i_2\text{-afttsg}_2$$

- The sum of the units received by a plant from all plants should be less than or unequal to the back order of that plant for a particular period 'p';

qp1_p2i1 = received by plant 2 to from plant1:

$$qp1_p2i1 \leq qp2i1_afttst_2_bo;$$

$$qp1_p2i2 \leq qp2i2_afttst_2_bo$$

$$qp2_p1i1 \leq qp1i1_afttst_2_bo;$$

$$qp2_p1i2 \leq qp1i2_aftts-2-bo;$$

- The sum of all transportation costs from all plants to a particular plant should be less than or equal to the transportation cost from any supplier to that plant plus the back ordering cost of that plant;

$$qp1_p2i1 * \text{unit transport cost from } p_2 \text{ to } p_1 \leq (qp2i1_afttsg_2_bo * \text{unit transport cost from } s_1) + (qp2i1_afttsg_2_bo * \text{unit backorder cost});$$

Table 6.6 Quantity present after stage 1

QUANTITY PRESENT AFTER STAGE1 IN BOTH PLANTS	QUANTITY
qp_1i_1	20000
qp_1i_2	20068
qp_2i_1	20000
qp_2i_2	20000

Table 6.7 Quantity present.

	Forecasted	Actual
qp_1i_1	20000	17000
qp_1i_2	20068	19000
qp_2i_1	20000	22000
qp_2i_2	20000	22000

UNIT TRANSPORTATION COST:

Table 6.8 Unit Transportation Cost between Supplier And Plant

UNIT TRANSPORTATION COST BETWEEN SUPPLIER – PLANT AND PLANT –PLANT (RUPEES)	p_1i_1	p_1i_2
Unit transportation cost from $p_2(rs)$	15.625	11.79
Unit transportation cost from Supplier 1 (s_1)	25	18.86
Unit transportation cost from supplier 2 (s_2)	37.5	28.3

Table 6.9 Unit Transportation Cost Internally Between Plants

UNIT TRANSPORTATION COST INTERNALLY BETWEEN PLANTS (RUPEES)	p_2i_1	p_2i_2
unit transportation cost from $p_1(rs)$	15.625	11.79
Unit transportation cost from supplier1 (s_1)	37.5	28.3
Unit transportation cost from suppliers 2 s_2	28.125	21.22

INVENTORY CARRYING COST:

Table 6.10 Inventory Carrying cost

Inventory carrying cost from plant (Rupees)	i_1	i_2
Inventory carrying cost from plant 1 (p_1)	125	150
Inventory carrying cost from plant 2 (p_2)	125	150

UNIT BACK ORDER COST:

Table 6.11 Unit back order cost

Unit Back order cost (Rupees)	i_1	i_2
Unit back order cost from plant 1 (p_1)	200	150
Unit back order cost from plant 2 (p_2)	200	150

OUTPUT TABLES FOR STAGE2

QUANTITY PRESENT IN BOTH THE PLANTS AFTER STAGE2:

Table 6.12 Quantity Present In Both the Plants after Stage 2

	Quantity excess	Quantity shortage
Quantity present in plant 1 for item 1 (p_1i_1) units	3000	
Quantity present in plant 1 for item 2 (p_1i_2) units	1068	
Quantity present in plant 2 for item 1 (p_2i_1) units		2000
Quantity present in plant 2 for item2 (p_2i_2) units		2000

Table 6.13 Quantity Transferred Internally Between Plants

Quantity transferred internally between plants (units)	i_1	i_2
Quantity transferred internally between plant 1 and plant2	2000	1068

FINAL QUANTITIES:

Table 6.14 Final Quantities

Plant	Item	Excess	Unfulfilled back orders
1	1	1000	0
1	2	0	0
2	1	0	0
2	2	0	932

We get the total cost in stage 2 as given below:

Total transportation internally between the plant: $tot_tr_p_p = 645800/-$

Total inventory cost internally between the plants; $tot_inv_p_p = 62500/-$

Total back – order cost internally between the plants: $tot_boc_p_p = 139800/-$

The second stage objective function is:

$$MIN TC_{p2} = tot_tr_p_p + tot_inv_p_p + tot_boc_p_p;$$

$$MIN TC_{p2} = 848100/-$$

$$\text{Total minimized value} = MIN TC_{p1} + MIN TC_{p2}$$

$$= 4506599/- + 848100 /-$$

$$= 5354699/-$$

DIRECT TRANSPORTATION:

