Conceptualizing the Dimensions of Mathematics Teacher's Knowledge for Teaching Mathematics at Secondary Level: A Critical Analysis

Rajendra Kunwar
Reader, Mathematics Education
Tribhuvan University
Faculty of Education
Mahendra Ratna Multiple Campus, Ilam, Nepal.

ABSTRACT

This study is conducted for conceptualizing the model of mathematics teachers' knowledge for secondary level. In this study, a brief introduction of mathematics, mathematics teaching, role of mathematics teacher and dimensions of mathematics teachers' knowledge are organized. The aim of the study is to establish the conceptual model of teacher's knowledge dimensions for secondary level mathematics teacher. The critical analysis method of the previous literature was adopted to conceptualize the model. Finally, the model was conceptualized in the form of integration of three knowledge dimensions: content, pedagogy and technology for mathematics teacher at secondary level.

Key words: Mathematics teachers' knowledge, content, pedagogy, technology

INTRODUCTION

Mathematics is the formal organized field of study. It is an interdisciplinary language and tool that it is considered as one of the fundamentals in the formal educational system. Mathematics is also an aid for representing and attempting to resolution the problem situations in all other disciplines. Mathematics provides an effective way of building mental discipline and encourages logical reasoning and mental rigor. In the same way, mathematical knowledge plays a crucial role in understanding the contents of other school subjects such as science, social studies, and even music and art. It is applied in various fields and disciplines, that is, mathematical concepts and procedures are used to solve problems in science, engineering and economics. Mathematical activity is an essential part of almost every profession, such as business, science, weather prediction, medicine, engineering, architecture, and economics (Charles & Lester, 1984). The study of mathematics provides students with certain basic life skills and processes that will prepare them to be creative members of society. The content of mathematics curriculum must focus on ways to equip students with an ability to learn things that no one yet knows; such focus implies a different role for mathematics teachers.

The main objective of the study is to establish the conceptual model of teacher's knowledge dimensions for secondary level mathematics teacher through the critical analysis method of the previous literature. The study presents the brief introduction of mathematics, mathematics teaching, role of mathematics teacher and the impact of ICT in education.

Mathematics and Mathematics Teaching

Mathematics is a universal and utilitarian subject that is needed for every people in their life. Mathematics is considered as a most, complex and difficult subject and the tendency for most students is to consider the subject as one that is boring. This creates a great challenge for mathematics teachers and educators, especially in school level, wherein a good study habit and a firm grasp of basic concepts should be developed. Hence, the core concept of teaching is being a plausible conception of the professional practice of teachers and the work of

teaching can further be defined through the mathematical tasks that teachers do in order to facilitate students' learning of mathematics (Hoover, Mosvold & Fauskanger, 2014).

In the traditional way of mathematics teaching, mathematics teachers have concentrated their efforts on helping their students acquire computational knowledge and skills, instead of emphasizing concept formation. But in the recent time, the effective teacher should focus on the multiple dimensions of knowledge so that; learning mathematics could be often more a matter of understanding than of memorizing. In such condition, mathematics education can be a platform to learn and teach mathematics in the better way. Mathematics education helps the learner to cultivate a mathematical way of thinking, understand the process and applying mathematics to real life problems, motivates the learning of structures through applications and concrete situation and develops a favorable attitude towards a study of mathematics. Therefore, the teaching of basic mathematical subject matter is essential for each school level students.

Role of Mathematics Teacher

Teachers play vital roles in the lives of the students in their classrooms. Teachers are best known for the role of educating the students that are placed in their care. Beyond that, teachers serve many other roles in the classroom. Teachers set the nature of their classrooms, build a warm environment, mentor and nurture students, become role models, and listen and look for signs of trouble. Mentoring is a natural role taken by the teachers, whether it is intentional or not. This again can have positive or negative effects on children. Mentoring is a way of encouraging students by the teacher to attempt to be the best they can. This also includes encouraging students to enjoy learning. The mentoring also consists of listening to the students. By taking time to listen to what students say, teachers impart to students a sense of ownership in the classroom. This helps build their confidence and helps them want to be successful. Mathematics teacher knowledge has become a fertile research field in mathematics education (Ponte & Chapman, 2016). Scholars have considered mathematics teacher knowledge from multiple perspectives, using various constructs and frameworks to describe and explain what makes mathematics teacher knowledge specialized.

