Model for Optimal Reserve Inventory Between Three Machines in Series with Reference to Repair Time Distribution

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Abstract: Consider an inventory system in which three machines are operating in series. The mathematical expressions for optimal reserve inventories between the first two machines as well as that between last two machines are derived based on the repair items of the first machine. The truncation point is also a random variable which follows any one of the probability distributions. Numerical values are obtained for optimal reserve inventory for varying holding costs, shortage costs and inter-arrival times between breakdowns and also graphically exhibited.

Key words: Repair time, truncation point, SCBZ property, Leibnitz rule and optimal reserve inventory.

1. Introduction

In inventory control theory, determining the optimal reserve inventory between two machines is an important model and these models have been studied by many researchers. The two machines problem may be conceptualized as follows.

A system in which there are two machines M_1 and M_2 in series and the output of machine M_1 is the input to the machine M_2. Whenever the machine M_1 goes in the breakdown state, it affects the productive level of Machine M_2 in order to avoid the idle time of machine M_2, a reserve inventory is to be maintained between the machines. Hence, there are two costs namely holding cost and idle time cost involved in this process and therefore to balance out the two costs an optimal reserve inventory needs to be derived.

This type of model is basically introduced by Hanssmann (1962) and these types of models have been discussed by many researchers under the assumption that the repair time of machine M_1 is a random variable.

Sachithanantham et al., (2006) have studied a model with the assumption that the probability function of the repair time undergoes a parametric change after the truncation point. The very basic concept of parametric change known as the Setting the Clock Back to Zero (SCBZ) property has been first discussed by RajaRao and Talwalker (1990). The model for optimal reserve inventory between two machines under the assumption that the repair time of machine M_1 satisfies the SCBZ property with the truncation point is being a random variable has been derived by Sachithanantham et al., (2007). The same type of model has been discussed by Srinivasan et al., (2007a) with the assumption that the inter arrival times between successive break downs of machine M_1 is a random variable, which satisfies SCBZ property. The same model with a modification of the probability function of truncation point has been discussed by Ramathilagam et al., (2014). The same model has been extended by Sachithanantham and Jagathesan (2017) and they authors have obtained the optimal reserve inventory under the assumption that the repair time of machine M_1 is a random variable which satisfies the SCBZ property. It is also assumed that the truncation of repair time distribution is itself a random variable which follows mixed exponential distribution. Rajagopal and Sathiyamoorthi (2003) have studied the model in which there are three machines in series and derived the optimal reserve inventories between "M_1 and M_2" and that between "M_2 and M_3" under the assumption that the repair time of machine M_1 is a random variables. Venkatesan et al., (2016) have discussed same model with the assumption that the consumption rates of the reserve inventories by the machines M_2 and M_3 are random variable. The improvement over this model is being studied in this paper, in which the repair time of M_1 is a random variable and its probability function undergoes a parametric change after the truncation point. It is also assumed that the truncation point itself is a random variable which follows uniform distribution.

2. Assumptions

There are three machines M_1,M_2 and M_3 in series.

(i) The output of M_1 is the input for M_2 and the output of M_2 is the input for M_3
(ii) M_1 is in down state where as M_2 and M_3 are in upstate.
(iii) The idle time cost of M_3 is very costly
(iv) The repair time of Machine M_1 is a random variable and its distribution satisfies the SCBZ property.

3. Notations

r_1, r_2 : Consumption rates of M_2 and M_3 respectively.
h_1,h_2 : Inventory holding costs per unit of S_1 and S_2 respectively
d_1,d_2 : Down time cost of M_2 and M_3 respectively
S_1,S_2 : Reserve inventory between M_1 and M_2, M_2 and M_3 respectively.
μ_1, μ_2 : Average inter arrival time between breakdowns of M_1 and M_2 respectively
t : Repair time of M_1

4. Model 1

In this model, M_1 is in the downstate and both M_2 and M_3 are in the upstate. The expected total cost of this model is given by

$$E(C) = h_1S_1 + h_2S_2 + \frac{d_1}{\mu_1}\int_{\frac{s_1}{r_1}}^{\infty}\left(t-\frac{s_1}{r_1}\right)g(t)dt + \frac{d_2}{\mu_2}\int_{\frac{s_1}{r_1}+\frac{s_2}{r_2}}^{\infty}\left(t-\frac{s_1}{r_1}-\frac{s_2}{r_2}\right)g(t)dt \quad (\mathbf{1})$$

where the first two terms are the inventory holding costs, the third term is the idle time cost of M_2 after the failure of M_1 and before it is repaired and the fourth term is the idle time cost of M_3 after the failure of M_1 and before it is repaired.

