SOURCE FLOW OF INCOMPRESSIBLE FLUID ALONG POROUS WALL WITH THE AID OF POHLHAUSEN PROFILE

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Abstract

In the present paper the momentum integral equation, the kinetic energy integral equation and the wall compatibility condition in non-dimensional form have been drived and used to find the numerical solution for the flow of viscous incompressible fluid in the laminar boundary layer along a wall in divergent channel with homogeneous suction at the wall. Runge-Kutta method has been used for the calculation at some initial point.

Keywords : *Divergent Channel, Suction Parameter, Velocity Profile, Momentum Integral and Energy Integral Equations, Potential Flow.*

Introduction

The joint use of the momentum integral and kinetic energy integral equation to solve boundary layer equation gives results with sufficient accuracy. The first attempt to use a method of this kind was by W.G.L. Sutton who restricted himself to the case of a flat plate at zero incidence. A modified version of this method was developed by M.R. Head, K. Wieghardt’s method was simplified by A. Walz who reverted to a one parameter set of velocity profiles by dropping the first compatibility condition at the wall and retaining both the energy and the momentum equations.

Schlichting was given one parameter system of velocity profiles especially suited to the approximate calculation of the boundary layer with suction. Pohlhausen calculated the point of separation for the in compressible layminar boundary layer in a divergent channel with straight solid Walls.

Notations :

X	=	Radial distance from the source flow along the Wall.
a	=	Distance of the edge of the Wall from the source.
$\bar{X}$	=	$\dfrac{x}{a}$ non-dimensional radial distance.
y	=	Distance Normal to the Wall.
u	=	Velocity in the boundary layer in x-direction.
v	=	Velocity in y-direction.
$U(x)$	=	Potential flow velocity.
U_0	=	Entrance velocity at the edge of the Wall.
$\bar{U}(x)$	=	$\dfrac{U(x)}{U_0}$

SOURCE FLOW :

SOLUTION WITH THE AID OF POHLHAUSEN'S PROFILE (P4)

In the case of laminar incompressible flow in a divergent channel (Source Flow), the potential velocity is represented by $U(x) = U_0 \dfrac{a}{x}$

The Wall is assumed to begin at a distance $x = a$, from the source where the stream velocity is U_0.

Let $\bar{x} = \dfrac{x}{a}$, the non-dimensional radial distance.

$$\bar{U} = \frac{U}{U_0} = \frac{U_0 \frac{a}{x}}{U_0} = \frac{a}{x} = \frac{1}{x} \qquad (1)$$

$$t^* = \left(\frac{\theta}{a}\right)^2 \frac{U_0 a}{\nu} \qquad (2)$$

where t^* is momentum thickness parameter and ν is kinematic viscosity.

$$\wedge = \frac{\theta^2}{\nu}\frac{du}{dx} = t^* \frac{d\bar{u}}{d\bar{x}} = -\frac{t^*}{\bar{x}^2} \tag{3}$$

$$\bar{V}s = \frac{Vs}{V_0}\sqrt{\frac{U_0 a}{\nu}} \tag{4}$$

and $$\lambda = \frac{Vs\theta}{\nu} = t^{*\frac{1}{2}}\bar{V}s \tag{5}$$

$\lambda < 0,$ Suction

$\lambda > 0,$ Injection

Introducing these dimensionless quantities the momentum integral equation

$$\frac{dt^*}{d\bar{x}} = \frac{2}{\bar{U}}\left[1-(2+H)\wedge+\lambda\right] \tag{A}$$

Kinetic energy equation

$$\frac{dH_\in}{d\bar{x}} = \frac{1}{t^*\bar{U}}\left[2D - H_\in\left\{l-(H-1)\wedge+\lambda\right\}\right] \tag{B}$$

and the Wall compatibility condition equation

$$m = -\wedge + \lambda l \tag{C}$$

where $m = \frac{\theta^2}{U}\left(\frac{\delta^2 u}{\delta y^2}\right)_{y=\infty}$

For the boundary layer in a divergent channel with porous wall reduces to

$$\frac{dt^*}{d\bar{x}} = f\left(\bar{x}, H_\in, t^*\right)$$

$$= 2\bar{x}\left\{1+\frac{(2+H)t^*}{\bar{X}^2} + t^{*\frac{1}{2}}\bar{V}_s\right\} \tag{6}$$

$$\frac{dH_{\in}}{d\,\bar{x}} = g\left(\bar{x}, H_{\in}, t^{*}\right)$$

$$= \frac{X}{t^{*}}\left[2D - H_{\in}\left\{1 + \frac{(H-1)t^{*}}{\bar{X}^{2}} + t^{*\frac{1}{2}}\bar{V}s\right\} + t^{*\frac{1}{2}}\bar{V}_{s}\right] \qquad (7)$$

and $$m = \frac{t^{*}}{\bar{x}^{2}} + t^{*\frac{1}{2}}\bar{V}_{s} l \qquad (8)$$

VALUES AT THE STARTING POINT

The boundary layer starts to develop from the leading edge $\bar{x} = a$ of the Wall. The point $\bar{x} = \frac{x}{a} = 1$ is taken as the starting point for the step by step calculation. At the starting point $t^{*} = 0$ and hence, $\lambda = t^{*\frac{1}{2}}\bar{V}s = 0$ for all values of the suction velocity.