The teacher inspires and teaches the student. Effective teachers convey knowledge, embody values, excite curiosity and never lose sight of the fact that they are not teaching reading or math, they are teaching humans. Teachers are, without any doubt, the single most important role for student successes in mathematics. As important as curriculum, assessment tools, well equipped and supported classroom may be, without an effective teacher to control and execute of all the variables in the classroom, these components amount to very little. So, the teacher, full of all knowledge dimensions is must to execute the knowledge transformation. The role of the teacher's is to help the learner achieve different knowledge, skills, and better understanding of the subject matter. In so doing, the teachers themselves need to have strong knowledge of subject matter and way of skill delivering to the learner for meaningful knowledge transfer. Another side, effective mathematics teachers should develop, applies, and analyzes that knowledge in the context of their own classroom situation so that knowledge and practice are integrated.

Dimensions of Mathematics Teacher's Knowledge

The dimensions of mathematics teachers' knowledge for secondary level consist of overall areas of information that is needed for the learner. Knowledge of mathematics includes knowledge of mathematical facts, concepts, procedures, and the relationships among them; knowledge of the ways that mathematical ideas can be represented; and knowledge of mathematics as a discipline, in particular, how mathematical knowledge is produced, the nature of discourse in mathematics, and the norms and standards of evidence that guide argument and proof (National Research Council, 2001). In some cases, personal characteristics such as gender, age, environment and education level were mostly used as individual difference; in educational studies, personal cognitive differences such as learning style, teaching style or perceived teaching style were focused (Lu & Lin,

2012; Lin, Lu & Fan, 2014). Several scholars also discovered that different cognitive styles would affect the learning behaviors of the learner. Hence, the teachers' have knowledge, beliefs, and understandings for both in planning to teach and in the act of teaching itself.

The mathematics teacher should have good knowledge about the curriculum, subject matter of the concern subject, pedagogical components (knowledge of motivation and classroom management, teaching methods, structuring for implementing curriculum and classroom assessment) related to the curriculum and psychological components (knowledge of learning processes and individual student characteristics). Learning occurs in a social context and learning success depends on the general cognitive and affective motivational characteristics of individual students (Guerriero, 2017). Psychological aspects such as general cognitive abilities, motivational and affective characteristics, and prior knowledge of the learner will differ in learning and thus impact on the success of individual learning. It is equally important to know about the different aspects of school (school location and area, physical facilities, class size, number of students etc.), students' individual and social aspects like their demographics, culture, society and physical condition etc. and the students learning behaviors for effective practice of the knowledge. The most important and effective medium of learning and instruction for teachers as well as students is the use of information communication technology (ICT). Without which the transfer of knowledge in effective and faster rate is simply impossible. The rapid development of information technology, STEM (Science, Technology, Engineering, Mathematics) education has become a trend affecting global educational systems and it highly focused on the development of knowledge and skills.

LITERATURE REVIEW

The development of teachers' knowledge also depends on the impact of growing body of knowledge. Sources contributing to teacher knowledge development include apprenticeship of observation, subject matter knowledge, teacher education, and classroom experience (Grossman, 1990). In the past, teacher content knowledge is very significant to the enhancement of teaching and learning. It was not given enough attention to its development (Leach & Moon, 2000). The focus was directed towards many aspects of teaching, but very little attention has been given to how teachers need to understand the subjects they teach. In this section, the literature related to different models and dimensions of teachers' knowledge for teaching at school level were reviewed and in the role of ICT in education were also reviewed.

Review of Different Models for Teacher's Knowledge

Shulman (1986) described about the development of teacher education as a process in which pedagogical knowledge is essential aspects along with subject matter content knowledge. The acknowledging of the importance of pedagogical subject matter in teachers' knowledge was recognized among different researchers before 1980s. The study about the structure and content of teacher knowledge category began with Shulman (1986) and Ernest, P. (1989). Shulman (1986), discussed about the three category of knowledge as: content knowledge, pedagogical content knowledge, and curricular knowledge in his article "Those Who Understand: Knowledge Growth in Teaching". He only discussed about the possibility of other category of knowledge but didn't categorize. Shulman (1987), again categorized the teachers knowledge into 7 categories in his article "Knowledge and Teaching: Foundations of the New Reform".

1. Content knowledge,
2. General pedagogical knowledge, with special reference to those broad principles and strategies of classroom management and organization that appear to transcend subject matter;
3. Curriculum knowledge, with particular grasp of the materials and programs that serve as "tools of the trade" for teachers
4. Pedagogical content knowledge, that special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding

5. Knowledge of learners and their characteristics
6. Knowledge of educational contexts, ranging from the workings of the group or classroom, the governance and financing of school districts, to the character of communities and cultures
7. Knowledge of educational ends, purposes, and values, and their philosophical and historical grounds

According to Shulman (1987), content knowledge concerns mathematical concepts, use of mathematical techniques, knowledge of subject matter and its organizing structures. Pedagogical knowledge is a unique kind of knowledge for a teacher based on the way teacher's associate pedagogy with the content of a discipline or what to teach Shulman (1987). According to Shulman (1987), curriculum knowledge means teachers' broad comprehension of school subjects and an understanding that the current one presents only one way of constructing a curriculum. Curriculum knowledge includes awareness of various instructional materials, teaching procedures, and learning objectives. In the same way, Geddis (1993) describe pedagogical content knowledge as a set of attributes that helped someone transfer the knowledge of content to others. Pedagogical content knowledge consists of subject knowledge, pedagogy, student knowledge and situational knowledge (Dan, 2016).