Equation (1) is partially differentiating with respect to S_1 and S_2 and equating them to zero, we obtain.

$$h_1 + \frac{d_1}{\mu_1 r_1}\left[1 - G\left(\frac{S_1}{r_1}\right)\right] - \frac{d_2}{\mu_1 r_1}\left[1 - G\left(\frac{S_1}{r_1}+\frac{S_2}{r_1}\right)\right] = 0 \quad (\mathbf{2})$$

and

$$h_2 - \frac{d_1}{\mu_1 r_1}\left[1 - G\left(\frac{S_1}{r_1}+\frac{S_2}{r_2}\right)\right] = 0 \quad (\mathbf{3})$$

Solving the equations (2) and (3), the optimal values of S_1 and S_2 can be obtained. This model has been discussed by Rajagopal and Sathiyamoorthi (2003).

5. Results

In this model, a novel concept, the so called SCBZ properly is applied to the probability function of the repair time of Machine M_1. In doing so, the probability function of the repair time of machine M_1 is assumed to follow exponential distribution and which takes a parametric change (SCBZ) after the truncation point. It is also assumed that the truncation point itself a random variable, which follows uniform distribution.

Thus the pdf of the repair time is

$$g(t,\theta) = \theta e^{-\theta t} \qquad , if\ t \le x_o$$
$$g(t,\theta) = \theta^* e^{-\theta^* t} e^{x_o(\theta^*-\theta)}, if\ t > x_o$$

It can be shown that $\int_o^\infty g(t)dt = 1$. where x_o is a random variable denoting that truncation point, which follows uniform distribution (a, b).
Hence the p.d.f of repair time can be stated as

$$f(t) = g(t,\theta)P(t \le x_o) + g(t,\theta^*)P(t > x_o) \quad \textbf{(4)}$$

$$P(t \le x_o) = P(x_o \le t) = \int_t^b \frac{1}{b-a} dx_o$$
$$= \frac{1}{b-a}\int_t^b dx_o$$
$$= \frac{1}{b-a}[b-t] \quad \textbf{(5)}$$

$$P(t > x_o) = P(x_o < t) = \int_a^t \frac{1}{b-a} dx_o \quad \textbf{(6)}$$

Substituting equations (5) and (6) in the equation (4), we get

$$f(t) = \theta^{-\theta t}\left(\frac{1}{b-a}\right)(b-t) + \theta^* e^{-\theta^* t} e^{x_o(\theta^*-\theta)}\left(\int_t^b \frac{1}{b-a} dx_o\right) \quad \textbf{(7)}$$

The expected cost is given by

$$E(C) = h_1 S_1 + h_2 S_2 + \frac{d_1}{\mu_1}\int_{\frac{S_1}{r_1}}^\infty \left(t - \frac{S_1}{r_1}\right) f(t)dt + \frac{d_2}{\mu_2}\int_{\frac{S_1}{r_1}+\frac{S_2}{r_2}}^\infty \left(t - \frac{S_1}{r_1} - \frac{S_2}{r_2}\right) f(t)dt \quad \textbf{(8)}$$

It can be shortly rewritten as

$$E(C) = h_1 S_1 + h_2 S_2 + T_1 T_2 \quad \textbf{(9)}$$

Here,

$$T_1 = \frac{d_1}{\mu_1}\int_{\frac{S_1}{r_1}}^\infty \left(t - \frac{S_1}{r_1}\right)\left\{\theta e^{-\theta t}\left(\frac{1}{b-a}\right)(b-t) + \theta^* e^{-\theta^* t} e^{x_o(\theta^*-\theta)}\left(\int_a^t \left(\frac{1}{b-a}\right) dx_o\right)\right\} dt$$

$$T_2 = \frac{d_2}{\mu_2}\int_{\frac{S_1}{r_1}+\frac{S_2}{r_2}}^\infty \left(t - \frac{S_1}{r_1} - \frac{S_2}{r_2}\right)\left\{\theta e^{-\theta t}\left(\frac{1}{b-a}\right)(b-t) + \theta^* e^{-\theta^* t} e^{x_o(\theta^*-\theta)}\left(\int_a^t \left(\frac{1}{b-a}\right) dx_o\right)\right\} dt$$