Total 6.15 Direct Transportation

	Q(units)	C(rs)	Fr(trips)	Time (days)
Quantity delivered from supplier 1 to plant 1 for item 1 ($q_{s_1p_1i_1}$)	10000	4000	125	3
Quantity delivered from supplier 1 to plant 2 for item 1 ($q_{s_1p_2i_1}$)	10000	4000	125	3
Quantity delivered from supplier 2 to plant 1 for item 1 ($q_{s_2p_1i_1}$)	10000	6000	125	3
Quantity delivered from supplier 2 to plant 2 for item 1 ($q_{s_2p_2i_1}$)	10000	6000	125	3
Quantity delivered from supplier 1 to plant 1 for item 2 ($q_{s_1p_1i_2}$)	10034	6000	95	3
Quantity delivered from supplier 1 to plant 2 for item 2 ($q_{s_1p_2i_2}$)	10000	6000	94	3
Quantity delivered from supplier 2 to plant 1 for item 2 ($q_{s_2p_1i_2}$)	10000	4500	94	3
Quantity delivered from supplier 2 to plant 2 for item 2 ($q_{s_2p_2i_2}$)	10034	4500	95	3

Total transportation cost in direct delivery = 3984500/-

Pipeline inventory cost in direct delivery = 275255/-

Plant inventory cost in direct delivery = 67700.91/-

Total cost in case of only direct delivery =4327456/-

CONCLUSIONS

This chapter presents the experimental results obtained from the two stage distribution inventory modelling. As it is considered a two stage distribution inventory model with 2 suppliers, 2 plants and I cross –dock with limited capacities.

- In first stage suppliers, plants and Cross-Dock and interaction between them are considered. Based on the forecasted demand values, plants met the need from all suppliers. The capacities of all suppliers and plants were considered in optimizing the inventory cost and transportation cost. Minimized value obtained in stage 1 is given below:
 - Total transportation cost in case of direct delivery: $g_{di}(X) = 2838288/-$
 - Total transportation cost in case of milk-run delivery: $g_{mi}(X) = 183788.7/-$
 - Total transportation cost in case of cross-dock delivery: $g_{cd}(X) = 1484723/-$
 - Now, the first stage objective function is given below:

$$\text{MIN } TC_{p1}(X) = g_{di}(X) + g_{mi}(X) + g_{cd}(X)$$
 - Total minimized value is stage 1 = $\text{MIN } TC_{p1} = 4506599/-$
- In second stage the interaction among plants is considered to meet the gap between actual requirement and forecasted projections. The optimal values from first stage are taken as input to minimize the inventories and back order cost at each plant by considering the lateral transshipment among plants. Minimized value obtained in stage 2 is given below:
 - Total transportation internally between the plant: $\text{tot_tr_p_p} = 645800/-$
 - Total inventory cost internally between the plants; $\text{tot_inv_p_p} = 62500/-$
 - Total back – order cost internally between the plants: $\text{tot_boc_p_p} = 139800/-$
- The second stage objective function is:

$$\text{MIN } TC_{p2} = \text{tot_tr_p_p} + \text{tot_inv_p_p} + \text{tot_boc_p_p};$$
 Total minimized value in stage 2 - $\text{MIN } TC_{p2} = 848100/-$

Total minimized value of the problem is given below:

Total minimized value of the problem = $\text{MIN } TC_{p1} + \text{MIN } TC_{p2} = 5354699/-$

It is then compared by considering only one distribution strategy i.e., is direct delivery and total cost by using only direct delivery is given below:

Total cost in case of only direct delivery = 4327456/-

A new approach to transport as part of a larger system. The logistics chain, led to the need to consider it in different ways. From the perspective of a systematic approach transport is a complex adaptive economic system consisting of interconnected in a single process of transport logistics services to regional human and material flows. Technological processes in the logistics chain for delivery of goods to the consumer, have their own characteristics, depending on the characteristics of the transport of cargo, quantity of goods, means of transport and its carrying capacity, the nature of production facilities.

Based on the foregoing, it should be noted that the main function of transport logistics is the management of material flows from the manufacturer to the recipient on schedule. The main element of the logistics is transport. The subject of transport logistics is a set of tasks associated with the organization moving cargo for general use. Transport is an important link in the logistics system; he must possess a number of desirable properties and meet certain requirements in order to create innovative systems for the collection and distribution of goods. He must be able to carry small batches at short intervals in accordance with changing user inventory.

Within the boundaries of international logistics systems, different modes of transport are used based on optimizing the contact graphs, when in the presence of long-term sustainable transport all those involved in these kinds of transport are managed from a single center.

The criteria for the choice of vehicles take the safety of goods, the best use of their capacity and capacity and reduce the cost of transportation. Logistics meet the objectives of such progressive methods of transportation, as batch, container, combined.

Achievement of significant cost savings and improvements in profitability requires a typical retail company to make long-term decisions regarding the structure of its supply chain network and bringing its facilities, suppliers and customers closer together under the strategic supply chain planning.

SCOPE FOR FUTURE WORK:

The results presented by the present work can be used directly for effective and economical growth of the industry by using different distribution strategies the total cost in any industry can be minimized.

In the future work, any other distribution strategies can be consider, By using different distribution strategies the total cost in any industry can be minimized.

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