INTUITIONISTIC FUZZY STRONG BI-IDEALS IN NEAR SUBTRACTION SEMIGROUPS

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ABSTRACT

Dheena discussed and derived some properties of near subtraction semigroups. The concept of fuzzy set was first initiated by Zadeh. In this paper we introduced the notation of intuitionistic fuzzy strong bi-ideal of near-subtraction semigroups.

Key words:

Intuitionistic fuzzy two sided X-Sub algebra, Intuitionistic fuzzy sub-near subtraction semigroups, Intuitionistic fuzzy bi-ideal, Intuitionistic fuzzy strong bi-ideal.

1. INTRODUCTION:

B.M.Schein [6] considered systems of the form (X; o ;/), where X is a set of functions closed under the composition "o" of functions (and hence (X; o) is a function semigroup) and the set theoretic subtraction "/" (and hence (X;/) is a subtraction algebra in the sense of [2]).Y.B.Jun et al [3] introduced the notation of ideals in subtraction algebras and discussed the characterization of ideals. In[3], Y.B.Jun and H.S.Kim established the ideal generated by a set,and discussed related results.The concept of fuzzy set was first initiated by Zadeh[7]. Narayanan et al.[5] defined the concept of generalized fuzzy ideals of near-rings. Mahalakshmi et al. [3] studied the notation of bi-ideals in near-subtraction semigroups. Manikandan [4] studied fuzzy fuzzy bi-ideals in near-rings. The concept of intuitionistic fuzzy set was introduced by Atanassov (1986) as a generalisation of the notion of fuzzy set.In this paper, we introduce the notion of a intuitionistic fuzzy strong bi-ideal of a near-subtraction semigroup and obtain the characterization of a strong bi-ideal in terms of a intuitionistic fuzzy strong bi-ideal of a near -subtraction semigroup. We establish that every intuitionistic fuzzyleft X-subalgebra or intuitionistic fuzzy left ideal of a near-subtraction semigroup is a intuitionistic fuzzy strong bi-ideal of a near-subtraction semigroup and also we establish that every intuitionistic left permutable fuzzy right X-subalgebra or intuitionistic left permutable fuzzy right ideal of a near- subtraction semigroup is a intuitionistic fuzzy strong bi-ideal of a near- subtraction semigroup.

2.Preliminaries

Definition:2.1

A nonempty set X together with two binary operation – and . is called **near subtraction algebra** if it satisfying the following:

(i) x-(y-x) = x

(ii) x-(x-y) = y-(y-x)

(iii) (x-y)-z = (x-z)-y

Definition:2.2

A nonempty set X together with two binary operation – and . is said to be **subtraction semigroup** if it satisfying the following:

(i) (x,-) is a subtraction algebra.

(ii) (x,.) is a semigroup.

(iii) x(y-z) = xy-xz and

(x-y)z = xz-yz $\forall x,y,z\in X$

Definition: 2.3

A near- subtraction semigroup X is called **zero-symmetric,** if x0 = 0, for all x in X.

Definition: 2.4

A non empty subset S of a subtraction algebra X is said to be a **Subalgebra** of X, if x-y ϵ S $\forall x,y \in S$.

Note: 2.5

1. Let X be a near- subtraction semigroup. Given two subsets A and B of X,

A*B = {ab/a∈A, b∈B}. Also we define another operation "∗ "

A∗B = {ab–a(a'-b) / a',a,∈A, b∈B}.

Definition: 2.6

A function A from a non-empty set X to the unit interval is called a **fuzzy subset** *of X.*

Definition: 2.7

A Sub algebra B of X is called **bi-ideal** if BXB$\cap BX * B \subseteq B$. *In case* of zero Symmetric,BXB$\subseteq B$.

Notation: 2.8

Let A and B be two fuzzy subsets of a semigroup X. We define the relation ⊆ between A and B , the union, intersection and product of A and B, respectively as follows:

1. *A⊆B if A(x) ≤ B (x), for all x∈X,*
2. *(A∪B) (x) = max{A(x), B(x)}, for all x∈X,*
3. *(A ∩B) (x) = min{ A(x), B (x)},*

For all x∈X.

4. $(A.B)(x) = \begin{cases} \sup_{x=yz} min\{A(y), B(z)\}, & \text{if } x=yz \text{ for all } x,y \in X \\ 0 & Otherwise \end{cases}$

5. $(A*B)(x) = \begin{cases} \inf_{x=yz} max\{A(y), B(z)\}, & \text{if } x=yz \text{ for all } x,y \in X \\ 0 & Otherwise \end{cases}$

Definition: 2.9:

A Fuzzy subalgebra *A of X is called fuzzy bi-ideal of X,if* $(AXA) \cap (AX*A) \subseteq A$.

In case of zero symmetric if $AXA \subseteq A$.

Definition: 2.10

Let (X,-,.) be a near subtraction semigroup.A non-empty subset I of X is called

(i) A Left ideal if I is a subalgebra of (X,-) and $xi\text{-}x(y-i) \in I$ *for every* $x,y \in X, i \in I$

(ii) A right ideal if I is a subalgebra of (X,-) and $IX \subseteq I$.

(iii) An Ideal if I is both a left and right.

Definition: 2.10

A Fuzzy subset *A of X is called* ***fuzzy ideal*** if it satisfying the following conditions:

(i) $A(x\text{-}y) \geq min\{A(x), A(y)\}$.

(ii) $A(xi\text{-}x(y\text{-}i) \geq A(i)$.

(iii) $A(xy) \geq A(x)$. ,*for every* $x,y \in X$.