To obtain the optimal values of S_1 and S_2, differentiate the equation (9) with respect to S_1 and S_2.

$$\frac{\partial E(C)}{\partial S_1} = 0 \Rightarrow h_1 + \frac{\partial T_1}{\partial S_1} + \frac{\partial T_2}{\partial S_1} \quad \textbf{(10)}$$

$$T_1 = \frac{d_1}{\mu_1}\int_{\frac{S_1}{r_1}}^\infty \left(t - \frac{S_1}{r_1}\right)\left\{\theta e^{-\theta t}\left(\frac{1}{b-a}\right)(b-t) + \theta^* e^{-\theta^* t} e^{x_o(\theta^*-\theta)}\left(\int_a^t \left(\frac{1}{b-a}\right) dx_o\right)\right\} dt$$

Using Leibnitz rule of differentiation of an integral, we get

$$\frac{\partial T_1}{\partial S_1} = \frac{d_1}{\mu_1}\int_{\frac{S_1}{r_1}}^\infty \left(-\frac{1}{r_1}\right)\left\{\theta e^{-\theta t}\left(\frac{1}{b-a}\right)(b-t) + \theta^* e^{-\theta^* t} e^{x_o(\theta^*-\theta)}\left(\int_a^t \left(\frac{1}{b-a}\right) dx_o\right)\right\} dt$$

$$= -\frac{d_1}{\mu_1 r_1(b-a)}\int_{\frac{s_1}{r_1}}^{\infty}\left\{\theta e^{-\theta t}(b-t) + \theta^* e^{-\theta^* t}\left(\int_a^t e^{x_o(\theta^*-\theta)}dx_o\right)\right\}dt$$

$$= \frac{d_1}{\mu_1 r_1(b-a)}\left[\left\{\int_{\frac{s_1}{r_1}}^{\infty}\theta e^{-\theta t}(b-t)dt\right\} + \left\{\int_{\frac{s_1}{r_1}}^{\infty}\theta^* e^{-\theta^* t}e^{\theta^* t}\left(\int_a^t e^{x_o(\theta^*-\theta)}dx_o\right)dt\right\}\right]$$

$$= \frac{d_1}{\mu_1 r_1(b-a)}\left[\theta\left\{\left(\int_{\frac{s_1}{r_1}}^{\infty} b\, e^{-\theta t} - te^{-\theta t}\right)dt + I_1\right\}\right]$$

$$= -\frac{d_1}{\mu_1 r_1(b-a)}\left[\theta\left\{\int_{\frac{s_1}{r_1}}^{\infty} b\, e^{-\theta t}dt - \int_{\frac{s_1}{r_1}}^{\infty} t\, e^{-\theta t}dt\right\} + I_1\right]$$

$$= -\frac{d_1}{\mu_1 r_1(b-a)}\left[\left\{be^{-\frac{s_1}{r_1}\theta} - \frac{s_1}{r_1}e^{-\frac{s_1}{r_1}\theta} - \frac{1}{\theta}e^{-\frac{s_1}{r_1}\theta}\right\} + I_1\right]$$

$$\frac{\partial T_1}{\partial S_1} = -\frac{d_1}{\mu_1 r_1(b-a)}\left[\left\{b - \frac{s_1}{r_1} - \frac{1}{\theta}\right\}e^{-\frac{s_1}{r_1}\theta} + I_1\right] \quad \mathbf{(11)}$$

where

$$I_1 = \int_{\frac{s_1}{r_1}}^{\infty}\theta^* e^{-\theta^* t}\left(\int_a^t e^{x_o(\theta^*-\theta)}dx_o\right)dt$$

$$= \frac{\theta^*}{(\theta^*-\theta)}\int_{\frac{s_1}{r_1}}^{\infty}\theta^* e^{-\theta^* t}\left[e^{t(\theta^*-\theta)} - e^{a(\theta^*-\theta)}\right]\mathrm{dt}$$

$$= \frac{\theta^*}{(\theta^*-\theta)}\left[\int_{\frac{s_1}{r_1}}^{\infty} e^{-\theta^* t}e^{t(\theta^*-\theta)}dt - \int_{\frac{s_1}{r_1}}^{\infty} e^{-\theta^* t}e^{a(\theta^*-\theta)}dt\right]$$

$$= \frac{\theta^*}{(\theta^*-\theta)\theta}\left[\int_{\frac{s_1}{r_1}}^{\infty} e^{-\theta^* t}dt - e^{a(\theta^*-\theta)}\int_{\frac{s_1}{r_1}}^{\infty} e^{-\theta^* t}dt\right]$$

$$I_1 = \frac{\theta^*}{(\theta^*-\theta)}\left[e^{-\frac{s_1}{r_1}\theta} - e^{a(\theta^*-\theta)-\frac{s_1}{r_1}\theta^*}\right] \quad \mathbf{(12)}$$

