TWO WAREHOUSE INVENTORY MODEL FOR DECAYING ITEMS WITH TIME DEPENDENT DEMAND AND SHORTAGES

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ABSTARCT

This paper deals with a two-warehouse inventory problem for deteriorating items has been developed and analyzed. Effect of shortages is also taken into account. In real life situation of any enterprise, we can observe that sometime it is better to purchase more goods than the capacity of its own warehouse (OW). Therefore, there is a need to hire other warehouse to store the remaining goods. The warehouse, in which remaining goods are stored, is called known as the rented warehouse (RW). Our objective is to find the excess stock to be kept in RW so as to minimize the total system cost.

Key Words: Two Warehouse, Decaying Items, time dependent Demand, shortages.

1. INTRODUCTION

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Functioning Nowadays, managers are faced with the need to deliver a high level of service with minimal warehouse and inventory cost. In addition, with the improvement in information technology, it becomes possible to develop tools which can help managers to handle warehouse. Warehouse managers have to tackle problems which can be divided into two broad classes: warehouse management and inventory management problems. A warehouse is a large building where goods are stored, and where they may be catalogued, shipped, or received, depending upon the type. Though in the past, many warehouses, often located in industrial areas sometimes next to major shipping ports, were teeming with workers, the modern warehouse may be either completely or totally automated depending upon how advance the company is. Sometimes a manufacturing facility also has an attached warehouse, where their manufactured goods are stored until shipped.

Most of the classical inventory models assumed the utility of the inventory remains constant during their storage period. But in a real life, deterioration does occur in storage. The problem of deteriorating inventory has received considerable attention in recent years. Deterioration is defined as change, damage, decay, spoilage, evaporation, obsolescence, pilferage, and loss of utility or loss of marginal value of a commodity that results in decreasing usefulness from the original one. Most products such as medicine, blood, fish, alcohol, gasoline, vegetables and radioactive chemicals have finite shelf life, and start to deteriorate once they are replenished. Ongoing deteriorating inventory has been studies by several authors in recent decades.

Along with deterioration, another concrete problem receipts the responsiveness of canvassers these days is that of restricted storage competence. The major facets associated to this problem are lack of large storage space at vital market places, forcing the vendor to own a small warehouse at vital market places. However, in order to take plus of striking price discounts vacant on bulk acquisitions, or in prevent progression in demand with time, it may be lucrative for the vendor to order an amount that outshines the capability of his own warehouse. Storage of deteriorating items entails expressly prepared storage ability so as to shrink the quantity of deterioration. The cost of building such storage ability is usually un due. Hence, it may be difficult for the vendor to have such storage ability of his own at the trade orifice. This gratifies the requisite of another storage space, providing the essential conditions. Sharma (1987) [12] extended his earlier model to the case of infinite replenishment rate with shortages. In his study, the deterioration item was first stowed in the OW and the spare quantity was stowed in the RW. Then in the inventory model an infinite replenishment rate was measured and shortages were permissible Yang (2004) [13] provided a two-warehouse inventory model for a single item with constant demand and shortages under inflation. Instead of the classical view of accumulating shortages at the end of each replenishment cycle, an alternative model in which each cycle begins with shortages has been proposed here. Zhou and Yang (2005) [15] recognized a two-warehouse inventory model on the basis of minimum cost. In the model the factors such as a constant deteriorating rate and demand trade credits were taken into consideration. Dye *et al.* (2008) [7] studied the two-warehouse inventory problem from the perception of traders. In their study, the storage capability of OW which is always situated at a busy market place is limited. Benkherouf, L., (1997) [1] develop an inventory model for deteriorating items with shortages and stock dependent demand under inflation for two shops. Ghosh and Chakrabarty (2009) [4] derived a deterministic two warehouse inventory model for deteriorating items with stock dependent demand and shortages under the conditions of permissible delay. Bhunia and Maiti (2001) [9] derived two storage inventory model of a deteriorating item with variable demand under partial credit period. Pakkala and Achary (1992) [11] derived effect of deterioration on two warehouse inventory model with imperfect quality. Yang, H.L., (2006) [14] develop two warehouse inventory models with partial backlogging.

The customary parameters such as holding cost, set up cost and demand rate in traditional inventory models generally are presumed to be fixed. Consequently, traditional models have some discrepancy with real life situations. This issue has roused lots of researchers to alter the inventory models to contest real world conditions. The assumption, holding cost is always fixed is not true in general and thus to represents real-life situations, in this chapter holding cost is presumed to be a varying function of time. In inaugurating of inventory models, an assortment of functions describing holding cost was careful by specific researchers like Goswami and Chaudhuri (1992, 1998) [5,6], Bhunia and Maiti (1994, 1998) [2,3], Benkherouf (1997) [1], Kar et al. (2001) [9] presented an inventory model for perishable items with variable holding cost and partial backlogging. Mishra, R.B., (1975-a) [10] presented an optimization of inventory model for decaying item with variable holding cost and power demand. Jaggi, C.K., Verma, P., (2010) [8] develop An EOQ model with variable holding cost and partial backlogging under credit limit policy and cash discount.

