A STUDY ON SINGLE SERVER RETRIAL QUEUE WITH SETUP TIME IN VACATION POLICY

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Abstract

This research article deals with single server retrial queue with setup time at the vacation policy. Arriving customers join the system when the server is idle with probability p or the server shows busy when the customer decide to join retrial group with probability 1-p. Head of the customer in the orbit get service and access to the system. If the system is free then the retrial customer gets service immediately, otherwise the customers again enter the orbit or leave the system because of impatience. If the system is totally free when no customer in the arrival and in the orbit then server fix the time to go to the vacation. In steady state, the probabilities of generating function and queue length have been obtained. Expected number of customer in the retrial group and expected waiting time of the customer in the orbit are also obtained. We have obtained expression for the performance measures of the system.

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1 Introduction

Queue is formed if the service required by customer is not immediately available (current demand for a particular service > the capacity to provide the service). More over there is less demand, in which there is too much idle facility time or too many facilities. There are many ways to increase the quality of service, some of which include increasing the resource (namely the service provides), increasing the service rates and controlling the arrivals. Queues with repeated attempts have been widely used to model many problems in telephone switching systems, computer and communication systems, local area network and online shopping to purchase the customer due to the time period. For detailed survey Yang and Templeton (1987), Falin (1992) and Choi and Chang(1999) investigated an M/G/1 retrial queue with two types customers in which the service time distribution for both types of customers at the same. Khalil et al.(1992) investigated the above model at Markovian level; in detail.

Falin et al.(1993) investigated a similar model in which they assumed different service time distribution for the two types of customers. Kalyanaraman and srinivasan (2004) studied an M/G/1 retrial queue with geometric loss and with type I batches arrivals and type II single arrivals. In 2011 the authors have analyzed feedback retrial queueing systems with two types of arrivals and type I arrival being in batches of fixed size. In 2009 Pazhanibalamurugan and Kalyanaraman authors have analyzed a vacation queue with additional optional service in batches. In 2014, Rajadurai et al. analyzed on M/G/1 feedback retrial queue with subject to server breakdown and repair. In 1990 H Takagi studied time dependent analysis of M/G/1 vacation models with exhaustive service queueing system. In 2000 Choudhury, analysized on An M[x]/G/1 queueing system with a setup period and vacation period. In 2011 Song fang ja et al. studied discrete queue with vacation and setup time. In 2014 Shymala and Ayyappan, studied on an analysis of retrial queueing system with second optional service random breakdown, setup time and Bernoulli vacation. Some of the authors put their contribution of a setup time which plays a significant role in the study of queueing systems and which is defined as at every ends of the busy period the server enters into a random time process before actually providing service to a new customers or batch of customers joins the system in the renewed busy period. In many queueing systems, before starting a service, the server may have to do some preparatory work or some alignment (setup) in the case of certain machines. Baker (1973) first proposed the N policy M/M/1 queueing system with exponential startup time. Borthakur et al. (1987) extended Bakers results to the general startup time. Queueing system considering this aspect is called queueing system with setup time. Levy and Kleinrock (1986) studied the delay analysis of queue with starter and vacation by using decomposition. Choudhury (2000) discussed the setup period and vacation period of M[X]/G/1 queue. Ke (2007b) considered a batch arrival queue under vacation policies with server breakdown and startup/closedown times. In many queueing systems, before starting a service, the server may have to do some preparatory work or some alignment (setup) in the case of certain machines. In this paper generalized the work of Shymala and Ayyappan, is studied for analyzing of retrial queueing system with second optional service random breakdown, setup time and Bernoulli vacation. To the author best of knowledge, there is no work published in the queueing literature with combination of an analysis on single server retrial Queue with setup time with vacations. Setup time plays a significant role in the study of queueing systems and which is defined as at every end of the busy period, the server enters into a random setup time process before actually providing service to a new customer or a batch of customers joins the system in the renewed busy period. Levy and Kleinrock (1986) studied queue with starter and vacations. Choudhury (2000) studied a batch arrival queueing system having a setup period and a vacation. Yang et al. (2008) studied optimal control policy for a unreliable system with second optional service and startup period. Songfang Jia et al. (2011) studied discrete queue with multiple vacation and set up times. Zhong Yu et al. (2010)analyzed the steady state solution and performed sensitivity analysis of a batch arrival queue under a threshold policy with single vacation and setup times.