A fuzzy subset with (i) and (ii) is called a fuzzy left ideal of X,Whereas a fuzzy subset with (i) and (iii) is called a fuzzy right ideal of X.

Definition: 2.11

A Fuzzy subset *A of X is called* ***fuzzy X subalgebra*** if it satisfying the following conditions:

(i) *A is a fuzzy subalgebra of (X,-).*

(ii) $A(xy) \geq A(x)$.

(iii) $A(xy) \geq A(y)$,*for every* $x,y \in X$.

A fuzzy subset with (i) and (ii) is called a fuzzy right subalgebra of X ,Whereas a fuzzy subset with (i) and (iii) is called a fuzzy left subalgebra of X.

Definition: 2.12

An intuitionistic fuzzy subset having the form A = $\{(x, A_\mu(x), B_\mu(x)) / x \in X\}$where the functions A_μ : X→(0,1) and B_μ: X→(0,1) denote the degree of membership and the degree of non membership of each element x∈X to the set μ, respectively. and

$0 \leq A_\mu(x) + B_\mu(x) \leq 1$ for all x∈X. For the sake of simplicity, we shall use the symbol μ= (A_μ , B_μ) for the intuitionistic fuzzy subsetμ={(x, $A_\mu(x), B_\mu(x)$) / xϵX}.

Definition: 2.13

An intuitionistic fuzzy subset said to be a **intuitionistic fuzzy subalgebra** of X if for all x,y∈X

(i) $A_\mu(x-y) \geq \min\{A_\mu(x), A_\mu(y)\}$

(ii) $B_\mu(x-y) \leq \max\{B_\mu(x), B_\mu(y)\}$

Definition: 2.14

An intuitionistic fuzzy subset μ = (A_μ, B_μ) of X is called an **intuitionistic fuzzy subnear-subtraction semigroup** of X if for all x,y ∈X,

(i) $A_\mu(x-y) \geq \min\{A_\mu(x), A_\mu(y)\}$

(ii) $A_\mu(xyz) \geq \min\{A_\mu(x), A_\mu(z)\}$

(iii) $B_\mu(x-y) \leq \max\{B_\mu(x), B_\mu(y)\}$

(iv) $B_\mu(xyz) \leq \max\{B_\mu(x), B_\mu(z)\}$

Definition: 2.15

An intuitionistic fuzzy subset μ = (A_μ, B_μ) of X is said to be an intuitionistic fuzzy two-sided X-subalgebra of X if

(i) μ is an intuitionistic fuzzy subalgebra of (X,-),

(ii) $A_\mu(xy) \geq A_\mu(x)$ for all x,y∈X

(iii) $A_\mu(xy) \geq A_\mu(y)$ for all x,y∈X

(iv) $B_\mu(xy) \leq B_\mu(x)$ for all x,y∈X

(v) $B_\mu(xy) \leq B_\mu(y)$ for all x,y∈X

If μ satisfies (i), (ii) and (iv), then μ is called an intuitionistic fuzzy right X-subalgebra of X. If μ satisfies (i), (iii) and (v), then μ is called an intuitionistic fuzzy left X-subalgebra of X.

Definition: 2.16

An intuitionistic fuzzy subset μ = (A_μ, B_μ) of X is said to be an **intuitionistic fuzzy ideal** of X if

(i) μ is an intuitionistic fuzzy subnear-subtraction semigroup of X,

(ii) $A_\mu(x-y) \geq \min\{A_\mu(x), A_\mu(y)\}$ for all x, y ∈ X,

(iii) $A_\mu(xi - x(x' - i)) \geq A_\mu$(i), for all x, x' ∈X,

(iv) $A_\mu(xy) \geq A_\mu(x)$, for all x,y∈X

(v) $B_\mu(x-y) \leq \max\{B_\mu(x), B_\mu(y)\}$ for all x, y ∈ X,

(vi) $B_\mu(xi - x(x' - i)) \leq B_\mu$(i), for all x, x'∈X,

(vii) $B_\mu(xy) \leq B_\mu(x)$, for all x,y∈X

If μ satisfies (i),(ii),(iii),(v) and (vi) is called an intuitionistic fuzzy right ideal of X. If m satisfies (i), (ii), (iv) and (vii) is called an intuitionistic fuzzy left ideal of X. Let A_μ B_μ and be two intuitionistic fuzzy subsets of X. we define an intuitionistic fuzzy subset

$$(A_\mu * B_\mu)(x) = \begin{cases} \sup_{x=ai-(a'-i)} \min\{A_\mu(a), A_\mu(a'), B_\mu(i)\} & \text{If } x=ai-a(a'-i), a,b,i \in X, \\ 0 & \text{Otherwise.} \end{cases}$$

Definition:2.17

An intuitionistic fuzzy subset $\mu = (A_\mu, B_\mu)$ of X is said to be an **intuitionistic fuzzy bi- ideal of X** if for all x, y∈X,

(i) $A_\mu(x - y) \geq \min\{A_\mu(x), A_\mu(y)\}$

(ii) $(A_\mu{}^\circ X^\circ A_\mu) \cap (A_\mu{}^\circ X) * A_\mu \subseteq A_\mu$

(iii) $B_\mu(x - y) \leq \max\{B_\mu(x), B_\mu(y)\}$

(iv) $(B_\mu{}^\circ X^\circ B_\mu) \cup (B_\mu{}^\circ X) * B_\mu \supseteq B_\mu$

3. Intuitionistic Fuzzy Strong Bi-ideals In Near-Subtraction Semigroups

Definition:3.1

An intuitionistic fuzzy bi-ideal $\mu = (A_\mu, B_\mu)$ of X is called an intuitionistic fuzzy strong bi-ideal of X, if

(i) $X^\circ A_\mu{}^\circ A_\mu \subseteq A_\mu$.