In this paper we develop inventory model for the two-warehouses with variable holding cost and demand rate is a general ramp-type function of time, deterioration is taken into consideration with constant rate. Replenishment rate is infinite. Shortages are permissible and a constant fraction of shortages is presumed to be backlogged. Numerical examples are presented to explain the model. Sensitivity and convexity analysis is carried out with respect to the parameters of the model.

2. Assumptions and Notations

The model is developed under the following assumptions:

- The demand rate is a ramp-type function of time t given by

$$D(t)=\begin{cases} f(t), & if \quad t \le u \\ f(u), & if \quad t > u \end{cases}$$

Where f (t), t∊ (0, u) is any increasing twice differentiable function, and u is a constant parameter denoting the time epoch up to which the general demand rate f(t) increases.

- Demand is satisfied initially from goods stored in RW and continues with those in OW once inventory stored at RW is exhausted. This implies that $t_1 < T_1$.
- Lead-time is zero, replenishment is instantaneous.
- Shortages are partially backlogged at a constant rate.
- Once an item deteriorates, it is withdrawn immediately from the stock. Hence holding cost is incurred only for good items.
- Transportation cost from RW to OW and transportation time are negligible.
- RW has infinite capacity.
- Holding cost assumed as a variable function of time and is given by H(t) = $(H_1 + tH_2)$ for OW, Where $H_1 > 0$, $H_2 > 0$ and F(t) = $(F_1 + tF_2)$ for RW, Where $F_1 > 0$, $F_2 > 0$
- Holding cost per unit time at RW is greater than that at OW.

The notations used in developing the model are as follows:

L_2 Denotes the inventory system with two warehouses.

The known parameters are:

H_1, H_2 Holding cost parameters of OW.

F_1, F_2 Holding cost parameters of RW.

C_1 Shortage cost per unit short per unit time.

C_2 Opportunity cost per unit of lost sales.

C_3 Deterioration cost per unit deteriorates.

B Fraction of shortages to be backlogged.

W Storages capacity of OW.

T The cycle length.

Π Total cost per cycle for the L_2 system.

The decision variables are**:**

t_1 Time epoch at which the inventory level at RW becomes zero.

T_1 Time epoch at which the inventory level at OW becomes zero.

S Initial inventory level for the L_2 -system.

3. Mathematical Modeling

In this section, we present analysis for the L_2 system.

At time t=0, the amount of inventory ordered arrives the system, a part of which is used to satisfy the shortages of the previous epoch. S units of inventory persist in the system, out of which W units are retained in OW and the remaining (S –W) units are retained in RW. Depending upon the relative values of u, t_1 and T_1, there are three distinct cases:

Case1: u≤ t_1 ; Case 2: t_1 <u≤ T_1 ; Case 3: T_1 <u .

Let $\prod_i$ denote the total cost for case i; i=1, 2, 3.We now discuss each of these cases separately.