According to Shulman (1987), content knowledge and pedagogical content knowledge are separate categories. But others do not agree with this definition. Shulman formally proposed the concept of "pedagogical content knowledge", in which he referred to knowledge that integrates the content knowledge of a specific subject and the pedagogical knowledge for teaching that particular subject. Among those 7 categories, the pedagogical content knowledge has given of special interest by Shulman. Because it represents the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction (Shulman, 1987). The normative and theoretical knowledge of ends, purposes, and values of education is perhaps the most important part of teachers' scholarly knowledge (Shulman, 1987). This includes images of what is possible, of how a well-functioning school might look, what the students should become, and what can be understood as comprising a good education. Baumert et al. (2010) investigated the specific impact of pedagogical content knowledge (of mathematics) about student achievement. The pedagogical content knowledge was operationalized into three assessment dimensions as below:

1. A "tasks" dimension, which assessed teachers' ability to identify multiple solution paths
2. A "student" dimension, which assessed teachers' ability to recognize students' misconceptions, comprehension difficulties and solution strategies; and
3. An "instruction" dimensions that assessed teachers' knowledge of different representations and explanations of standard mathematics problems.

The multi level analysis of the student showed that, the teachers' pedagogical content knowledge was a significant predicator in explaining differences in student achievement between classes. It was also found that, the relationship between pedagogical content knowledge and mathematics achievement was linear.
Ball, Thames & Phelps, (2008) developed a practice-based theory of mathematics knowledge for teaching that includes the following domains empirically generated through factor analysis:

1. *Common Content Knowledge* (math knowledge used in a wide variety of settings that is not exclusive to teaching)
2. *Specialized Content Knowledge* (involves knowledge that goes beyond a conceptual understanding of mathematical ideas. It constitutes the knowledge and skills that are unique to math teaching as it requires teachers to understand math content with a strategic focus on pedagogy)

3. *Knowledge of Content and Students* (comprises of teachers' knowledge about students as well as math content. Understanding common errors and misconceptions made by students, and interpreting students' mathematical thinking)
4. *Knowledge of Content and Teaching* (involves the combination of pedagogical knowledge and math content. This requires teachers to understand instructional design, such as how to represent mathematical concepts, sequence content, select examples, and explain methods and procedures).

According to National Council of Teachers of Mathematics (NCTM, 2010), mathematical knowledge for teaching has been categorized into two domains:

1. Subject matter knowledge
2. Pedagogical content knowledge

The subject matter knowledge has also been categorized into 3 sub domains as: common content knowledge specified content knowledge and knowledge at the mathematical horizon. Similarly, pedagogical content knowledge is also categorized into 3 sub domains as: knowledge of content and students, knowledge of contents and teaching and knowledge of contents and curriculum. Subject matter represents simply a given body of truth to be employed in locating new problems, instituting new researches, and carrying them through to a verified outcome. Shulman (1987) defined pedagogical content knowledge as the distinctive bodies of knowledge for teaching. It represents a mix of pedagogy and content into an understanding of how certain topics, problems or issues are organized, represented, and adapted to the different interests and needs of learners, and presented for instruction.

Teacher knowledge is most often discussed as being comprised of three strands: content knowledge, pedagogical knowledge, and didactical knowledge (Durand-Guerrier & Winslow, 2005). These three categories of teachers' knowledge are same as the Shulman (1987) teachers' knowledge category respectively, as content knowledge, pedagogical knowledge, and pedagogical content knowledge. Didactical knowledge is the knowledge regarding the conditions and ways of mathematics teaching and learning (Bloch, 2005; Durand-Guerrier & Winslow, 2005) and "captures both the link and the distinction between knowing something for oneself and being able to enable others to know it" (Rowland, Thwaites, & Huckstep, 2005). According to Amade-Escot (2006) the "didactic system" is defined as an irreducible three way relationship linking teacher, students and knowledge taught. Didactical knowledge is the specific kind of knowledge that is needed for the teaching of specific mathematical concepts (Garcia & Sanchez, 2005). Such elaborations start with a topic and work outwards to encompass specific strategies, tasks, and assessment instruments that will facilitate the learning of that topic. In general, the three categories of teachers' knowledge can be seen as knowing the subject matter of mathematics, knowing way of teaching, and knowing how and when to teach mathematics.