(ii) $X^\circ B_\mu{}^\circ B_\mu \supseteq B_\mu$.

Example:3.2

-	0	a	b	c
0	0	0	0	0
a	a	0	a	0
b	b	b	0	0
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	a	0	a
b	0	b	0	b
c	0	c	0	c

Let X={0,a,b,c} be a near-subtraction semigroup with two binary operations '-'and '•' is defined as follows.

Define a fuzzy subset $\mu = (A\mu, B\mu)$ where $A\mu : X \to (0,1)$ by $A\mu(0) = 0.9$, $A\mu(a) = 0.8$, $A\mu(b) = 0.5 = A\mu(c)$. Then $(A\mu^\circ X^\circ A\mu)(0) = 0.9$, $(A\mu^\circ X^\circ A\mu)(a) = 0.8$, $(A\mu^\circ X^\circ A\mu)(b) = 0.5$, $(A\mu^\circ X^\circ A\mu)(c) = 0.5$, $(X^\circ A\mu^\circ A\mu)(0) = 0.9$, $(X^\circ A\mu^\circ A\mu)(a) = 0.8$, $(X^\circ A\mu^\circ A\mu)(b) = 0.5$, $(X^\circ A\mu^\circ A\mu)(c) = 0.5$and so $A\mu$ is a intuitionistic fuzzy strong bi-ideal of X and

$B\mu : X \to (0,1)$ by $B\mu(0) = 0.3$, $B\mu(a) = 0.5$, $B\mu(b) = 0.7 = B\mu(c)$ Then $(B\mu^\circ X^\circ B\mu)(0)=0.5$, $(B\mu^\circ X^\circ B\mu)(a)=0.5$, $(B\mu^\circ X^\circ B\mu)(b) = 0.7$, $(B\mu^\circ X^\circ B\mu)(c) = 0.7$, $(X^\circ B\mu^\circ B\mu)(0) = 0.5$, $(X^\circ B\mu^\circ B\mu)(a) = 0.5$, $(X^\circ B\mu^\circ B\mu)(b)=0.7$, $(X^\circ B\mu^\circ B\mu)(c)=0.7$ and so $B\mu$ is an intuitionistic fuzzy strong bi-ideal of X. Thus μ = (Aμ,Bμ) is an intuitionistic fuzzy strong bi-ideal of X.

Theorem: 3.3

Let $\{\mu_i\} = \{(A_{\mu_i}, B_{\mu_i}) : i \in I\}$ be any family of intuitionistic fuzzy strong bi-ideals in a near-subtraction semigroup X. Then $\bigcap_{i\in I} \mu_i$ is an intuitionistic fuzzy strong bi-ideal of X, where $\bigcap_{i\in I} \mu_i = \{(\bigcap_{i\in I} A_{\mu_i}, \bigcup_{i\in I} B_{\mu_i})\}$.

Proof:

Let $\{\mu_i : i \in I\}$ be any family of intuitionistic fuzzy strong bi-ideals of X.

Now for all x,y ∈X,

$\bigcap_{i\in I} A_{\mu_i}(x - y) = \min\{A_{\mu_i}(x - y) / i\in I\}$

$\geq \min\{\min\{A_{\mu_i}(x), A_{\mu_i}(y) / i\in I\}$

(since A_{μ_i} is an intuitionistic fuzzy subalgebra of X)

$= \min\{\bigcap_{i\in I} A_{\mu_i}(x), , \bigcap_{i\in I} A_{\mu_i}(y) / i\in I\}$

$\bigcup_{i\in I} B_{\mu_i}(x - y) = \max\{B_{\mu_i}(x - y) / i\in I\}$

$\leq \max\{\max\{B_{\mu_i}(x), B_{\mu_i}(y) / i\in I\}$

(since B_{μ_i} is an intuitionistic fuzzy subalgebra of X)

$= \max\{\bigcup_{i\in I} B_{\mu_i}(x), \bigcup_{i\in I} B_{\mu_i}(y) / i\in I\}$

Therefore $\bigcap_{i\in I} \mu_i$ is an intuitionistic fuzzy subalgebra of X.

To Prove: $\bigcap_{i\in I} \mu_i$ is an intuitionistic fuzzy bi-ideal of X..

Now for all x∈X, since $A_\mu = \bigcap_{i\in I} A_{\mu_i} \subseteq A_{\mu_i}$, for every i∈I, we have

$((A_\mu \circ X \circ A_\mu) \cap (A_\mu \circ X) * A_\mu))(x) \leq ((A_{\mu_i} \circ X \circ A_{\mu_i}) \cap (A_{\mu_i} \circ X) * A_{\mu_i}))(x)$

(since A_{μ_i} is an intuitionistic fuzzy bi-ideal of X).

$\leq A_{\mu_i}(x)$ for every i∈I.

It follows that

$((A_\mu \circ X \circ A_\mu) \cap (A_\mu \circ X) * A_\mu))(x) \leq \inf\{A_{\mu_i}(x) : i \in I\}$

$= \bigcap_{i\in I} A_{\mu_i}(x)$

$= A_\mu(x)$

Thus $(A_\mu \circ X \circ A_\mu) \cap (A_\mu \circ X) * A_\mu) \subseteq A_\mu$

So A_μ is an intuitionistic fuzzy bi-ideal of X.