3.3.1 Case 1: $u \le t_1$.

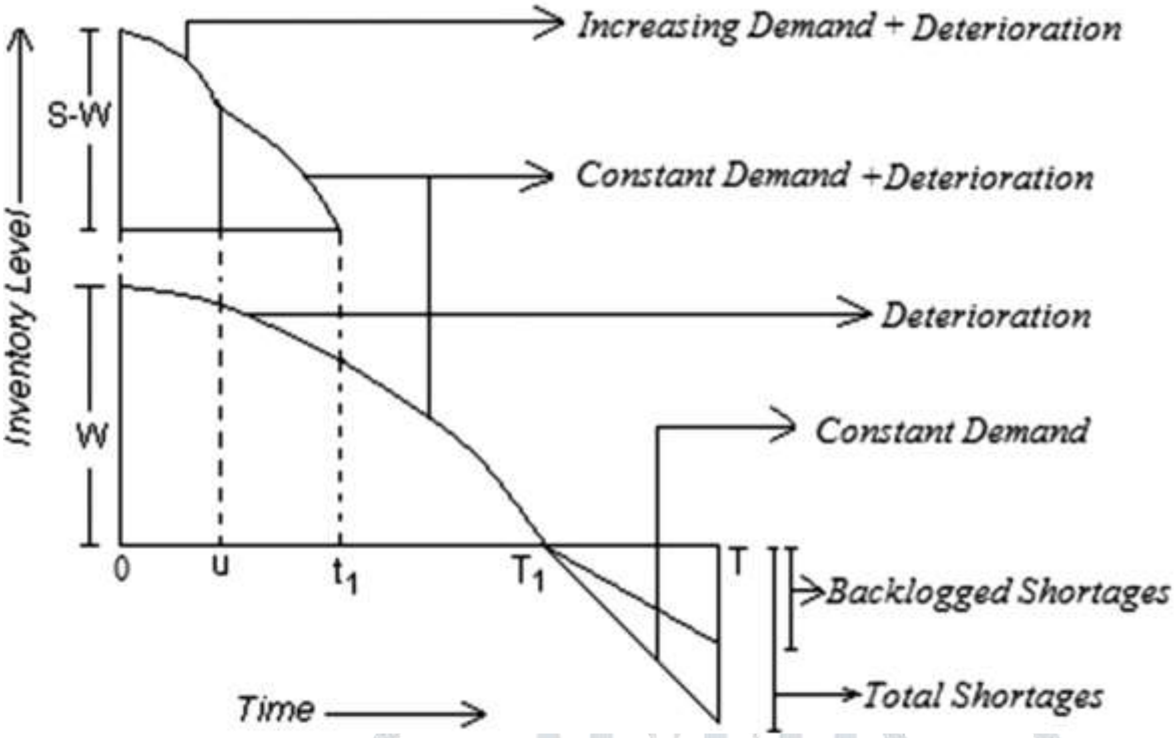

Figure 3.1: Time inventory graph for L_2 system case 1.

In this case, the inventory level at RW decreases because of the increasing demand rate and constant deterioration rate in the interval (0,u) and because of the constant demand rate and constant deterioration rate in the interval (u, t_1) (figure 3.1).

Hence, the inventory level in RW for t ϵ (0, t_1) satisfies the differential equations:

$$\frac{dI_1(t)}{dt}+\theta I_1(t)=-f(t); \qquad 0\le t\le u \qquad ...(3.1)$$

$$\frac{dI_2(t)}{dt}+\theta I_2(t)=-f(u); \qquad u\le t\le t_1 \qquad ...(3.2)$$

With the boundary conditions $I_1(u)=I_2(u)\ and\ I_2(t_1)=0$, the solutions of the differential equations (3.1) and (3.2) are given, respectively, by

$$I_1(t)=-\int f(t)dt+\frac{f(u)}{\theta}(e^{\theta(t_1-t)}-e^{\theta(u-t)})+\int f(u)e^{\theta(u-t)}du; \qquad 0\le t\le u \qquad ...(3.3)$$

and $I_2(t)=\frac{f(u)}{\theta}\left(e^{\theta(t_1-t)}-1\right); \qquad u\le t\le t_1 \qquad ...(3.4)$ further, since $I_1(0)=$ $S-W$, we get

$$S=W+\frac{f(u)}{\theta}\left(e^{\theta t_1}-e^{\theta u}\right)+\int f(u)e^{\theta u}du \qquad ...(3.5)$$

Inventory level at OW decreases, due to deterioration over the interval (0, t_1), and due to constant demand rate and constant deterioration rate over the interval (t_1, T_1). Thus it satisfies the following differential equation:

$$\frac{dI_3(t)}{dt}+\theta I_3(t)=0; \qquad 0\le t\le t_1 \qquad ...(3.6)$$

$$\frac{dI_4(t)}{dt}+\theta I_4(t)=-f(u); \qquad t_1\le t\le T_1 \qquad ...(3.7)$$

with boundary conditions $I_3(0)=W$ and $I_4(T_1)=0$ .The solutions of (3.6) and (3.7) are, respectively

$$I_3(t)=We^{-\theta t}; \qquad 0\le t\le t_1 \qquad ...(3.8)$$

$$I_4(t)=\frac{f(u)}{\theta}\left(e^{\theta(T_1-t)}-1\right); \qquad t_1\le t\le T_1 \qquad ...(3.9)$$

Further, since $I_4(t_1)=I_3(t_1),$ we obtain

W = $\frac{f(u)}{\theta}\left(e^{\theta T_1}-e^{\theta t_1}\right)$.