Thus,

$$\frac{\partial T_1}{\partial S_1} = -\frac{d_1}{\mu_1 r_1(b-a)}\left[\left\{b - \frac{s_1}{r_1} - \frac{1}{\theta}\right\}e^{-\frac{s_1}{r_1}\theta} + \frac{\theta^*}{(\theta^*-\theta)\theta}\left\{e^{-\frac{s_1}{r_1}\theta} - e^{a(\theta^*-\theta)-\frac{s_1}{r_1}\theta^*}\right\}\right] \quad \mathbf{(13)}$$

Similarly, $\frac{\partial T_2}{\partial S_1}$ can be obtained as

$$\frac{\partial T_2}{\partial S_1} = \frac{d_2}{\mu_2 r_1(b-a)}\left[\left\{b - \left(\frac{s_1}{r_1} + \frac{s_2}{r_2}\right) - \frac{1}{\theta}\right\}e^{-\left(\frac{s_1}{r_1}+\frac{s_2}{r_2}\right)\theta} + \frac{\theta^*}{(\theta^*-\theta)\theta}\left\{e^{-\left(\frac{s_1}{r_1}+\frac{s_2}{r_2}\right)\theta} - e^{a(\theta^*-\theta)-\left(\frac{s_1}{r_1}+\frac{s_2}{r_2}\right)\theta^*}\right\}\right] \quad \mathbf{(14)}$$

Substituting (13) and (14) in (10), the resultant equation is,

$$\frac{\partial E(C)}{\partial S_1} = 0 \Rightarrow$$

$$h_1 - \frac{d_1}{\mu_1 r_1(b-a)}\left[\left\{b - \frac{s_1}{r_1} - \frac{1}{\theta}\right\}e^{-\frac{s_1}{r_1}\theta} + \frac{\theta^*}{(\theta^*-\theta)\theta}\left\{e^{-\frac{s_1}{r_1}\theta} - e^{a(\theta^*-\theta)-\frac{s_1}{r_1}\theta^*}\right\}\right]$$

$$-\frac{d_2}{\mu_2 r_1(b-a)}\left[\left\{b-\left(\frac{s_1}{r_1}+\frac{s_2}{r_2}\right)-\frac{1}{\theta}\right\}e^{-\left(\frac{s_1}{r_1}+\frac{s_2}{r_2}\right)\theta}+\frac{\theta^*}{(\theta^*-\theta)\theta}\left\{e^{-\left(\frac{s_1}{r_1}+\frac{s_2}{r_2}\right)\theta}-\ e^{a(\theta^*-\theta)-\left(\frac{s_1}{r_1}+\frac{s_2}{r_2}\right)\theta^*}\right\}\right]=0\ \mathbf{(15)}$$

In a similar way it can be shown that

$$\frac{\partial E(C)}{\partial S_2}=0\Rightarrow$$

$$h_2-\frac{d_2}{\mu_2 r_2(b-a)}\left[\left\{b-\left(\frac{s_1}{r_1}+\frac{s_2}{r_2}\right)-\frac{1}{\theta}\right\}e^{-\left(\frac{s_1}{r_1}+\frac{s_2}{r_2}\right)\theta}\right.$$
$$\left.+\frac{\theta^*}{(\theta^*-\theta)\theta}\left\{e^{-\left(\frac{s_1}{r_1}+\frac{s_2}{r_2}\right)\theta}-\ e^{a(\theta^*-\theta)}e^{-\left(\frac{s_1}{r_1}+\frac{s_2}{r_2}\right)\theta^*}\right\}\right]=0 \qquad \mathbf{(16)}$$

Solving the equations (15) and (16), we get

$$h_1r_1-h_2r_2\frac{d_1}{\mu_1(b-a)}\left[\left\{b-\frac{s_1}{r_1}-\frac{1}{\theta}\right\}e^{-\frac{s_1}{r_1}\theta}+\frac{\theta^*}{(\theta^*-\theta)\theta}\left\{e^{-\frac{s_1}{r_1}\theta}-\ e^{a(\theta^*-\theta)}e^{-\frac{s_1}{r_1}\theta^*}\right\}\right]=0 \quad \mathbf{(17)}$$

Using the equations (16) and (17) the optimal value of $\hat{S}_1$ and $\hat{S}_2$ can be obtained for fixed values of h_1, h_2, d_1, d_2 , μ_1, μ_2 ,r_1, r_2 , θ, θ^*, a and b.