Bergsten and Grevholm (2005) speak of teacher knowledge as being comprised of disciplinary knowledge and pedagogical knowledge. The disciplinary knowledge is the substantive knowledge of facts, procedures, concepts, and so forth, as well as knowledge of mathematics as a discipline. Pedagogical knowledge is pedagogical content knowledge and curriculum knowledge as well as knowledge of general issues in education, such as learning, developmental psychology, and socialization. Pedagogical knowledge is subject independent and deals with general principles of education such as theories of learning; sociological, psychological, and ethical aspects of education and its functions (Durand-Guerrier & Winslow, 2005).

Adler & Davis (2005) view the acquisition of teacher knowledge as learning to teach and learning mathematics for teaching. Rowland et al. (2005) introduced the teachers' knowledge into 4 categories which is a tool for thinking about the ways that a teacher's subject knowledge comes into play in the classroom. It is mentioned below.

1. Foundation (teachers' knowledge, beliefs, and understandings acquired in the academy)
2. Transformation (teachers' knowledge in action as demonstrated both in planning to teach and in the act of teaching itself)
3. Connection (binds together certain choices and decisions that are made for the more or less discrete parts of mathematics education)
4. Contingency (witnessed in classroom events that are almost impossible to prepare for)

Variation and further partitioning of each of the above mentioned knowledge strands allow for more fine grain discussion of the particularities of teachers' knowledge. The different three dimensions of teacher knowledge are represented by individual and discrete strands. As teacher education progresses, these strands are braid together to form a tighter experience in which, although still distinguishable from one another, the different strands are integrated Liljedahl, Bloch, Winslow, & Grevholm, (2009). In ideal circumstances this braid tightens towards the end of the initial teacher experience to form a unified fiber, the content of which is teacher knowledge. These three dimensions of teachers' knowledge are; Content knowledge, pedagogical knowledge and didactical knowledge. These three dimensions of teachers' knowledge are same as Durand-Guerrier & Winslow (2005).

In the model defined by Voss & Kunter, (2013), students and teachers are considered interconnected and interdependent in the teaching and learning process. Their model is domain general and the main theme of the model was "knowledge is needed to create and optimize teaching learning situations". This model also addresses an important issue brought up by Grossman and McDonald (2008) about the centrality of relationships in teaching and the lack of research investigating this pedagogical relationship and how it influences student engagement. They have divided the pedagogical/psychological knowledge into 8 parts are stated below:

1. Educational objective
2. Classroom assessment
3. Shaping the learning environment
4. Teaching methods and concepts
5. Students' heterogeneity
6. Learning processes
7. Classroom management
8. Communication and interaction

Another cognitive model also based on student learning is proposed by Konig et al. (2011). They used a task-based framework of teacher competence to define and operationalise “general pedagogical knowledge” by adopting Slavin’s (1994) model of effective teaching as a theoretical framework. The general pedagogical knowledge is comprised of four dimensions:

1. Structure (structuring of learning objectives, lesson planning and structuring the lesson process, and lesson evaluation)
2. Motivation and classroom management (achievement motivation, strategies to motivate a single student or a whole group, strategies to prevent and counteract interferences, and effective use of allocated time and routines)
3. Adaptivity (strategies of differentiation and use of a wide range of teaching methods)
4. Assessment (assessment types and functions, evaluation criteria and teacher expectation effects)

In this framework, effective teachers are proposed to have acquired general pedagogical knowledge if they showed competency in tasks requiring them to prepare, structure and evaluate lessons; to motivate and support

students and make effective use of time to manage the classroom; to deal with heterogeneous learning groups in the classroom by making use of differentiated strategies and methods of instruction; and to assess students.
In a study investigating the quality of teachers' content knowledge of mathematics, Hill, Rowan and Ball (2005) reported a significant relationship to student achievement gains. In this study, the knowledge category of interest was conceptualized as "content knowledge for teaching mathematics" and comprised two components as under.

1. Common knowledge of content (the knowledge of the subject that most adults would have)
2. Specialized knowledge used in teaching mathematics to students (the knowledge needed for evaluating solutions, at the level of a mathematician).

Kaiser, Blomeke, Konig, Busse, Dohrmann, & Hoth, (2016) has developed the model for the performance of teachers in the classroom, integrating both perspectives on teachers' perspectives, the cognitive and the situated, and providing a more comprehensive view of teachers' professional competences and activities.

