A Two Warehouse Probabilistic Order-Level Inventory Model for Deteriorating Items for Exponentially Increasing Demand

M.Madhavilata
Senior Assistant Professor
Dept of Humanities and Mathematics,
G.Narayanamma Institutute of Technology and Science (For women), Shaikpet, Hyderabad, India.

Abstract : This paper deals with an order-level probabilistic inventory model for deteriorating items with two warehouse is developed. Here the demand is assumed to be exponentially increasing. It is assumed that the production rate is infinite and shortages are allowed. The rate of deterioration of items in the two warehouses is different. Also a single warehouse version for deteriorating items is discussed. An attempt was made to find the optimal order level by minimizing the cost function.

IndexTerms - **Two levels of storage, exponentially increasing demand, Deterioration**

I. INTRODUCTION

This paper deals with an order-level probabilistic inventory model for deteriorating items with two warehouses assuming that the production rate is infinite and shortages are allowed. The effect of deterioration cannot be ignored in many inventory systems; deterioration is defined as decay, damage or spoilage such that the items cannot be used for the original purpose. Food items, photographic films, drugs and Pharmaceuticals, chemicals, electronic components and radioactive substances are some examples of items in which sufficient deterioration may occur during the normal storage period of the units and consequently and this loss must be taken into account while analyzing the inventory systems.

Inventory models for deteriorating items subject to exogenous demands have been studied by various authors in different periods. Most of the authors have assumed a constant rate of deterioration and also constant demand. These include Shah and Jaiswal [15], Aggarwal and Goel [3], Aggarwal [1], Aggarwal and Veena Jain [2].

Efforts in analyzing mathematical models of inventory in which a constant or variable proportion of the on hand inventory gets deteriorated per unit time have been studied. Ghare and Schrader [7] and Emmons [6] considered the case of a constant fraction of the on-hand inventory gets deteriorated per unit of time. This work has been extended by Covert and Philip [5], who developed EOQ model for units with a variable rate of deterioration of items in the inventory. In all these models shortages are not allowed and the demand is assumed to be deterministic. Shah [14] has generalized the work of Ghare and Schrader [7] and Philip [11] have developed an inventory model by considering general deteriorating function with shortages. Probabilistic model for deteriorating items with general deteriorating function are developed by Shah and Jaiswal [15].

Here it is assumed that the deterioration times of items in two warehouses are exponentially distributed with parameters α and β in OW and RW respectively.

II. ASSUMPTIONS AND NOTATIONS

The assumptions and notations are given here under.

a) The scheduling period T is constant and there is no supply lead time.
b) f(x) is the probabilistic density function of demand x which is a random variable during scheduling period $T(0 \leq x \leq \infty)$.
c) At the beginning of every period the initial stock is raised to level S.
d) Shortages are allowed and backlogged.
e) C_2 is unit shortage cost per unit time.
f) H is the unit holding cost per unit time in OW.
g) F is the unit holding cost per unit time in RW.
h) The age –specific failure rate is α for OW and β for RW.
i) W is the maximum capacity of OW and RW has unlimited capacity.
j) C is the known cost of deteriorated unit which includes any disposal cost or salvages value.
k) $Q_0(t)$ is the inventory level at time t in RW.

III. MODEL DEVELOPMENT AND ANALYSIS

The model is considered in which initially OW is filled to its capacity and RW is filled by (S-W) units so that the order level is S. Demands are met from RW first and when it is empty the inventory in OW is used for meeting the demand .It is further assumed that the items deteriorate during the scheduling period and are not replaced by good ones. The model is analyzed with the objective of determining optimal order level S^0 which minimizes the expected total average cost.

The stock depletion in the system is mainly due to demand and partly due to deterioration in both warehouses. As a result, RW gets emptied at time T_1, OW is emptied at T_2 and shortages build up until the end of the period T.

This process depends upon the level of demand occurring during the period. If, at the end of the scheduling period, positive stock remains then the old stock is carried to the next cycle and (S-W) units in RW. This will not affect the analysis because the residual life has the same exponential distribution as that of original lifetime, regardless of how long the item was stored.

The differential equation describing the state of inventory in RW is given by

$$\frac{d}{dt}Q_r(t) + \beta Q_r(t) = \frac{-xe^{at}}{T} \quad 0 \le t \le T_1 \tag{1}$$

Solving equation (6.1)

$$Q_r(t).e^{\beta t} = \frac{-x}{T}\frac{e^{(a+\beta)t}}{a+\beta} + c' \tag{2}$$

$$Q_r(0) = S - W \tag{3}$$

Substituting (3) in equation (2)

$$Q_r(t) = \frac{-x}{T}\frac{e^{(a+\beta)t}}{a+\beta} + (S - W) + \frac{x}{T(a+\beta)} \tag{4}$$

$$Q_r(T_1) = 0 \tag{5}$$

Substituting the condition (6.5) in equation (6.4) and solving for T_1

$$T_1 = \frac{1}{(a+\beta)}\log\left[\frac{T}{x}(S - W)(a + \beta) + 1\right] \tag{6}$$