This implies that

$$T_1=\frac{1}{\theta}\log\left(e^{\theta t_1}+\frac{\theta W}{f(u)}\right)=T_1^1(t_1).$$

Hence

$$\frac{dT_1}{dt_1}=\frac{dT_1^1(t_1)}{dt_1}=e^{-\theta\left(T_1^1(t_1)-t_1\right)}<1 \qquad ...(3.10)$$

The amount of backlogged shortage during the interval (T_1,T) satisfies the differential equation

$$\frac{dI_5(t)}{dt}=-Bf(u); \qquad T_1\le t\le T \qquad ...(3.11)$$

Using the boundary condition $I_5(t)=0,$ the solution of the differential equation (3.11) is given by

$$I_5(t)=-Bf(u)(t-T_1); \quad T_1\le t\le T \qquad ...(3.12)$$

The amount of lost sales during the interval (T_1,T) is

L = $\int_{T_1}^{T}(1-B)\,f(u)\,dt$ …(3.13)

L = $(1-B)\,f(u)(T-T_1).$

The total demand during the interval $(0,T_1)\ is \int_0^u f(t)\,dt+\int_u^{T_1} f(u)\,dt.$

Hence amount of inventory deteriorated during $(0,T_1)$ is $D=S-\left(\int_0^u f(t)\,dt+\int_u^{T_1} f(u)\,dt\right).$

Thus, the total cost during the cycle is given by

$$\Pi_1(t_1)=[(F_1+tF_2)\left(\int_0^u I_1(t)\,dt+\int_u^{t_1} I_2(t)\,dt\right)+(H_1+H_2t)\left(\int_0^{t_1} I_3(t)\,dt+\int_{t_1}^{T_1} I_4(t)\,dt\right)$$

$$+C_1\int_{T_1}^{T}\left(-I_5(t)\right)dt+C_2L+C_3D \qquad ...(3.14)$$

$$\Pi_1(t_1) = (F_1 + tF_2)[\int_0^u \{-\int f(t)\,dt + \frac{f(u)}{\theta}\left(e^{\theta(t_1-t)} - e^{\theta(u-t)}\right) + \int f(u) e^{\theta(u-t)} du\} dt +$$

$$\int_u^{t_1} \frac{f(u)}{\theta}\left(e^{\theta(t_1-t)} - 1\right)dt] + (H_1 + H_2 t)[\int_0^{t_1} \frac{f(u)}{\theta}\left(e^{\theta(T_1-t)} - e^{\theta(t_1-t)}\right)dt + \int_{t_1}^{T_1} \frac{f(u)}{\theta}\left(e^{\theta(T_1-t)} - 1\right)dt]$$

$$+C_1 \int_{T_1}^{T} Bf(u)(t - T_1)dt + C_2(1-B) f(u)(T - T_1)$$

$$+C_3[\frac{f(u)}{\theta}(e^{\theta T_1} - e^{\theta u}) + \int f(u) e^{\theta u} du - \int_0^u f(t)\,dt - \int_u^{T_1} f(u)\,dt]$$

$$\frac{d\Pi_1(t_1)}{dt_1} = F_1(\frac{f(u)}{\theta}(e^{\theta t_1} - 1)) + F_2(\frac{-f(u)}{\theta^2}(\theta t_1 + 1) + \frac{f(u)}{\theta^2} e^{\theta t_1}) + H_1(\frac{f(u)}{\theta} - \frac{f(u)}{\theta}\frac{dT_1}{dt_1})$$

$$+H_2(\frac{f(u)}{\theta^2}(\theta t_1 + 1) - \frac{f(u)}{\theta^2}\frac{dT_1}{dt_1}(\theta T_1 + 1)) + C_1(Bf(u)\frac{dT_1}{dt_1}(T_1 - T)) + C_2((B-1) f(u)\frac{dT_1}{dt_1})$$

$$+C_3(f(u) e^{\theta t_1} - f(u)\frac{dT_1}{dt_1})$$

3.3.2 Case 2: $t_1 < u \le T_1$.

In this case, the inventory level at RW becomes zero before the demand stabilizes (figure 3.2). Thus, the inventory at RW decreases only because of the increasing demand and constant deterioration rate in the interval (0, t_1) where t_1 is the epoch where the inventory level in RW becomes zero. Hence, the instantaneous change in inventory level in RW at any time t in the interval (0, t_1) is given by the following differential equation:

$$\frac{dI_1(t)}{dt} + \theta I_1(t) = -f(t), \qquad 0 \le t \le t_1 \qquad ...(3.15)$$

With the boundary condition $I_1(t) = 0$,the solution of differential equation (3.15) is given by