Now for all x ЄX,since $B_\mu = \bigcup_{i\in I} B_{\mu_i} \supseteq B_{\mu_i}$ for some iЄI,

we have

$((B_\mu \circ X \circ B) \cup (B_\mu \circ X) * B))(x) \geq ((B_{\mu_i} \circ X \circ B_{\mu_i}) \cup (B_{\mu_i} \circ X) * B_{\mu_i}))(x)$

(since B_{μ_i} is an intuitionistic fuzzy bi-ideal of X).

$\geq B_{\mu_i}(x)$ for every iєI.

It follows that

$((B_\mu \circ X \circ B) \cup (B_\mu \circ X) * B))(x) \geq \sup\{B_{\mu_i}(x) : i \in I\}$

$= \bigcup_{i\in I} B_{\mu_i}(x) = B_\mu(x)$

Thus $(B_\mu \circ X \circ B) \cap (B_\mu \circ X) * B) \supseteq B_\mu$

So B_μ is an intuitionistic fuzzy bi-ideal of X.
Thus$\bigcap_{i\in i} \mu_i$ is an intuitionistic fuzzy bi-ideal of X.

Next we prove: $\bigcap_{i\in i} \mu_i$ is an intuitionistic fuzzy strong bi-ideal of X.

Now for all xєX, Since $A_\mu = \bigcap_{i\in I} A_{\mu_i} \subseteq A_{\mu_i}$, for every iЄI, we have

$(X \circ A_\mu \circ A_\mu)(x) \leq (X \circ A_{\mu_i} \circ A_{\mu_i})(x)$

$\leq A_{\mu_i}(x)$ for every iєI

(since A_{μ_i} is an intuitionistic fuzzy strong bi-ideal
of X)
It follows that,

$(X \circ A_\mu \circ A_\mu)(x) \leq \inf\{A_{\mu_i}(x) : i \in I\} = \bigcap_{i\in I} A_{\mu_i}$

$= A_\mu(x)$

Thus $(X \circ A_\mu \circ A_\mu) \subseteq A_\mu$. *So* A_μ is an intuitionistic fuzzy strong bi-ideal of X.

Now for all xЄX, since $B_\mu = \bigcup_{i\in I} B_{\mu_i} \supseteq B_{\mu_i}$, *for some* $i \in I$, we have

$(X \circ B_\mu \circ B_\mu)(x) \geq (X \circ B_{\mu_i} \circ B_{\mu_i})(x)$

$\geq B_{\mu_i}(x)$ for every iєI

(since B_{μ_i} is an intuitionistic fuzzy strong bi-ideal

of X)
It follows that,

$(X^{\circ} B_{\mu}{}^{\circ} B_{\mu})(x) \quad \geq \sup \{B_{\mu_i}(x): i \in I\}$

$= (\bigcup_{i \in I} B_{\mu_i}(x))$

$= B_{\mu}(x)$

Thus $X^{\circ} B_{\mu}{}^{\circ} B_{\mu} \supseteq B_{\mu}$. So B_{μ} is an intuitionistic fuzzy strong bi-ideal of X.

Thus $\bigcap_{i \in i} \mu_i$ is an intuitionistic fuzzy strong bi-ideal of X.

Theorem: 3.4

Every left permutable intuitionistic fuzzy right X-subalgebra of X is an intuitionistic fuzzy strong bi-ideal of X.

Proof:

Let μ = (A_{μ}, B_{μ}) be a left permutable intuitionistic fuzzy right

X-subalgebra of X.

To prove: μ is an intuitionistic fuzzy strong bi-ideal of X.

First we prove: μ is an intuitionistic fuzzy bi-ideal of X.

Choose a,b,c,x,y,i,b_1,b_2,x_1,x_2,y_1, y_2 in X such that a = bc = $(xi - x(x' - i))$, b =b_1,b_2 , x =x_1,$x_2$and y = y_1, y_2. Then

$(A_{\mu}{}^{\circ}X^{\circ}A_{\mu}) \cap ((A_{\mu}{}^{\circ}X) * A_{\mu})(a) = \min\{(A_{\mu}{}^{\circ}X^{\circ}A_{\mu})(a), ((A_{\mu}{}^{\circ}X) * A_{\mu}))(a)\}$

$= \min\{\sup_{a=bc} \min(A_{\mu}{}^{\circ}X)(b), A_{\mu}(c)\}, ((A_{\mu}{}^{\circ}X) * A_{\mu})(xi - x(x' - i))\}$

$= \min\{\sup_{a=bc} \min\{\sup_{b=b_1 b_2} \min\{A_{\mu}(b_1), X(b_2)\}, A_{\mu}(c)\}, ((A_{\mu}{}^{\circ}X) * A_{\mu})\big(xi - x(x' - i)\big)\}$

(Since X(z)=1,for all z∈X)

$= \min\{\sup_{a=bc} \min\{\sup_{a=b_1 b_2} \min\{A_{\mu}(b_1), A_{\mu}(c)\}, ((A_{\mu}{}^{\circ}X) * A_{\mu})\big(xi - x(x' - i)\big)\}$

(Since $A_{\mu}$is an intuitionistic fuzzy right X-subalgebra of X,

$A_{\mu}(bc) = A_{\mu}(b_1 b_2 c) = A_{\mu}(b_1(b_2 c)) \geq A_{\mu}(b_1))$

$\leq \min\{\sup_{a=bc} \min\{A_{\mu}(bc), X(c)\}, X(xi - x(y - i))\}$

$= \min\{\sup_{a=bc} \min\{A_{\mu}(bc),, X(xi - x(x' - i))\}$

$= A_{\mu}(bc)$

$= A_{\mu}(a)$

Thus $(A_\mu{}^\circ X^\circ A_\mu) \cap ((A_\mu{}^\circ X) * A_\mu)) \subseteq A_\mu$.