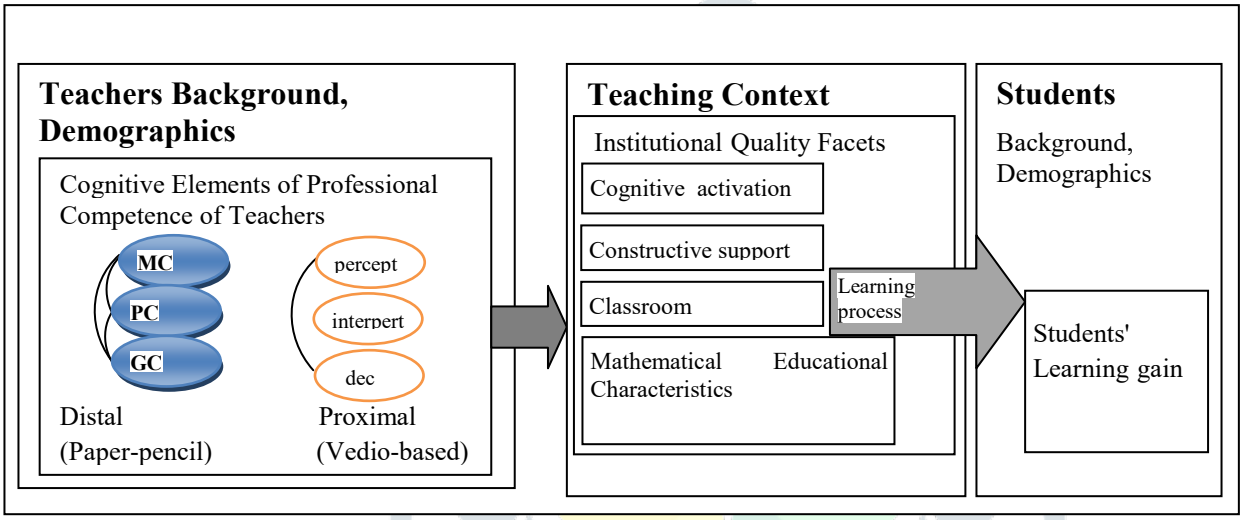

Figure 1. Research and impact model of TEDS-Instruct and TEDS-Validate. Legend:MCK mathematical content knowledge, MPCK mathematics pedagogical content knowledge and GPK general pedagogical knowledge, percept (perception of classroom situations), interpret (interpretation), dec (decision making) Figure: adapted from Kaiser, et al. (2016)

Dohrmann, Kaiser, & Blomeke, (2012) had given the teacher competencies model by combining the models given by Shulman, (1986) and (Richardson 1996; Thompson 1992). In this model, teacher competencies are categories into two dimensions as: cognitive abilities and professional knowledge and the other affective-motivational characteristics (*Figure* 2).

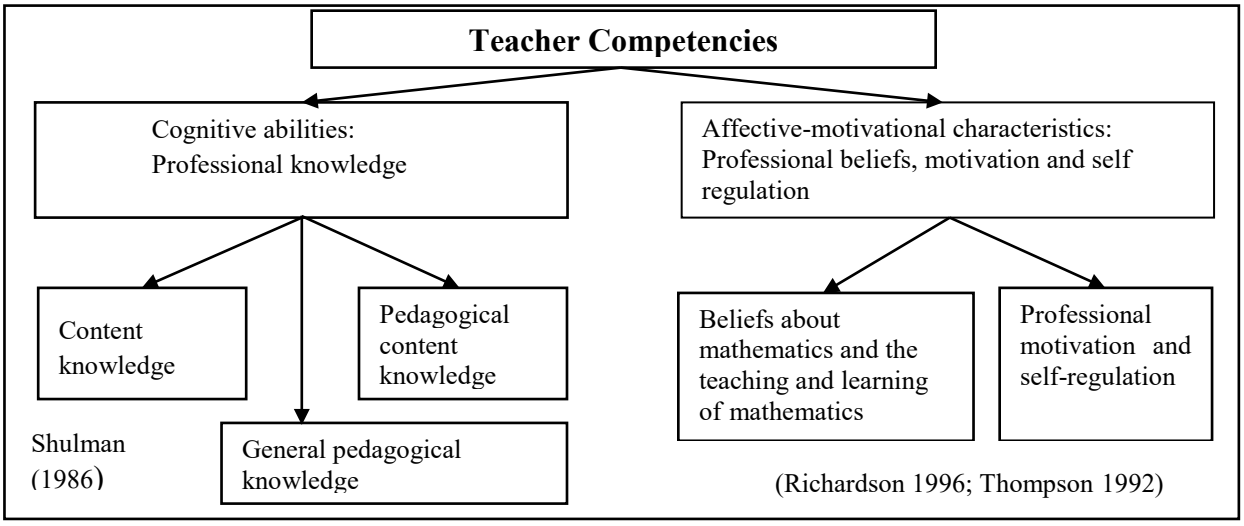

Figure 2 Conceptual model of teachers' professional competencies (Dohrmann, Kaiser, & Blomeke, 2012. Figure: adapted from Kaiser, et al (2016).