During the interval $(0,T_1)$ all the W units in OW are not used but they are subject to deterioration at the rate of θ. The differential equation describing the state of inventory during the time interval $(0,T_1)$ in OW is

$$\frac{d}{dt}Q_0(t) + \alpha Q_0(t) = 0 \quad 0 \le t \le T_1 \tag{7}$$

$$Q_0(0) = W \tag{8}$$

$$Q_0(t) = We^{-\alpha t} \quad \text{for } 0 \le t \le T_1 \tag{9}$$

The stock level at T_1 is

$$W' = We^{-\alpha T_1} \tag{10}$$

During (T_1,T_2) the inventory position is described by the differential equation

$$\frac{d}{dt}Q_0(t) + \alpha Q_0(t) = \frac{-xe^{at}}{T} \quad T_1 \le t \le T_2 \tag{11}$$

The solution of equation (11) is given by

$$Q_0(t) = \frac{-x}{T}\frac{e^{\alpha t}}{\alpha+a} + (W + \frac{xe^{(a+\alpha)t}}{T(\alpha+a)})e^{-\alpha t} \tag{12}$$

$$Q_0(T_2) = 0 \tag{13}$$

Substituting (13) in (12)

$$T_2 = \frac{1}{(a+\alpha)}\log\left[\frac{WT}{x}(\alpha + a) + e^{(a+\alpha)T_2}\right] \tag{14}$$

The entire demand for the scheduling period is met only from RW whenever $Q_r(T)\ge 0$; which will happen if

$$x \le \frac{(S-W)T(\alpha+\beta)}{1-e^{(a+\beta)t}} = A$$

Further, if x is such that $Q_0(T) > 0$

$$\frac{-x}{T}\frac{e^{\alpha T}}{(\alpha+a)} + (W + \frac{x}{T}\frac{e^{(\alpha+a)T_1}}{(\alpha+a)})e^{-\alpha t} > 0$$

Let B be the root of the following equation in x

Then for all $x \le B$ shortage does not occur. Whenever $x > B$ shortage occurs and the average shortage during the period is

$$\frac{x}{2}\left(1 - \frac{T_2}{T}\right)^2 \tag{15}$$

The Quantity which has deteriorated in RW is

$$D_r(x)=\begin{cases} S-W-x-Q_r(T) & \text{if } x \leq A \\ S-W-\frac{xT_1}{T} & \text{if } x > A \end{cases} \quad (16)$$

The total inventory held in RW is

$$I_r(x) = \frac{D_r(x)}{\beta} \quad (17)$$

Because the quantity deteriorated in RW is β times the inventory held in RW. Also, the quantity deteriorated in OW is

$$D_0(x) = \begin{cases} W - We^{-\alpha t} \text{ if } x \leq A \\ W - \frac{x}{T}(T-T_1) - e^{\alpha T}\left[W + \frac{x}{T}\frac{e^{(a+\alpha)T_2}}{(\alpha+a)}\right] + \frac{x}{T}\frac{e^{\alpha t}}{(a+\alpha)} \text{ if } A \leq x \leq B \\ W - \frac{x}{T}(T_2 - T_1) \text{ if } x > B \end{cases} \quad (18)$$

$$I_0(x) = \frac{D_0(x)}{\alpha}$$

$$C(S) = \left[\frac{\left(C+\frac{F}{\beta}\right)}{T}\right]\int_0^\infty I_r(x)f(x)dx + \left[\frac{\left(C+\frac{H}{\alpha}\right)}{T}\right]\int_0^\infty I_0(x)f(x)dx + C_2\int_s^\infty \frac{x}{2}\left(1-\frac{T_2}{T}\right)^2 f(x)dx \quad (19)$$

Where $D_r(x)$ and $D_0(x)$ are taken from equations (16) and (18).

The optimal order level S^o for the system is obtained by minimizing the above cost function. For a given demand density the optimal S^o can be determined by numerical techniques.

If the optimal order level S^o is not greater than W for the model discussed in the previous section, then the optimal order level is determined using a single-warehouse system. Also the model in the above is a more of general situation. However, when items in stock are non –deteriorating in nature, the same model can be modified and used. This is done by obtaining the limiting form of the above model as $\alpha \to 0$ and $\beta \to 0$

To obtain optimum value of S^0 of S then the necessary condition is

$$\frac{dC(S)}{dS} = 0$$

The above differential equation is very difficult to solve explicitly and hence numerical methods are used in general.

IV. CONCLUSION

In this paper an optimum order level inventory for deteriorating items when the demand is exponentially increasing under probabilistic situation is attempted. However, the closed form of solution for the order level S^0 could not be obtained because of the complex nature of the cost function. To overcome this difficulty one can use non-parametric approach as suggested by Strijbosch and Heuts [16].

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