Hence A_μ is an intuitionistic fuzzy bi − ideal of X.

Choose a,b,c,x,y,i,b_1,b_2,x_1,x_2,y_1, y_2 in X such that a = bc = $(xi - x(x' - i))$, b =b_1,b_2 , x =x_1,$x_2$and y = y_1, y_2. Then

$(B_\mu{}^\circ X^\circ B_\mu) \cup ((B_\mu{}^\circ X) * B_\mu))(a) = \max\{(B_\mu{}^\circ X^\circ B_\mu)(a), ((B_\mu{}^\circ X) * B_\mu))(a)\}$

$= \max\{\inf_{a=bc} \max(B_\mu{}^\circ X)(b), B_\mu(c)\}, ((B_\mu{}^\circ X) * B_\mu)(xi - x(x' - i))\}$

$= \max\{\inf_{a=bc} \max\{\inf_{b=b_1 b_2} \max\{B_\mu(b_1), X(b_2)\}, B_\mu(c)\}, ((B_\mu{}^\circ X) * B_\mu)\left(xi - x(x' - i)\right)\}$

(Since X(z)=0,for all z∈X)

$= \min\{\inf_{a=bc} \max\{\inf_{a=b_1 b_2} \{B_\mu(b_1), B_\mu(c)\}, ((B_\mu{}^\circ X) * B_\mu)\left(xi - x(x' - i)\right)\}$

(Since B_μ is an intuitionistic fuzzy right X-subalgebra of X,

$B_\mu(bc) = B_\mu(b_1 b_2 c) = B_\mu(b_1(b_2 c)) \geq B_\mu(b_1))$

$\geq \max\{\inf_{a=bc} \max\{B_\mu(bc), X(c)\}, X(xi - x(x' - i))\}$

$= \max\{\inf_{a=bc} \max\{B_\mu(bc),, X(xi - x(x' - i))\}$

$= B_\mu(bc)$

$= B_\mu(a)$

Thus $(B_\mu{}^\circ X^\circ B_\mu) \cup ((B_\mu{}^\circ X) * B_\mu)) \supseteq B_\mu$.

Hence B_μ is an intuitionistic fuzzy bi − ideal of X.

Thus μ = (A_μ, B_μ) is an intuitionistic fuzzy bi − ideal of X.

Next we prove : μ is an intuitionistic fuzzy strong bi − ideal of X.

Choose a,b,c,b_1,b_2 ∈ X such that a = bc and b =$b_1 \circ b_2$, Then

$X^\circ A_\mu{}^\circ A_\mu(a) = \sup_{a=bc} \min\{X^\circ A_\mu(b), A_\mu(c)\}$

$= \sup_{a=bc} \min\{\sup_{b=b_1 b_2} \min\{X(b_1)^\circ A_\mu(b_2)\}, A_\mu(c)\}$

$= \sup_{a=bc} \min\{\sup_{b=b_1 b_2} \min\{A_\mu(b_2), A_\mu(c)\}$

(Since A_μ is a left permutable intuitionistic fuzzy right X-subalgebra of X, $A_\mu(bc) = A_\mu((b_1 b_2)c) =$

$A_\mu((b_2 b_1)c) > A_\mu(b_2))$ and $X(c) \geq A_\mu(c)$

$\leq \sup_{a=bc} \min\{A_\mu(bc), X(c)\}$

$= \sup_{a=bc} \min\{A_\mu(bc), 1\}$

$= {}_{a=bc}^{sup} \ A_\mu(bc)$

$= \ A_\mu(a)$

Therefore X° A_μ° A_μ(a)$\subseteq A_\mu$.

Hence A_μ is an intuitionistic fuzzy strong bi − ideal of X.

Choose a,b,c,b_1,b_2 ∈ X such that a = bc and b =$b_1 \circ b_2$, Then

X° B_μ° B_μ(a) = ${}_{a=bc}^{inf}$ max {(X° B_μ)(b), $B_\mu(c)$}

$= {}_{a=bc}^{inf}$ max {${}_{b=b_1b_2}^{inf}$ max { X(b_1)° $B_\mu(b_2)$}, $B_\mu(c)$}

$= {}_{a=bc}^{inf}$ max {${}_{b=b_1b_2}^{inf}$ { $B_\mu(b_2)$, $B_\mu(c)$}

(Since $B_\mu$is a left permutable intuitionistic fuzzy right

X-subalgebra of X, $B_\mu(bc)$= $B_\mu((b_1b_2)c) = \ B_\mu((b_2b_1)c) \leq \ B_\mu(b_2))$

$\geq {}_{a=bc}^{inf}$ max$\{ B_\mu(bc), X(c)\}$

$= {}_{a=bc}^{inf}$ max$\{ B_\mu(bc), 0\}$

$= {}_{a=bc}^{sup} \ B_\mu(bc)$

$= \ B_\mu(a)$

Therefore X° B_μ° B_μ(a)$\supseteq B_\mu$

Hence B_μ is an intuitionistic fuzzy strong bi − ideal of X.

Thus μ = (A_μ, B_μ) is an intuitionistic fuzzy strong bi − ideal of X

Theorem: 3.5

Every intuitionistic fuzzy left X-subalgebra of X is an

intuitionistic fuzzy strong bi-ideal of X.