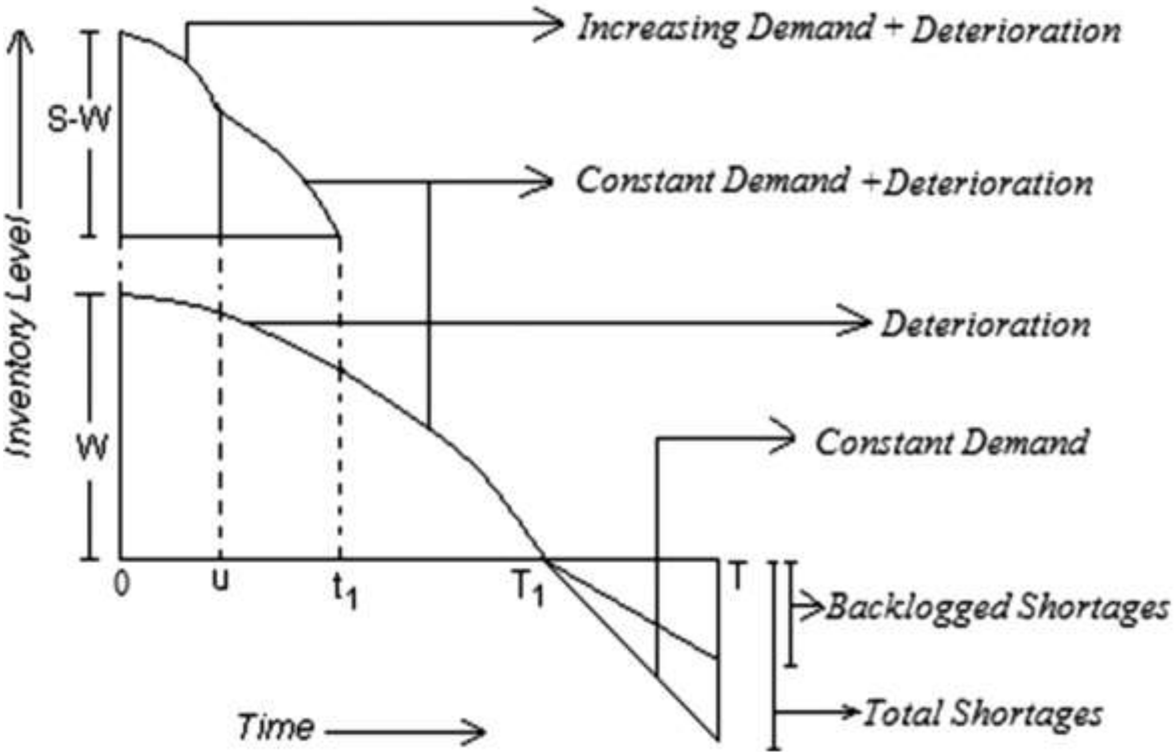

Figure 3.2: Time inventory graph for L_2 **-system case 2.**

$$I_1(t) = -\int f(t)\,dt + \int f(t_1)e^{\theta(t_1-t)}dt_1 \qquad ...(3.16)$$

Further, since $I_1(0) =$ S-W, we get

$$S = W + \int f(t_1)e^{\theta t_1}dt_1 \qquad ...(3.17)$$

Similar opinions applied to OW over the time interval $(0,T_1)$ lead to the following differential equations

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = 0; \qquad 0 \le t \le t_1 \qquad ...(3.18)$$

$$\frac{dI_3(t)}{dt} + \theta I_3(t) = -f(t); \quad t_1 \le t \le u \qquad ...(3.19)$$

$$\frac{dI_4(t)}{dt} + \theta I_4(t) = -f(u); \quad u \le t \le T_1 \qquad ...(3.20)$$

With boundary conditions $I_2(0) = W, I_2(t_1) = I_3(t_1)$ *and* $I_4(T_1) = 0,$ the solutions of these differential equations are respectively

$$I_2(t) = W e^{-\theta t}; \qquad 0 \le t \le t \qquad ...(3.21)$$

$$I_3(t) = -\int f(t)\,dt + We^{-\theta t} + \int f(t_1)e^{\theta(t_1-t)}dt_1; \qquad t_1 \le t \le u \qquad ...(3.22)$$

$$I_4(t) = \frac{f(u)}{\theta}\left(e^{\theta(T_1-t)} - 1\right); \qquad u \le t \le T_1. \qquad ...(3.23)$$

Further, since $I_3(u) = I_4(u),$ we get

$$-\int f(u)\,du + We^{-\theta u} + \int f(t_1)e^{\theta(t_1-u)}dt_1 = \frac{f(u)}{\theta}\left(e^{\theta(T_1-u)} - 1\right)$$

$$W = \frac{f(u)}{\theta}\left(e^{\theta T_1} - e^{\theta u}\right) + \int f(u)e^{\theta u}du - \int f(t_1)e^{\theta t_1}dt_1 \qquad ...(3.24)$$

From this relation, it is evident that

$$T_1 = \frac{1}{\theta}\log\left(e^{\theta u} + \frac{\theta}{f(u)}\left(-\int f(u)e^{\theta u}du + W + \int f(t_1)e^{\theta t_1}dt_1\right)\right) = T_1^2(t_1) \qquad ...(3.25)$$

This implies that

$$\frac{dT_1}{dt_1} = \frac{dT_1^2(t_1)}{dt_1} = \frac{f(t_1)e^{\theta t_1}}{f(u)e^{\theta T_1}} \qquad ...(3.26)$$