The Impact of ICT in Education and Its Role in Mathematics Teaching

The development of ICT and other new innovations through the new technology not only assist the people but also make them crazy. Even the rural located people are being habitual on ICT and they also are taking advantages. In such prevailing situation, the classroom learning can be more enthusiastic and fantastic through using the ICT. Hence, the knowledge of teaching through using ICT could be the best pedagogical dimension for the teacher. This dimension refers to any interactive teaching work for meaningful learning, creating meaningful and active participation of the students. This also helps to promote innovative learning, critical thinking and reflection. According to Trucano, (2005), the teacher with good knowledge and skills of using ICT in the classroom learning situation can help to create more learner-centric learning environment, support change and to extend existing teaching practices and help to perform more knowledge better in a short time.

Information and communication technology (ICT) has huge potential to support teacher quality improvement and knowledge delivery in the classroom. It can help in management, communications, research analysis and reporting, record keeping and file archiving, and delivery of training for capacity development (Asian Development Bank, 2017). ICT is a tool to support teacher development and delivery such as online learning, blended learning, and hybrid models with a mix of online and face-to-face modalities. The use of ICT reduced the cost and time of delivering goods through online delivery of teachers and students teaching and learning materials. So, ICT helps the teacher as well as the student to enhance their ability, skills and performance for their professional competencies. The teacher as well as the students get benefit from the global database of online information from the web and also can take advantages on the pedagogical activities like; visualization for abstract concepts, develop self-directed learning, simulations for dynamic and interactive learning, student-centered learning and self-learning etc.

Therefore, it is concluded that technologies can help teachers to teach differently as well as providing more complex kinds of tasks for students to engage. The role of information and communication technology is seen as supporting and enhancing the ability of the student teachers to solve mathematics problems. Most importantly, it changed the way the teachers see the problems and devise ways of teaching mathematical problem solving using technology in order to offer new and effective learning environment for the future generations. The objective of the study is to conceptualizing the model of mathematics teachers' knowledge for secondary level.

CONCEPTUAL FRAMEWORK

The conceptual framework of the study is presented in *Figure* 3. The visual display of the ideas of conducting this study was conceptualized as a three dimensional model of mathematics teacher's knowledge. The combination of these three knowledge dimensions can make an effective teacher for the sake of knowledge delivery.

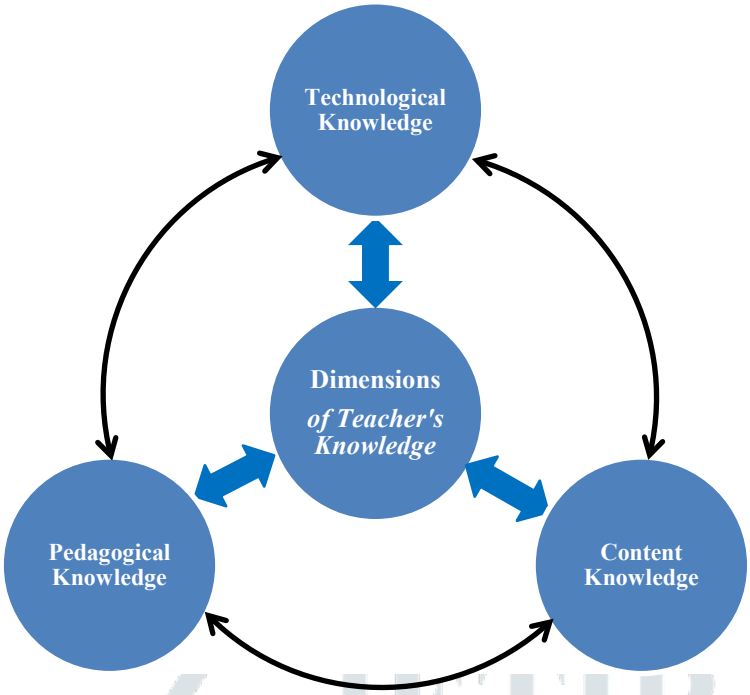

Figure 3: Conceptual Model of Teacher's Knowledge Dimensions.