An M/M/1 queue with customers balking was proposed by Haight[9],Sumeet Kumar Sharma [10] studied the M/M/1/N queuing system with retention of reneged customers, Kumar and Sharma [11] studied a single server queueing system with retention of reneged customers and balking. Kumar and Sharma [14] consider a single server, finite capacity Markovian feedback queue with balking, balking and retention of reneged customers in which the inter-arrival and service times follow exponential distribution. Krishnakumar[10] studied the transient solution M/M/1 queue with catastrophes, failures and repairs. Thangaraj[18]studied the transient analysis of M/M/1 feedback queue with catastrophes using continued fraction methods. Setup time: The unreliable server may complete the service and go to the vacation. When the server go to the vacation, it takes some time known as setup time before going to vacation. The setup time and vacation policy are i.i.d and general distributed.

2 MATHEMATICAL MODEL AND MAIN RESULTS

In this we consider single server retrial queue, setup time with vacation policy.

The assumptions of the system model are as follows:

- The customers arrive according to Poisson process with rate λ.
- Exponential service to customer on a FCFS served basis with service rate μ.

• Arriving customers join the system when the server is idle with probability p or the server shows busy when the customer decide to join retrial group with probability 1-p.

• N is number of customers in the system (number of customer in the queue+ Number of customer being served).

• Let $\{X(t), t \geq 0\}$ be the number of customer in the system at time.

• $P_n(t) = P(X(t) = n), n = 1,2,3,.......$ denote the probability that there are n customers in the system at time.

• When the bus period is ended the server to serves the head of the customer in the orbit before going to vacation. The service time of the orbit follows exponential distribution with density function as follows $O(t) = \omega e^{-\omega t}, t \geq 0, \omega \geq 0$ Where ω is the service rate of orbit customers.

• Server go to vacation when the time for duration of vacation has an exponential distribution with density function as follows $S(t) = \varsigma e^{-\varsigma t}, t \geq 0, \varsigma \geq 0$ Where ς is the vacation rate of the server.

• Server go to vacation when the time for duration of vacation has an exponential distribution with density function as follows $V(t) = \eta e^{-\eta t}, t \geq 0, \eta \geq 0$ Where η is the vacation rate of the server.

The stochastic process related to the model is $X(t) = \{A(t), O(t), S(t), t \geq 0\}$

where A(t) =Number of arrival customer in the system at time t.

O(t)= Number of retrial customer in the orbit at time t.

V (t)= Server setup time before goes to vacation at time t.

$$S(t) = \begin{cases} 0, \text{server is idle} \\ 1, \text{if the server is busy to arrival customer} \\ 2, \text{if the server is busy to the retrial customer from the orbit} \\ 3, \text{if the server set time before go for vacation} \end{cases}$$

Let N(t) be the system size of the server at time t.

We introduced the supplementary variable

$$w(t) = \begin{cases} A(t) \text{ if } C(t) = 0,1 \\ O(t) \text{ if } C(t) = 2 \\ V(t) \text{ if } C(t) = 3 \end{cases}$$

Where A(t)elapsed service time of the bulk customer arrives at time t.

O(t) elapsed service time of the regular customer in service at time t.

V (t)elapsed vacation time of the server when he is setup time before goes to vacation.

The process $\{S(t), N(t), \omega(t), t \geq 0$ is a continuous time Markov process.

We defined the probabilities,

1. $P_{0,0}(t) = P\{S(t) = 0, N(t) = 0\}$

2. $P_{1,t}(n)$ is the probability that there are n(n=0,1,2,) arrival customer

3. $P_{2,t}(n)$ is the probability that there are n(n=0,1,2,) orbit customers.

4. $P_{3,t}(n)$ is the probability that there are n(n=0,1,2,) server setup time before going to vacation.