Proof:

Let μ = (A_μ, B_μ)be an intuitionistic fuzzy left X-subalgebra of X.

To prove: μ is an intuitionistic fuzzy strong bi-ideal of X.

First we prove: μ is an intuitionistic fuzzy bi-ideal of X.

a,b,c,x,y,i,c_1,c_2,x_1,x_2,y_1, y_2 in X such that a = bc =

$(xi - x(x' - i))$, c =$c_1 \circ c_2$, x =x_1,$x_2$and y = y_1, y_2. Then

(A_μ°X°A_μ)∩((A_μ°X)∗ A_μ))(a) = min{(A_μ°X°A_μ)(a), ((A_μ°X)∗ A_μ))(a)}

= min{${}_{a=bc}^{sup}$ min{(A_μ)(b) , $(X^o A_\mu)(c)$}, ((A_μ°X)∗ A_μ) $(xi - x(y - i))$}

= min{${}_{a=bc}^{sup}$ min{(A_μ)(b),${}_{c=c_1c_2}^{sup}$ min{$X(c_1)$, A_μ (c_2)}, ((A_μ°X)∗ A_μ) $\left(xi - x(x' - i)\right)$}

(Since X(z)=1,for all z∈X)

= min{$\sup_{a=bc}$ min{ A_μ(b),$\sup_{c=c_1c_2}$, A_μ (c_2)}}, ((A_μ∘X)∗ A_μ) $(xi - x(x' - i))$}

(Since A_μ is an intuitionistic fuzzy right X-subalgebra of X,

A_μ(bc)= $A_\mu(bc_1c_2) = A_\mu((bc_1)c_2) \geq A_\mu(c_2)$)

≤ min{$\sup_{a=bc}$ min{ X(b),A_μ(bc),},X$(xi - x(x' - i))$}

= min{$\sup_{a=bc}$ min{ A_μ(bc),X$(xi - x(x' - i))$}

= $A_\mu(bc)$

=$A_\mu(a)$

Thus (A_μ∘X∘A_μ)∩((A_μ∘X)∗ A_μ))⊆ A_μ.

Hence A_μ is an intuitionistic fuzzy bi − ideal of X.

Choose a,b,c,x,y,i,c_1,c_2,x_1,x_2,y_1, y_2 in X such that a = bc = $(xi - x(x' - i))$, c = c_1∘c_2, x =x_1,$x_2$and y = y_1, y_2. Then

(B_μ∘X∘B_μ)∪((B_μ∘X)∗B_μ))(a) = max{(B_μ∘X∘B_μ)(a), ((B_μ∘X)∗B_μ))(a)}

= max{$\inf_{a=bc}$ max {(B_μ)(b) , $(X^{o}B_\mu)(c)$}, ((B_μ∘X)∗ B_μ) $(xi - x(x' - i))$}

= max {$\inf_{a=bc}$ max {(B_μ)(b),$\inf_{c=c_1c_2}$ max {$X(c_1)$, B_μ (c_2)}, ((B_μ∘X)∗ B_μ) $(xi - x(x' - i))$}

(Since X(z)=1,for all z∈X)

= max{$\inf_{a=bc}$ max{ B_μ(b),$\inf_{c=c_1c_2}$, B_μ (c_2)}}, ((B_μ∘X)∗ B_μ) $(xi - x(x' - i))$}

(Since B_μ is an intuitionistic fuzzy right X-subalgebra of X,

B_μ (bc)= $B_\mu(b(c_1c_2)) = B_\mu((bc_1)c_2) \leq B_\mu(c_2)$)

≥ max{$\inf_{a=bc}$ max{ X(b), B_μ (bc),},X$(xi - x(x' - i))$}

= max{$\sup_{a=bc}$ max{ B_μ (bc),X$(xi - x(x' - i))$}

= $B_\mu(bc)$

=$B_\mu(a)$

Thus (B_μ∘X∘B_μ)∪((B_μ∘X)∗B_μ)) ⊇ B_μ.

Hence B_μ is an intuitionistic fuzzy bi − ideal of X.

Thus μ = (A_μ, B_μ) is an intuitionistic fuzzy bi − ideal of X.

Next we prove : μ is an intuitionistic fuzzy strong bi − ideal of X.

Choose a,b,c,c_1,c_2 ∈ X such that a = bc and c = c_1,c_2 Then

$$X^{\circ} A_\mu{}^{\circ} A_\mu(a) = \sup_{a=bc} \min\{ X(b),(A_\mu{}^{o} A_\mu)(c)\}$$

$$= \sup_{a=bc} \min\{X(b), \sup_{c=c_1c_2} \min\{ A_\mu(c_1), A_\mu(c_2)\}$$

$$= \sup_{a=bc} \min\{1, \sup_{c=c_1c_2} \min\{ A_\mu(c_1), A_\mu(c_2)\}$$

Since A_μ *is an* intuitionistic fuzzy left X-subalgebra of X

$A_\mu(bc) = A_\mu(bc_1c_2) = A_\mu((bc_1)c_2) \geq A_\mu(c_2))$

$\leq \sup_{a=bc} \min\{X(c_1), A_\mu(bc)\}$

$= \sup_{a=bc} \min\{1, A_\mu(bc)\}$

$= A_\mu(bc)$

$= A_\mu(a)$

Therefore $X^{\circ} A_\mu{}^{\circ} A_\mu(a) \subseteq A_\mu$

Hence A_μ is an intuitionistic fuzzy strong bi − ideal of X.