6. Numerical illustrations

The variations in $\hat{S}_1$ and $\hat{S}_2$ in h_1, h_2 , d_1, d_2 , μ_1 and μ_2 have been studied by assuming other parameters has

$h_1 = 15$	$d_1 = 50$	$r_1 = 2$	$\mu_1 = 0.8$	$\theta = 1.2$	$a = 1$
$h_2 = 10$	$d_1 = 60$	$r_2 = 2$	$\mu_2 = 1.0$	$\theta^* = 1.0$	$b = 2$

The computed values and their curves are presented here.

Table 1 The optimal reserve inventory for varying h_1

h_1	15	16	17	18
$\hat{S}_1$	0.9274	0.7528	0.6036	0.4731

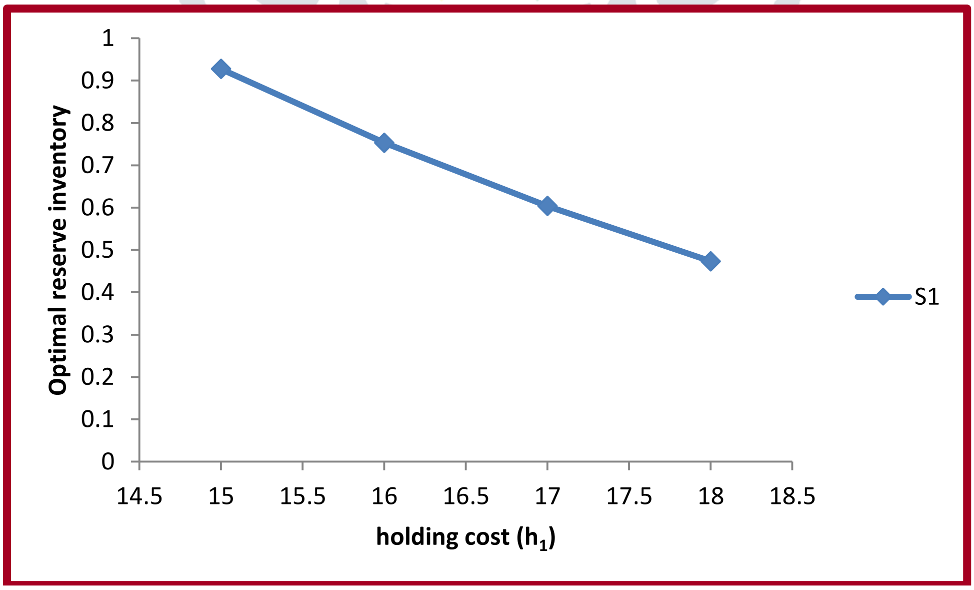

Fig 1 The optimal reserve inventory for varying h_1

Table 2 The optimal reserve inventory for varying h_2

h_2	9	10	11	12
$\hat{S}_1$	0.7528	0.9274	1.1383	1.4065
$\hat{S}_2$	0.5262	0.3856	0.2591	0.1441