The process $\{N(t), X(t), \omega(t); t \geq 0\}$ is a continuous time Markov process.

We defined the probabilities,

$P_{1,n}(t)dx = P\{S(t) = 1, N(t) = n, x \leq \tau_1(t) < x + dx\}, x \geq 0, n \geq 1$

$P_{2,n}(t)dx = P\{S(t) = 2, N(t) = n, x \leq \tau_2(t) < x + dx\}, x \geq 0, n \geq 0$

$P_{3,t}(n)dx = P\{S(t) = 3, N(t) = n, x \leq \tau_3(t) < x + dx\}, x \geq 0, n \geq 0$

3. Governing Equations

The queueing model is governed by the following set of differential-difference equations:

$$\frac{d}{dt} S_n(t) = -(\lambda + v) S_n(t) + \lambda \sum_{i=1}^{n} c_i S_{n-i}(t) + \lambda c_n O(t), n \geq 1 \quad \ldots\ldots(1)$$

$$\frac{d}{dt} P_n^{(1)}(x,t) + \frac{\partial}{\partial x} P_n^{(1)}(x,t) + (\lambda + \mu(x) + \alpha) P_n^{(1)}(x,t) = \lambda \sum_{i=1}^{n-1} c_i P_{n-i}(x,t), n \geq 1 \quad \ldots\ldots(2)$$

$$\frac{d}{dt} V_n(x,t) + \frac{\partial}{\partial x} V_n(x,t) + (\lambda + v(x)) V_n(x,t) = \lambda \sum_{i=1}^{n-1} c_i V_{n-i}(x,t), n \geq 1 \quad \ldots\ldots(3)$$

$$\frac{d}{dt} V_0(x,t) + \frac{\partial}{\partial x} V_0(x,t) + (\lambda + v(x)) V_0(x,t) = 0 \quad \ldots\ldots(4)$$

Assume the server become idle when there is no customer in the system,

$$P_n(0) = 0, n = 0,1,2,3\ldots\ldots$$
$$V(0) = V_n(0) = 0,1,2,\ldots\ldots \quad \ldots\ldots(5)$$
$$O(0) = 1$$

Probability generating function follows,

$$P_q^{(1)}(x,z,t)=\sum_{n=0}^{\infty} z^n P_n^{(1)}(x,t)$$

$$P_q^{(1)}(z,t)=\sum_{n=0}^{\infty} z^n P_n^{(1)}(t)$$

$$V_q^{(1)}(x,z,t)=\sum_{n=0}^{\infty} z^n V_n^{(1)}(x,t)$$

$$V_q^{(1)}(z,t)=\sum_{n=0}^{\infty} z^n V_n^{(1)}(t) \qquad \ldots\ldots(6)$$

$$O_q^{(1)}(x,z,t)=\sum_{n=0}^{\infty} z^n O_n^{(1)}(x,t)$$

$$S_q^{(1)}(x,z,t)=\sum_{n=0}^{\infty} z^n S_n(t)$$

Which we converge inside the circle given by |z|≤ 1 and we define Laplace transform of a function f(t) as $\overline{f(s)}=\int_0^{\infty} f(t)e^{-st}dt$ taking Laplace transform on the above,

$$(S+\lambda+v)\overline{S_n(t)}=\lambda\sum_{i=1}^{n} c_i\overline{S_{n-i}(s)}+\lambda c_n\overline{O(s)} \qquad \ldots\ldots(7)$$

$$\frac{\partial}{\partial x}P_n^{(1)}(x,s)+(S+\lambda+\mu(x))\overline{P_n^{(1)}(x,s)}=\lambda\sum_{i=1}^{n} c_i\overline{P_{n-i}^{(i)}(x,s)}, n\geq 1 \qquad \ldots\ldots(8)$$

$$\frac{\partial}{\partial x}\overline{V_n(x,s)}+(S+\lambda+v(x))\overline{V_n(x,s)}=\lambda\sum_{i=1}^{n} c_i\overline{V_{n-i}(x,s)}, n\geq 1 \qquad \ldots\ldots(9)$$