Choose a,b,c,c_1,c_2 ∈ X such that a = bc and c = c_1,c_2 Then

$$X^{\circ} B_\mu{}^{\circ} B_\mu(a) = \inf_{a=bc} \max\{ X(b),(B_\mu{}^{o} B_\mu)(c)\}$$

$$= \inf_{a=bc} \max\{X(b), \inf_{c=c_1c_2} max\, B_\mu(c_1), B_\mu(c_2)$$

$$= \inf_{a=bc} \max\{1, \inf_{c=c_1c_2} \max\{ B_\mu(c_1), B_\mu(c_2)\}\}$$

$$= \inf_{a=bc} \max\{ B_\mu(c_1), B_\mu(c_2)\}$$

Since B_μ *is an* intuitionistic fuzzy left X-subalgebra of X

$B_\mu(bc) = B_\mu(bc_1c_2) = B_\mu((bc_1)c_2) \leq B_\mu(c_2))$

$\geq \inf_{a=bc} \max\{X(c_1), B_\mu(bc)\}$

$= \inf_{a=bc} \max\{0, B_\mu(bc)\}$

$= B_\mu(bc)$

$= B_\mu(a)$

Therefore $X^{\circ} B_\mu{}^{\circ} B_\mu(a) \supseteq B_\mu$

Hence B_μ is an intuitionistic fuzzy strong bi − ideal of X.

Thus μ = (A_μ, B_μ) is an intuitionistic fuzzy strong bi − ideal of X.

Theorem: 3.6

Every left permutable intuitionistic fuzzy two-sided X subalgebra of X is an intuitionistic fuzzy strong bi-ideal of X.

Proof:

The proof is straight forward from the above Theorem 3.4 and Theorem 3.5.

Theorem: 3.7

Every left permutable intuitionistic fuzzy right ideal of X is an intuitionistic fuzzy strong bi-ideal of X.

Proof:

The proof is similar to that of Theorem 3.4.

Theorem: 3.8

Every intuitionistic fuzzy left ideal of X is an intuitionistic fuzzy strong bi-ideal of X.

Proof:

Let μ = (A_μ, B_μ) be an intuitionistic fuzzy left ideal of X.

To prove: μ = is an intuitionistic fuzzy strong bi-ideal of X.

First we prove: μ is an intuitionistic fuzzy bi-ideal of X.

Choose a,b,c,x,y,i,b_1,b_2,x_1,x_2,y_1, y_2 in X such that a = bc = $(xi - x(x' - i))$, b =$b_1 \circ b_2$, x =$x_1 \circ x_2$ and y = $y_1 \circ y_2$. Then

$(A_\mu \circ X \circ A_\mu) \cap ((A_\mu \circ X) * A_\mu)(a) = \min\{(A_\mu \circ X \circ A_\mu)(a), ((A_\mu \circ X) * A_\mu)(a)\}$

$= \min\{\sup_{a=bc} \min(A_\mu \circ X)(b), A_\mu(c)\}, ((A_\mu \circ X) * A_\mu)(xi - x(x' - i))\}$

$= \min\{\sup_{a=bc} \min\{(A_\mu \circ X)(b_1 b_2), (A_\mu(c)\}, \sup_{a=(xi-x(x'-i))} \min((A_\mu \circ X)(x), A_\mu\big(xi - x(x' - i)\big)(x')$

(since A∘**X** a⊆X and since A_μ is an intuitionistic fuzzy left ideal of X, $A_\mu\big(xi - x(x' - i)\big) \geq A_\mu(i)$

$\leq \min\{\sup_{a=bc} \min\{X(b_1 \circ b_2), X(c)\}, \{\sup_{a=(xi-x(x'-i))} \min\{X(x), X(x'), A_\mu(xi - x(x' - i))\}$

$= A_\mu(xi - x(x' - i))\}$

$= A_\mu(a)$.

Thus $(A_\mu \circ X \circ A_\mu) \cap ((A_\mu \circ X) * A_\mu)(a) \subseteq A_\mu$.

Hence A_μ is an intuitionistic fuzzy bi-ideal of X.

Choose a,b,c,x,y,i,b_1,b_2,x_1,x_2,y_1, y_2 in X such that a = bc

$=(xi - x(x' - i))$, b =$b_1 \circ b_2$, x =$x_1 \circ x_2$ and y = $y_1 \circ y_2$. Then

$(B_\mu \circ X \circ B_\mu) \cup ((B_\mu \circ X) * B_\mu)(a) = \max\{(B_\mu \circ X \circ B_\mu)(a), ((B_\mu \circ X) * B_\mu)(a)\}$

$= \max\{\inf_{a=bc} \max(B_\mu \circ X)(b), B_\mu(c)\}, ((B_\mu \circ X) * B_\mu)(xi - x(x' - i))\}$

$= \max \inf_{a=bc} \max(B_\mu \circ X)(b_1 \circ b_2), B_\mu(c)\}, \inf_{a=(xi-x(x'-i))} max\{(B_\mu \circ X(x), (B_\mu \circ X)(x'), \{(B_\mu \circ X)(i)$

(since $B_\mu \circ$**X** ⊇X and since B_μ is an intuitionistic fuzzy left ideal of X, $B_\mu\big(xi - x(x' - i)\big) \leq B_\mu(i)$

$\geq \max\{\inf_{a=bc} \max\{X(b_1 b_2), X(c)\}, \{\inf_{a=(xi-x(x'-i))} \max\{X(x), X(y), B_\mu(xi - x(x' - i))\}$

$=B_\mu(xi - x(x' - i))\}$

$=B_\mu(a)$.