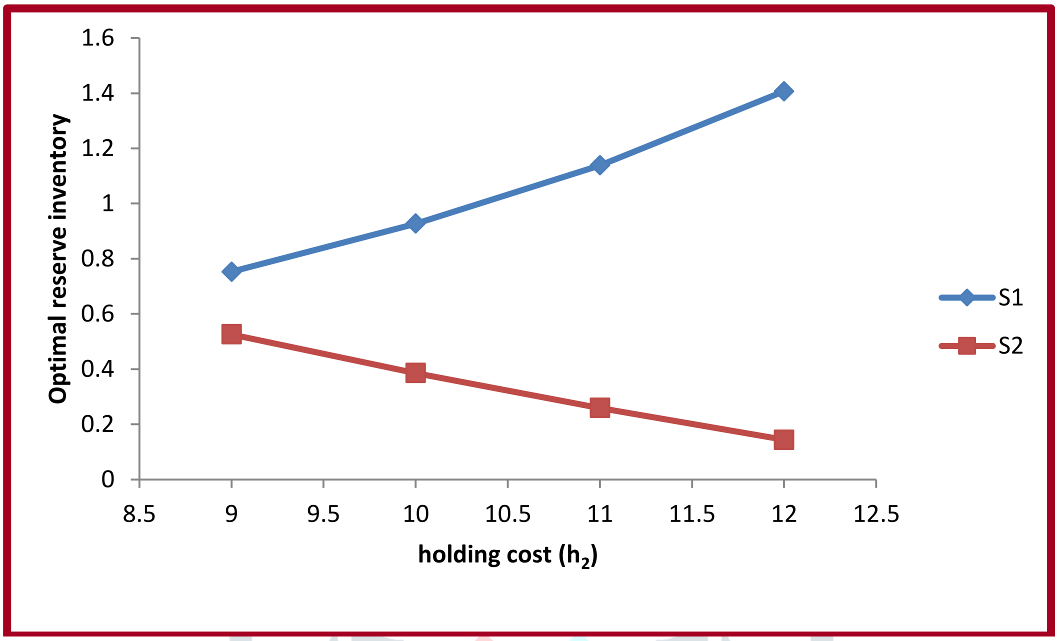

Fig. 2 The optimal reserve inventory for varying h_2

Table 3 The optimal reserve inventory for varying d_1

d_1	40	50	60	70
$\hat{S}_1$	0.7134	0.9274	1.1000	1.2445

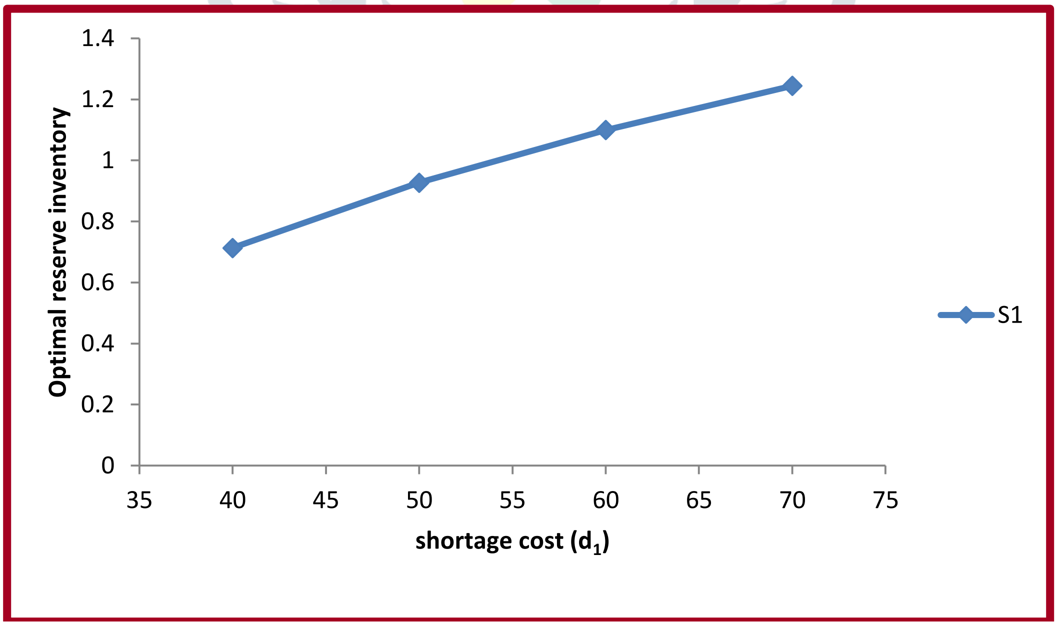

Fig. 3 The optimal reserve inventory for varying d_1

Table 4 The optimal reserve inventory for varying d_2

d_2	60	70	80	90
$\hat{S}_2$	0.3856	0.5916	0.7717	0.9320

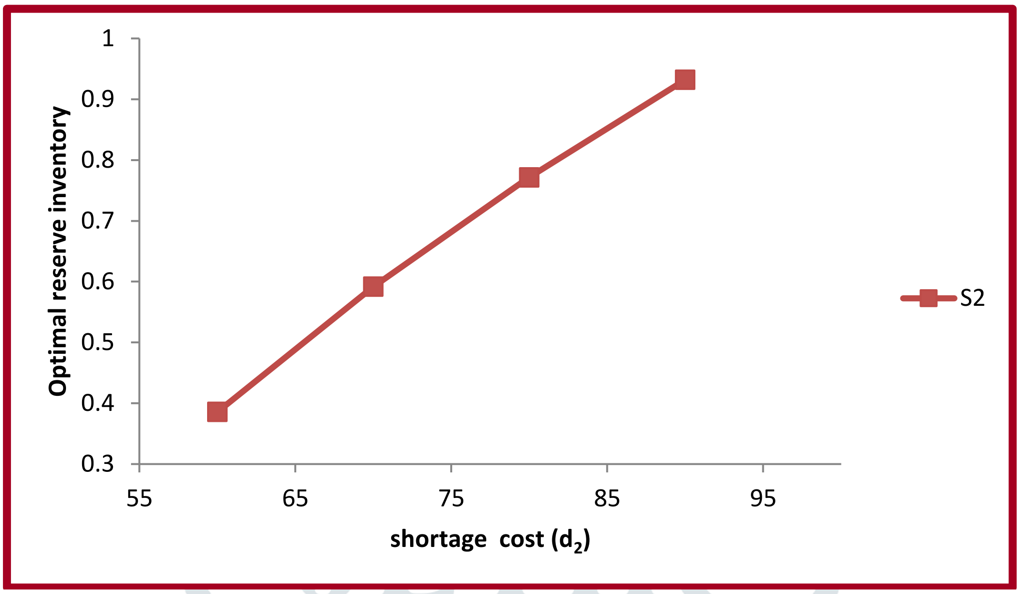

Fig. 4 The optimal reserve inventory for varying d_2

Table 5 The optimal reserve inventory for varying μ_1

μ_1	0.8	0.9	1.0	1.1
$\hat{S}_1$	0.9272	0.8148	0.7143	0.6211

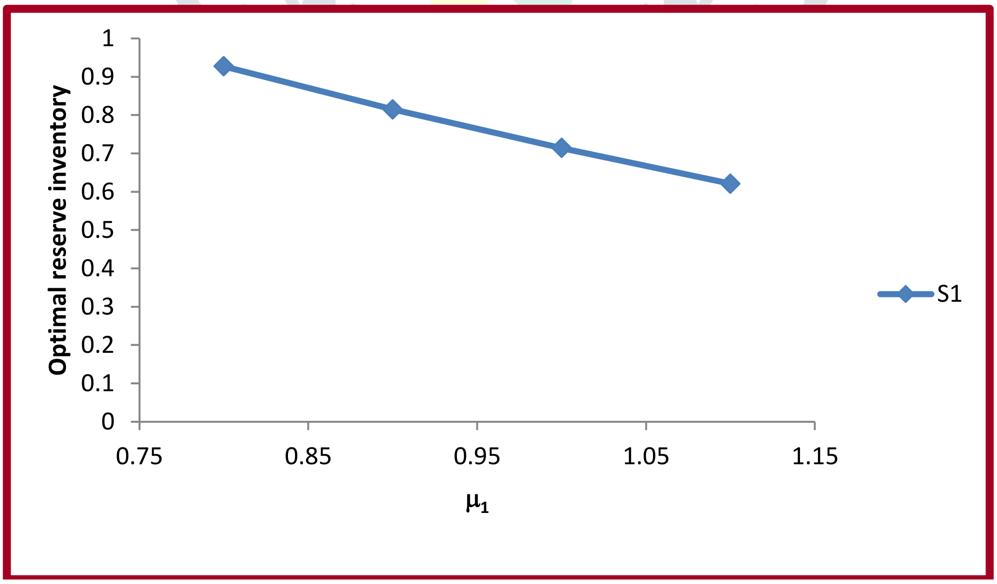

Fig. 5 The optimal reserve inventory for varying μ_1

Table 6 The optimal reserve inventory for varying μ_2

μ_2	1.0	1.1	1.2	1.3
$\hat{S}_2$	0.3856	0.2591	0.1441	0.0387

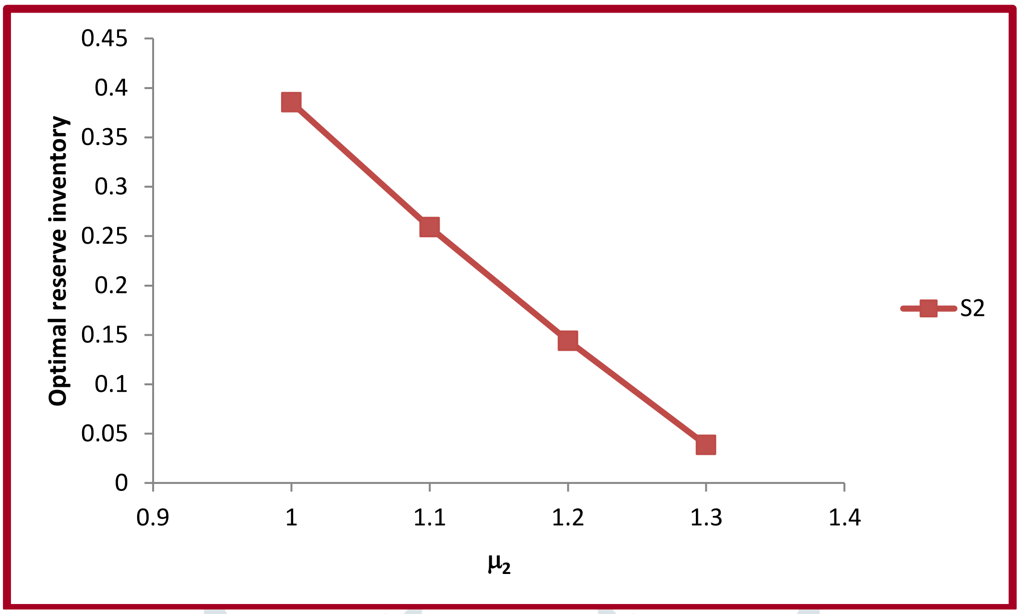

Fig. 6 The optimal reserve inventory for varying μ_2

7 Conclusion

It is seen from the above numerical illustration that the determination of optimal $\hat{S}_1$ and $\hat{S}_2$ depends upon various parameters μ_1, μ_2 and also on the values of the costs d_1,d_2, h_1 and h_2. Also two rates of consumption M_2 and M_3 namely r_1 and r_2 play a role in the optimal size of S_1 and S_2. The