For the boundary conditions are

$$\overline{P_n^{(1)}(0,s)}=\int_0^{\infty}\overline{P_n}(x,s)v(x)dx+v\overline{S_n(s)}, n\geq 1$$
$$\overline{V_n(0,s)}=\int_0^{\infty}\overline{V_n}(x,s)\mu(x)dx, n\geq 0 \qquad \ldots\ldots(10)$$

Now multiply (1) by z^n summing over n from 1 to ∞ and using the definition of PGF, we obtain $(S+\lambda-\lambda c(z)+\gamma)\overline{S_q(z,s)}=\lambda c(z)\overline{O(s)}, n\geq 1$ ……(11)

Continuous like this

$$\frac{\partial}{\partial x}P_q^{(1)}(x,z,s)+(S+\lambda-\lambda c(z)+\mu(x))\overline{P_q^{(1)}(x,z,s)}=0 \qquad \ldots\ldots(12)$$

$$\frac{\partial}{\partial x}\overline{V_n(x,z,s)}+(S+\lambda-\lambda c(z)+v(x))\overline{V_q(x,z,s)}=0 \qquad \ldots\ldots(13)$$

Multiply boths side (10) in z^{n+1} summing over 1 to ∞ and using the definition of PGF we get

$$z\overline{P_q^{(1)}}(0,z,s) = (1-p)\int_0^\infty \overline{P_q^{(1)}}(x,z,s)\mu(x)dx + z\int_0^\infty \overline{V_n(x,z,s)}v(x)dx + z\gamma \overline{S_q}(x,z,s) + z[1-s\overline{O}(s)] + \lambda z(c(z)-1)\overline{O(s)}$$

……………..(14)

Performing the equation (10) we get

$$\overline{P_q^{(n)}}(0,z,s) = \int_0^\infty \overline{P_n^{(1)}}(x,z,s)v(x)dx \qquad \text{……………………………………………………….(15)}$$

$$\overline{V_q}(0,z,s) = \int_0^\infty \overline{V_n}(x,z,s)dx \qquad \text{…………………………………………………………..(16)}$$

Integrate (12) from 0 to ∞, x yields,

$$\overline{P_q^{(1)}}(x,z,s) = \overline{P_q^{(1)}}(0,z,s)e^{-(s+\lambda-\lambda c(z)+\alpha)x-\int_0 \mu(t)dt} \qquad \text{……………………………………….(17)}$$

Where $\overline{P_q^{(1)}}(0,z,s)$ is given integrate (16) by parts with respect to x,

$$\overline{P_q^{(1)}}(z,s) = \overline{P_q^{(1)}}(0,z,s)\left[\frac{1-\overline{B_1}(s+\lambda-\lambda c(z))}{s+\lambda-\lambda c(z)}\right]$$

where $B_1(s+\lambda-\lambda c(z)) = \int_0 e^{-(s+\lambda-\lambda c(z))x}dB_1(x)$ Laplace-stieltjes transform of the same time $B_1(x)$. Now multiply the both side of (17) by μ(x) and integrate x we get

$$\int_0^\infty \overline{P_q^{(1)}}(x,z,s)\mu(x)dx = \overline{P_q^{(1)}}(0,z,s)\overline{B_1}(s+\lambda-\lambda c(z))$$

Similarly $\overline{V_q}(x,z,s) = \overline{V_q}(0,z,s)e^{-(s+\lambda-\lambda c(z)+\alpha)x-\int_0^x v(t)dt}$ Where $\overline{P_q^{(1)}}(0,z,s)$ and $\overline{V_q}(0,z,s)$ are given by(10). From(11) we get

$$\overline{S_q}(z,s) = \left[\frac{\lambda c(z)\overline{O}(s)}{s+\lambda-\lambda c(z)}\right]$$

$$\overline{P_q^{(1)}}(z,s) = (f_2(z,s)f_3(z,s))\left\{z(1-S\overline{O}(s)+\lambda z\overline{O}(s)) * \frac{vc(z)}{s+\lambda-\lambda c(z)} - 1\right\} * \left[1-\overline{B_1}(f_1(z,x))\right]$$