METHODOLOGY

The methodology adopted in this study is conducting a deep study of different previous literature, comparison and their critical reviews and also mix up with the author's empirical experiences about the mathematics teachers' knowledge at secondary level with a view to form a conceptual model of mathematics teacher knowledge dimensions.

DISCUSSION

In this study, it is discussed about the simple introduction of the mathematics teacher's knowledge for teaching school level mathematics. It is also critically appraised what the current literature implicitly represents as making mathematics teacher knowledge dimensions. In this study, it is attempted to make explicit limitations of the grounds on which at least three general tendencies stand, and which seem to have been inherent in the current discussion on the mathematics teachers' knowledge. In this study, after reviewing the different related literatures, different dimensions of mathematics teachers' knowledge were conceptualized through the critical analysis. This critical analyzing process provides a new light on the discussion of the nature of mathematics teacher knowledge that allows us to better integrate knowledge and action. It articulates mathematics teacher knowledge more as a mindset rather than as some static traits or dispositions (Scheiner et al. 2017). Although it is difficult to provide an explicit answer about the different knowledge perspectives of mathematics teacher as there are various orientations toward teachers' knowledge.

The study starts by portraying different dimensions of mathematical knowledge discussed in the literature as being essential for mathematics teachers. Then, the only selected learning dimensions that makes clear or essential subject matter knowledge for teaching mathematics in secondary level, particularly with reference to mathematical knowledge for teaching is considered. Afterwards, these areas of knowledge that are considered as the heart of teaching are discussed.

Content Knowledge/Subject Matter Knowledge

Content knowledge is teachers' knowledge about the subject matter to be learned or taught (Koehler & Mishra, 2009). Knowledge of content is of critical importance for teachers. As Shulman (1986) noted, this knowledge

would include knowledge of concepts, theories, ideas, organizational frameworks, knowledge of evidence and proof, as well as established practices and approaches toward developing such knowledge. The content knowledge which is only for the specific content area is known as specific content knowledge and the knowledge used in a wide variety of settings that is not exclusive to teaching is considered as general content knowledge. Content knowledge is considered as the primary knowledge for the mathematics teacher. Specifically, the knowledge of mathematical subject matter or content related to mathematical knowledge is known as the content knowledge.

Mathematical Knowledge

The literature foregrounds different aspects of mathematical knowledge as important for teachers. Bromme (1994), then again, acknowledged that, school subjects have a life of their own, with their own logic and meaning of the concepts taught cannot be explained simply by the logic of the respective scientific disciplines. In recognizing school mathematics as a special kind of mathematics, Bromme (ibid.) suggested school mathematical knowledge and academic content knowledge as further dimensions of mathematical knowledge. Buchholtz et al. (2013) set forth a kind of knowledge that comprises school mathematics, but goes beyond it and relates it to the underlying advanced academic mathematics. Knowledge of mathematics includes consideration of the goals of mathematics instruction and provides a basis for discriminating and prioritizing those goals (Scheiner et al 2017). Knowing mathematics for teaching also entails more than knowing mathematics for oneself. Teachers certainly need to be able to understand concepts correctly and perform procedures accurately, but they also must be able to understand the conceptual foundations of that knowledge.

Mathematics teacher needs multidimensionality of mathematical knowledge for knowledge transmission to the students. Ball and her colleagues' idea of specialized content knowledge put emphasis on a kind of knowledge distinctive to mathematics teachers not to teachers of other subjects. There should be qualitative differences between scholars (other subjects) and mathematics teachers' subject matter knowledge for teaching (Scheiner et al. 2017). Mathematics teachers need to know more than the subject matter they teach; mathematics teachers need to know subject matter in a qualitatively different way than other practitioners of mathematics (Scheiner et al. 2017) that is, mathematicians, physicists, engineers, among others. It is important that mathematics teachers need to know how to organize or structure the subject matter in teaching-oriented action. Mathematics teacher knowledge is construed as knowledge that is needed only for teaching mathematics that is, knowledge not required for other purposes than teaching and not needed for teaching other subjects than mathematics (Scheiner et al. 2017).

Pedagogical Knowledge

Pedagogical knowledge is teachers' deep knowledge about the processes and practices or methods of teaching and learning (Koehler & Mishra, (2009). They encompass, among other things, overall educational purposes, values and aims. This generic form of knowledge applies to understanding how students learn, general classroom management skills, lesson planning, and student assessment. Pedagogical knowledge is subject independent and deals with general principles of education such as theories of learning; sociological, psychological, and ethical aspects of education and its functions (Durand-Guerrier & Winslow, 2005).