The backlogged shortages at tε (T_1, T) is the same as in case 1, i.e.

$$I_5(t) = -Bf(u)(t - T_1); \qquad T_1 \le t \le T \qquad ...(3.27)$$

And the amount of lost sales during the interval (T_1, T) is

$$L = \int_{T_1}^{T}(1-B)\,f(u)\,dt$$

$$L = (1-B)\,f(u)(T - T_1) \qquad ...(3.28)$$

the amount of deteriorated inventory during the interval $(0, T_1)$ is

$$D = S - \left(\int_0^u f(t)\,dt + \int_u^{T_1} f(u)\,dt\right)$$

$$D = \frac{f(u)}{\theta}\left(e^{\theta T_1} - e^{\theta u}\right) + \int f(u)e^{\theta u}du - \left(\int_0^u f(t)\,dt + \int_u^{T_1} f(u)\,dt\right)$$

Thus, the total cost for the cycle is given by

$$\Pi_2(t_1)=(F_1+tF_2)\left(\int_0^{t_1} I_1(t)\,dt\right)+(H_1+tH_2)\left(\int_0^{t_1} I_2(t)\,dt+\int_{t_1}^{u} I_3(t)\,dt+\int_u^{T_1} I_4(t)\,dt\right)$$

$$+C_1\left(\int_{T_1}^{T}(-I_5(t))\,dt\right)+C_2L+C_3D \qquad ...(3.29)$$

$$\Pi_2(t_1)=(F_1+tF_2)\left(\int_0^{t_1}\left(-\int f(t)\,dt+\int f(t_1)e^{\theta(t_1-t)}dt_1\right)\right)+$$

$$(H_1+tH_2)\left(\int_0^{t_1}\left(\frac{f(u)}{\theta}\left(e^{\theta(T_1-t)}-e^{\theta(u-t)}\right)+\int f(u)e^{\theta(u-t)}du-\int f(t_1)e^{\theta(t_1-t)}dt_1\right)dt\right)$$

$$+(H_1+tH_2)(\int_{t_1}^{u}\left(-\int f(t)dt+\frac{f(u)}{\theta}\left(e^{\theta(T_1-t)}-e^{\theta(u-t)}\right)+\int f(u)e^{\theta(u-t)}du\right)dt+\int_u^{T_1}\frac{f(u)}{\theta}\left(e^{\theta(T_1-t)}-1\right)dt)$$

$$+C_1\int_{T_1}^{T}Bf(u)(t-T_1)\,dt+C_2(1-B)f(u)(T-T_1)+C_3(\frac{f(u)}{\theta}\left(e^{\theta(T_1)}-e^{\theta u}\right)+\int f(u)e^{\theta u}du-$$

$$\int_0^{u} f(t)\,dt-\int_u^{T_1} f(u)\,dt)$$

$$\frac{\Pi_2(t_1)}{dt_1}=F_1(\frac{f(t_1)}{\theta}\left(e^{\theta t_1}-1\right))+F_2(\frac{-f(t_1)}{\theta^2}(\theta t_1+1)+\frac{f(t_1)}{\theta^2}e^{\theta t_1})+H_1\left(\frac{f(t_1)}{\theta}-\frac{f(u)}{\theta}\frac{dT_1}{dt_1}\right)$$

$$+H_2\left(\frac{f(t_1)}{\theta^2}(\theta t_1+1)-\frac{f(u)}{\theta^2}\frac{dT_1}{dt_1}(\theta T_1+1)\right)+C_1Bf(u)\frac{dT_1}{dt_1}(T_1-T)+C_2(B-1)f(u)\frac{dT_1}{dt_1}+$$

$$C_3\left(f(t_1)e^{\theta t_1}-f(u)\frac{dT_1}{dt_1}\right)$$

3.3.3 Case 3: $T_1 < u$

In this case, the progression of stock level in the system is illustrated by Fig. 3.3.The inventory levels at both RW and OW becomes zero before the demand alleviates. Thus, the inventory levels at both the warehouses decrease due to increasing demand rate and constant deterioration rate.