$$\overline{V_q}(z,s) = (f_1(z,s)f_2(z,s)f_3(z,s))\overline{B_1}(f_1(z,x)\overline{B_2}(f_2(z,x) * \left\{\begin{matrix} z(1-S\overline{O}(s)+ \\ \lambda z\overline{O}(s)) * \dfrac{vc(z)}{s+\lambda-\lambda c(z)} - 1\end{matrix}\right\} * \left[\frac{1-\overline{V}(s+\lambda-\lambda c(z))}{s+\lambda-\lambda c(z)}\right]4$$

The steady state analysis

We assume the probability distribution from our queueing model to determine the steady state probability where ever it dependent time, →0sf(s) = ltt→0f(t)

$$\underset{s\to\infty}{lt}\, s\overline{f}(s) = \underset{t\to 0}{lt}\, f(t)$$

$$S_q(z) = \frac{\lambda Oc(z)}{\lambda - \lambda c(z) + v}$$

where $\overline{P_q}(z) = \dfrac{f_2(z) f_3(z)(\lambda + v)[\lambda c(z) - 1][1 - \overline{B_1} f_1(z)]O}{D_r f_4(z)}$

$$V_q(z) = \frac{P[f_1(z) f_2(z) f_3(z)(\lambda + v)]\overline{B_1} f_1(z) f_2(z)[\overline{v}(\lambda - \lambda c(z) - 1]O}{D_r f_4(z)}$$

Where

$$D_r f_4(z) = f_1(z) f_2(z) f_3(z)\{z - (1-P) - P\overline{v}(\lambda - \lambda c(z))\} * \overline{B}1(f_1(z))\overline{B}2(f_2(z) - \beta\eta(1 - \overline{B}1(f_1(z))\overline{B}2(f_1(z))))$$

$$f_1(z) = \lambda - \lambda c(z) + \beta$$

$$f_2(z) = \lambda - \lambda c(z) + \eta$$

$$f_1(z) = \lambda - \lambda c(z) + v$$

$$P(z) = S_q(z) + P_q^{(1)}(z) + V_q(z)$$

$$P(1) + O = 1$$

$$\overline{B_i}(0) = 1, i = 1,2 \;\; ,\overline{V}(0) = 1, S_q(1) = \frac{\lambda Q}{v}$$

In order to obtain, $P_q^1(z) = \dfrac{(\lambda + v)\lambda\beta\eta Q(1 - \overline{B_1}(\alpha))}{dr}$

$$V_q(1) = \frac{(\lambda + v)\lambda\beta\eta Q(\overline{B_1}(\alpha)\overline{B_2}(\alpha))}{dr}$$

$$dr = v\{\alpha\beta\eta\overline{B_1}(\alpha)\overline{B_2}(\alpha)\}[\alpha\beta + \beta\eta + \alpha\eta][1 - \overline{B_1}(\alpha)\overline{B_2}(\alpha)] - p\alpha\beta\eta\overline{B_1}(\alpha)\overline{B_2}(\alpha)$$

5 The measures of system performance

Let

1. $L_q = \dfrac{d}{dz} P(z)_{z=1}$

2. $W_q = \dfrac{L_q}{\lambda}$ by Little formula

3. $The\, blocking\, the\, \Pr obability = 1 - \{P_{00} + P_0(1)\}$

4. Expected waiting time in the system $W_s = \dfrac{L_s}{\lambda}$

6 Concluding Remarks

We have proposed a single server with setup time in vacation. At the end of each busy period, the server takes a setup time before giving proper service to the customer. In steady state, the probabilities of generating function and queue length have been obtained. Expected number of customer in the retrial group and expected waiting time of the customer in the orbit are also obtained. We have obtained expression for the performance measures of the system. The probability generating function of transient solutions is obtained explicitly, and along with this, the steady state has also been analyzed. Further performance measures like average number of customers in the queue and the average waiting time of a customer in the queue are obtained.

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