Hence, at the starting point the boundary layer parameters are :

$$\bar{X} = 1, \bar{U} = 1, t^{*} = 0$$
$$\lambda = 0, \wedge = 0$$

Hence $m = 0$ from equation (8)

$$H_{\in} = 1.571, \; H = 2.554, \; l = 0.235, \; D = 0.1745$$

NUMERICAL SOLUTION OF THE MOMENTUM AND THE KINETIC ENERGY INTEGRAL EQUATIONS

The momentum integral equation (6) and the kinetic energy integral equation (7) have been simultaneously solved with the aid of the compatibility condition (8) by employing the Runge-Kutta method for the two ordinary first order simultaneous differential equation.

The Runge-Kutta method for the two ordinary first order simultaneous differential equation is

$$K_{1} = f\left(\bar{x}_{0}, t_{0}^{*}, H_{\in 0}\right)\Delta\bar{X}$$
$$K_{2} = f\left(\bar{x}_{0} + \frac{\Delta\bar{x}}{2}, t_{0}^{*} + \frac{k_{1}}{2}, H_{\in 0} + \frac{l_{1}}{2}\right)\Delta\bar{X}$$

$$K_3 = f\left(\bar{X}_0 + \frac{\Delta \bar{x}}{2}, t_0^* + \frac{k_2}{2}, H_{\in 0} + \frac{l_2}{2}\right)\Delta \bar{X}$$

$$K_4 = f\left(\bar{X}_0 + \Delta\bar{X}, t_0^* + K_3, H_{\in 0} + l_3\right)\Delta \bar{X}$$

$$\Delta t^* = \frac{1}{6}\left(K_1 + 2K_2 + 2K_3 + K_4\right)\Delta \bar{X}$$

For *f* and *g* are computed and the calculation proceeds step by steps up to the point of separation. (*f*, *g* is functions as in equation of Runge-Kutta method)

SOURCE FLOW : (SOLUTION WITH THE AID OF SCHLICHING'S PROFILE) ALONG A SOLID WALL

For Schlichting's profile the compatibility condition

$$\left(K+1\right)\left(\frac{\theta}{\delta}\right)^2\left[1+\left(1-\frac{\pi}{6}\right)K\right]\frac{\theta}{\delta}\lambda - \wedge = 0 \text{ becomes}$$

$$\left(K+1\right)\left(\frac{\theta}{\delta}\right)^2 + \bar{V}s\,t^{*\frac{1}{2}}\frac{\theta}{\delta}\left[1+\left(1-\frac{\pi}{6}\right)K\right] + \frac{t^*}{\bar{X}^2} = 0 \qquad (9)$$

At the starting point the boundary layer parameters are

$$\bar{X} = 1, \bar{U} = 1, t^* = 0$$
$$\lambda = 0, \wedge = 0$$

Hence by equation (9), K = -1, then corresponding to K = -1 the values of the boundary layer parameters are :

$$\frac{\theta}{\delta} = 0.4098$$
$$l = 0.2145$$
$$H = 2.6600$$
$$H_{\in} = 1.5532$$
$$D = 0.1685$$

The momentum integral equation (6) and Kinetic energy equation (7) have been solved with the aid of Wall compatibility condition (9) by Runge-Kutta method.

The point of separation for solid Wall $\left(\bar{V}s = 0\right)$ by this method is $\bar{X}s = 1.2060$ which is very close to the known result of Pohlhansen's

RESULTS :

Calculation have been made for three different constant values of $\bar{V}s = 0, -0.2, -0.3$ when solved with the aid of Pohlhausen profile (P_4). For $\bar{V}s = 0$, the equations reduce to the equations for the solid Wall problem. The point of separation for $\bar{V}s = 0$ is found to be at $\bar{X} = 1.1612$ which is inclose agreement with the value $\bar{X} = 1.12130$ obtain by Pohlhausen.

DISCUSSION OF THE RESULTS :

Calculation have been made for

(i) $\bar{V}s = 0, -0.2, -0.3$ when solved with the aid of pohlhausen's profile (P_4).

and

(ii) $\bar{V}s = 0, -0.5$ when solved with the aid of schlichting profile.

The results of calculations are shown in the following comparision table.

COMPARISION TABLE

	Method	**Suction parameter** $\bar{V}s$	**Point of Separation** $\bar{X}s$
1.	Pohlhansen	0.0	1.2130
2.	Present method with the	0.0	1.1612
	aid of Pohlhansen's	-0.2	1.1947
	Profile	-0.3	1.2115
3.	Present method with	0.0	1.2060
	the aid of Schlichting's	-0.5	1.3260
	profile		

From above we observe that with increasing rate of suction parameter $\bar{V}s$ the point of separation moves further down stream.

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