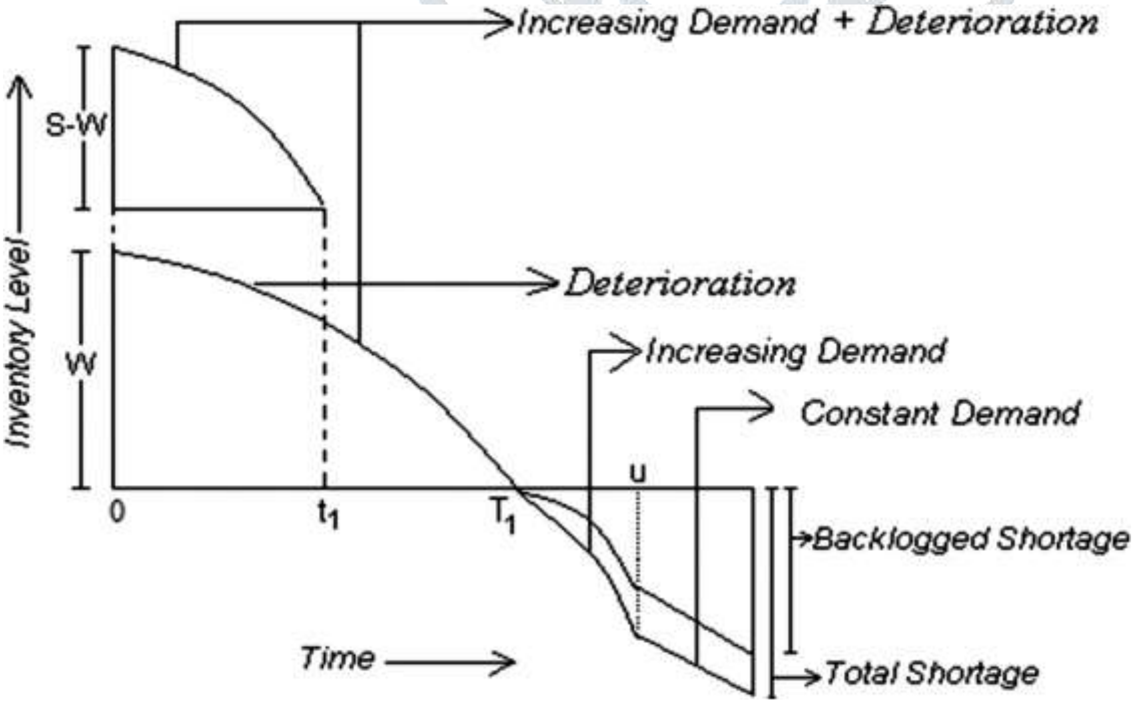

Figure 3.3: Time inventory graph for Case 3 of L_2 - system.

The differential equation for the inventory level at RW over $(0,t_1)$ is given by

$$\frac{dI_1(t)}{dt}+\theta I_1(t)=-f(t); \qquad 0\le t\le t_1 \qquad ...(3.30)$$

With the boundary condition $I_1(t_1)=0$, the solution of differential equation (3.30) is given by

$$I_1(t)=-\int f(t)\,dt+\int f(t_1)e^{\theta(t_1-t)}dt_1 \qquad ...(3.31)$$

Substituting $I_1(0)=S-W$, we get

$$S=W+\int f(t_1)e^{\theta(t_1)}dt_1 \qquad ...(3.32)$$

At OW, the inventory level decreases due to deterioration over (0, t_1) and over (t_1, T_1) due to increasing demand rate and constant deterioration. Thus, the differential equations related to inventory levels in the OW in the two sub intervals are

$$\frac{dI_2(t)}{dt}+\theta I_2(t)=0; \qquad 0\le t\le t_1 \qquad ...(3.33)$$

$$\frac{dI_3(t)}{dt}+\theta I_3(t)=-f(t); \qquad t_1\le t\le T_1 \qquad ...(3.34)$$

With boundary conditions $I_2(0)=W$ and $I_3(T_1)=0$, the solutions of these differential equations are:

$$I_2(t)=We^{-\theta t}; \qquad 0\le t\le t_1 \qquad ...(3.35)$$

and

$$I_3(t)=-\int f(t)\,dt+\int f(T_1)e^{\theta(T_1-t)}dT_1; \qquad t_1\le t\le T_1 \qquad ...(3.36)$$

Further, since $I_2(t_1)=I_3(t_1)$, we get

$$W=-\int f(t_1)e^{\theta t_1}dt_1+\int f(T_1)e^{\theta T_1}dT_1 \qquad ...(3.37)$$

From this relation we obtain $T_1=T_1^3(t_1)$. Differentiating T_1 with respect to t_1 and simplifying, we get

$$\frac{dT_1}{dt_1}=\frac{dT_1^3(t_1)}{dt_1}=\frac{f(t_1)e^{\theta t_1}}{f(T_1)e^{\theta T_1}}<1 \qquad ...(3.38)$$