It includes knowledge about techniques or methods used in the classroom; the nature of the target audience; and strategies for evaluating student understanding (Koehler & Mishra, (2009). A teacher with deep pedagogical knowledge understands how students construct knowledge and acquire skills and how they develop habits of mind and positive dispositions toward learning. As such, pedagogical knowledge requires an understanding of cognitive, social, philosophical, moral and developmental theories of learning and how they apply to students in the classroom. It deals with knowledge of students about how they learn mathematics and develop mathematical ideas. It also includes the common difficulties that students have with certain mathematical

concepts and procedures, and it encompasses knowledge about learning and about the sorts of experiences, designs, and approaches that influence students' thinking and learning. (Scheiner et al, 2017).

Pedagogical knowledge consists of knowledge of curriculum, knowledge of tasks and tools for teaching important mathematical ideas, knowledge of how to design and manage classroom discourse, and knowledge of classroom norms that support the development of mathematical proficiency. Context and knowledge of classroom practice, and student evaluation is also equally important aspects of pedagogical knowledge. Hence, it is considered as the one of the most important dimensions of teachers' knowledge.

Pedagogical Content Knowledge

Pedagogical content knowledge is the blended form of content and pedagogical knowledge. Shulman (1986) proposed a kind of knowledge which goes beyond knowledge of subject matter for every dimension of subject matter knowledge for teaching that is pedagogical content knowledge. Pedagogical content knowledge has given most priority by the different scholars. According to Shulman (1986), pedagogical content knowledge consists of two dimensions: knowledge of representations of subject matter and knowledge of specific learning difficulties and students' conceptions. Marks (1990) clarified pedagogical content knowledge in the context of mathematics by identifying four dimensions, including knowledge of students' understanding, knowledge of subject matter for instructional purposes, knowledge of media for instruction, and knowledge of instructional processes.

Pedagogical content knowledge is interwoven pedagogy and subject matter knowledge necessary for good disciplinary teaching (Okanlawon, Agboade & Akanni, 2009). Shulman (1987) believes that pedagogical content knowledge is also a form of subject matter knowledge. It is based on teaching, which has special forms of content knowledge (Dan, 2016). Analyzing the different literatures and definitions given by the different scholars, pedagogical content knowledge comprises the two knowledge dimensions for teaching in a integrated form as content and pedagogy.

Technological Knowledge

Technological knowledge is way of using techniques and processes in any field in efficient way. Technological knowledge is always in a state of change because it is changing in the faster rate. Thus, defining the technological knowledge is difficult due to its faster changing character. Technological knowledge is in fact still a fairly new knowledge globally in terms of teaching and classroom practice. Therefore, it requires a deeper, more essential understanding and mastery of information technology for information processing, communication, and problem solving than does the traditional definition of computer literacy (Koehler & Mishra, (2009).

Technological knowledge enables a teacher to accomplish a variety of different tasks related to teaching mathematics using ICT quickly as well as effectively. It helps to deliver the knowledge in efficient and more advance way. This conceptualization of technological knowledge does not posit an "end state," but rather sees it developmentally, as evolving over a lifetime of generative, open-ended interaction with technology (Koehler & Mishra, (2009). The use of technology makes the teaching interactive, students learn by doing, researching, and receiving feedback. It encourages the use of real-world problems in the classroom. The use of the internet or software tools, students can create online groups, web pages, and virtual communities that connect them in real time with students and teachers anywhere around the world. So it also helps to develop positive attitude towards mathematics learner. It gives the opportunity for students to interact with others around the world. Thus, technology in the classroom is really like a venture into modern invention and you get to be the expedition leader. Hence the technological knowledge is also the most necessary knowledge dimension for mathematics teacher.

Thus, the knowledge dimensions of mathematics teachers establishes through the mutual relationships between content-pedagogy-technology as presented in the *Figure 3*. This mutual integration of the given knowledge dimensions for the teachers' teaching at secondary level that may creating novel styles of knowing and may help to maintain the lack of mathematics teachers' knowledge.

CONCLUSION

The teacher education is primarily concerned with developing proficiency with a number of different dimensions of teacher knowledge. Integrated forms of knowledge, skills and behaviors is needed for a competent teacher to help students develop transferable knowledge that can be applied to solve new problems or respond effectively to new situations. To meet the learners need and interest, catch the recent trends and technology and balance the context of the society, culture, and ethical status, different knowledge perspective like mathematical core subject matter, subject matter related to society and culture, and technological as well as pedagogical knowledge should be implemented. In a nutshell, the mathematics teachers have knowledge of mathematical content or subject matter, knowledge of pedagogy and knowledge of technology with new trends. Although, very low number of teachers have enriched getting such integrated different knowledge dimensions it is also recommended that, an integrated perspective that construes teacher knowledge as blends of various knowledge facets that emerge dynamic structure.

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