Therefore $(B_\mu \circ X \circ B_\mu) \cup ((B_\mu \circ X) * B_\mu))(a) \supseteq B_\mu$.

Hence B_μ is an intuitionistic fuzzy bi-ideal of X.
Thus μ = (A_μ, B_μ) is an intuitionistic fuzzy bi − ideal of X.
Next we prove : μ is an intuitionistic fuzzy strong bi − ideal of X.

Choose a,b,c,b_1,b_2 ∈ X such that a = bc =bi-b(c-i).Then

$$X \circ A_\mu \circ A_\mu(a) = \sup_{a=bc} \min\{ (X \circ A_\mu)(b),\ A_\mu(c)\}$$

$$= \sup_{a=bc} \min\{ \sup_{b=b_1 b_2} \min\{ X(b_1), A_\mu(b_2) \}, A_\mu(c)\}$$

$$= \sup_{a=bc} \min\{ \sup_{b=b_1 b_2} \{A_\mu(b_2), A_\mu(c)\}$$

Since A_μ *is an* intuitionistic fuzzy left ideal of X

$A_\mu(bc) = A_\mu(bi - b(c - i)) > A_\mu(c)$ and $X(b_2) \geq A_\mu(b_2)$

$\leq \sup_{a=bc} \min\{ X(b_2), A_\mu(bi-b(c-i)\}$

$= \sup_{a=bc} A_\mu(bi-b(c-i))$

$= A_\mu(bc)$

$= A_\mu(a)$

Therefore $X \circ A_\mu \circ A_\mu(a) \subseteq A_\mu$

Hence A_μ is an intuitionistic fuzzy strong bi − ideal of X.

Choose a,b,c,b_1,b_2 ∈ X such that a = bc and b= $b_1 \circ b_2$, Then

$$X \circ B_\mu \circ B_\mu(a) = \inf_{a=bc} \max\{ (X \circ B_\mu)(b),\ B_\mu(c)\}$$

$$= \inf_{a=bc} \max\{ \inf_{b=b_1 b_2} \max\{ X(b_1),\ B_\mu(b_2) \}, B_\mu(c)\}$$

$$= \inf_{a=bc} \max\{ \inf_{b=b_1 b_2} \{B_\mu(b_2), B_\mu(c)\}$$

Since B_μ *is an* intuitionistic fuzzy left ideal of X

$B_\mu(bc) = B_\mu(bi - b(c - i)) \leq B_\mu(c)$ and $X(b_2) \leq B_\mu(b_2)$

$\geq \inf_{a=bc} \max\{ X(b_2), B_\mu(bi-b(c-i)\}$

$= \inf_{a=bc} B_\mu(bi-b(c-i))$

$= B_\mu(bc)$

$= B_\mu (a)$

Therefore X∘ B_μ∘ B_μ(a) $\supseteq B_\mu$

Hence B_μ is an intuitionistic fuzzy strong bi − ideal of X.

Thus μ = (A_μ, B_μ) is an intuitionistic fuzzy strong bi − ideal of X.

Theorem: 3.9
Every left permutable fuzzy ideal of X is a fuzzy strong biideal of X.

Proof:
The proof is straight forward from the Theorem 3.7 and Theorem 3.8.

Theorem: 3.10
Let μ = (A_μ, B_μ) is an intuitionistic fuzzy bi − ideal of X..Then A_μ (axy) ≥min{ A_μ (x), A_μ(y)} and $B_\mu \leq$ max{B_μ (x), B_μ (y)} for all a, x, y ∈X.
Proof:
Assume that (A_μ, B_μ) is an intuitionistic fuzzy strong bi-ideal of X. Then X∘ A_μ∘ A_μ(a) $\subseteq A_\mu$ and X∘ B_μ∘ B_μ(a) $\supseteq B_\mu$

.Let a, x and y be any element of X. Then
A_μ (axy) ≥ (X∘ A_μ∘ A_μ) (axy)
$= \sup_{axy=pq} \min\{ (X \circ A_\mu)(p), A_\mu(q)\}$
$\geq \min\{ (X \circ A_\mu)(ax), A_\mu(y)\}$
$= \min \{\sup_{ax=z_1z_2} \min\{X(z_1), A_\mu(z_2)\}, A_\mu(y)\}$
$\geq \min \{\min\{X(a), A_\mu(x)\}, A_\mu(y)\}$
$= \min\{\min\{1, A_\mu(x), A_\mu(y)\}$
$= \min \{A_\mu(x), A_\mu(y)\}$
This shows that A_μ (axy) ≥ min $\{A_\mu(x), A_\mu(y)\}$ for all a,x,y ϵ X.
B_μ (axy) ≤(X∘ B_μ∘ B_μ) (axy)
$= \inf_{axy=pq} \max\{ (X \circ B_\mu)(p), B_\mu(q)\}$
$\leq \max \{ (X \circ B_\mu)(ax), B_\mu(y)\}$
$= \max \{\inf_{ax=z_1z_2} \max\{X(z_1), B_\mu(z_2)\}, B_\mu(y)\}$
$\leq \max\{\max\{X(a), B_\mu(x)\}, B_\mu(y)\}$
$= \max\{\max\{1, B_\mu(x), B_\mu(y)\}$
$= \max \{ B_\mu(x), B_\mu(y)\}$
This shows that B_μ (axy) ≤ max $\{ B_\mu(x), B_\mu(y)\}$ for all a,x,y∈X

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