changes in the values of $\hat{S}_1$ and $\hat{S}_2$ due to changes of μ_1, μ_2 h_1, h_2, d_1 and d_2 have been studied. The optimal reserve inventory level increases in the first inventory when increasing both inventory costs, but the second inventory level does not altered due to the changes of its corresponding inventory cost. On the other hand, second inventory level decreases for increasing its inventory cost. Due to the changes of down time costs, the corresponding inventory level changes in the same direction, but others remain the same. Similarly, the levels of both inventories changes in opposite direction due to the changes of average inter-arrival times, but others do not changes.

References

1. Hanssmann, F 1962, ‘Operations Research in Production and Inventory Control’, John Wiley and Sons, Inc New York, pp. 45-46.
2. Raja Rao, B & Talwalkar, 1990, ‘Setting the Clock Back to Zero Property of a Family of Life Distributions’, Journal of Statistical Planning and Inference, vol.24, pp. 347-352.
3. Rajagopal, K & Sathiyamoorthi, R 2003, ‘An extension of the optimal Reserve Inventory level between machines model’, Second National Conference on Mathematical and Computational Models, Coimbatore PSG December 11-12, 2003. pp. 57-62.
4. Ramathilagam, S, Henry, L & Sachinthanantham, S 2014, ‘A model for optimal Reserve inventory between two machines in series with repair time undergoes a parametric change’, Ultra scientist vol.26, no.3, pp.227-237.
5. Sachithanantham, S, & Jagadeesan, R, 2017, ‘A model for Optimal Reserve inventory Between Two Machines with reference to the distribution of repair time’, International Journal of Mathematics Trends and Technology (IJMTT), vol.47 no.1, pp.74-80.
6. Sachithanantham, S, Ganesan, V & Sathiyamoorthi, R 2007, ‘A model for Optimal Reserve inventory Between Two Machines in Series;, Journal of Indian Acad. Math, vol.29, no.1, pp.59-70.

7. Sachithanantham, S, Ganesan, V, & Sathiyamoorthi, R, 2006, 'Optimal Reserve between Two Machines with Repair Time having SCRZ Property', Bulletin of Pure and Applied Science, vol. 25E, no.2, pp. 487-497.
8. Srinivasan, S, Sulaiman, A & Sathiyamoorthi, R 2007a, 'On the Determination of Optimal Inventory with Demand distribution as order statistics', Bulletin of Pure and Applied Sciences, pp. 25-32.
9. Venkatesan, T, Muthu, C & Sathiyamoorthi, R 2016, 'determination of optimal reserve between three machines in series', International Journal of Advanced Research in Mathematics and Applications, vol.1, no.1,pp.74-82.