In this case, the shortage period (T_1, T) comprises of two different demand functions in the two sub intervals (T_1,u) and (u,T), where in first sub interval, demand rate increases with time while in the second, it is constant. Thus, the backlogged inventory levels satisfy the following differential equations:

$$\frac{dI_4(t)}{dt}=-Bf(t); \qquad T_1\le t\le u \qquad ...(3.39)$$

$$\frac{dI_5(t)}{dt}=-Bf(u); \qquad u\le t\le T \qquad ...(3.40)$$

With boundary conditions $I_4(T_1)=0, I_4(u)=I_5(u)$ and the solutions are

$$I_4(t)=-B\int f(t)\,dt+B\int f(T_1)\,dT_1; \qquad T_1\le t\le u \qquad ...(3.41)$$

$$I_5(t)=Bf(u)(u-t)-B\int f(u)\,du; \qquad u\le t\le T \qquad ...(3.42)$$

The amount of lost

sales during the interval (T_1, T) in this case is given by

$$L = (1-B)\left(\int_{T_1}^{u} f(t)\,dt + \int_{u}^{T} f(u)\,dt\right) \quad ...(3.43)$$

Total demand during the interval $(0, T_1)$ is $\int_0^{T_1} f(t)\,dt$.

Hence the amount of inventory deteriorated during the interval $(0, T_1)$ is

$$D = S - \int_0^{T_1} f(t)\,dt$$

Thus the total cost during the cycle is given by

$$\Pi_3(t_1) = (F_1 + tF_2)\left(\int_0^{t_1} I_1(t)\,dt\right) + (H_1 + H_2 t)\left(\int_0^{t_1} I_2(t)\,dt + \int_{t_1}^{T_1} I_3(t)\,dt\right) +$$

$$C_1\left(\int_{T_1}^{u} (-I_4(t))\,dt + \int_{u}^{T} (-I_5(t))\,dt\right) + C_2 L + C_3 D \quad ...(3.44)$$

$$\Pi_3(t_1) = (F_1 + tF_2)\int_0^{t_1}\left(-\int f(t)\,dt + \int f(t_1) e^{\theta(t_1 - t)} dt_1\right)dt + (H_1 + H_2 t)$$

$$\left(\left(\int_0^{t_1}\left(-\int f(t_1)\,dt_1 + \int f(T_1) e^{\theta(T_1 - t_1)} dT_1\right)dt\right) + \left(\int_{t_1}^{T_1}\left(-\int f(t)\,dt + \int f(T_1) e^{\theta(T_1 - t)} dT_1\right)dt\right)\right)$$

$$+C_1\left(\int_{T_1}^{u}\left(B\int f(t)\,dt - B\int f(T_1)\,dT_1\right)dt + \int_{u}^{T}\left(Bf(u)(t-u) + B\int f(u)\,du\right)dt\right)$$

$$+C_2\left(\int_{T_1}^{u}(1-B) f(t)\,dt + \int_{u}^{T}(1-B) f(u)dt\right) + C_3\left(\int f(T_1) e^{\theta T_1} dT_1 - \int_0^{T_1} f(t)\,dt\right)$$

$$\frac{d\Pi_3(t_1)}{dt_1} = F_1\left(\frac{f(t_1)}{\theta}\left(e^{\theta t_1} - 1\right)\right) + F_2\left(\frac{-f(t_1)}{\theta^2}(\theta t_1 + 1) + \frac{f(t_1)}{\theta^2} e^{\theta t_1}\right) + H_1\left(\frac{f(t_1)}{\theta} - \frac{f(T_1)}{\theta}\frac{dT_1}{dt_1}\right)$$

$$+H_2\left(\frac{f(t_1)}{\theta^2}(\theta t_1 + 1) - \frac{f(T_1)}{\theta^2}\frac{dT_1}{dt_1}(\theta T_1 + 1)\right) + C_1 B f(T_1)(T_1 - u)\frac{dT_1}{dt_1} + C_2(B-1) f(T_1)\frac{dT_1}{dt_1}$$

$$+C_3\left(f(t_1) e^{\theta t_1} - f(T_1)\frac{dT_1}{dt_1}\right)$$

Combining (3.14), (3.29) and (3.44), the cost function $\Pi(t_1)$ of the problem results in the following three branches function

$$\Pi(t_1) = \begin{cases} \Pi_1(t_1), & u \le t_1 \le T_1 \\ \Pi_2(t_1), & t_1 < u \le T_1 \\ \Pi_3(t_1), & t_1 < T_1 \le u \end{cases} \quad 1$$

Corresponding to the three cases. In the above expression for the function $\Pi(t_1)$ the T_1 appearing in the three branches are different and equal to T_1^1, T_1^2 and T_1^3 respectively.

3.4 The Optimal Policy for the Two Warehouses

We now proceed to find the first and second order derivatives of this objective function. The first derivatives with respect to the decision variable t_1 for the three branches are

$$\frac{d\Pi_1(t_1)}{dt_1} = f(u)G_1(t_1), \quad \ldots(3.45) \qquad \frac{d\Pi_2(t_1)}{dt_1} = f(t_1)G_2(t_1), \quad \ldots(3.46)$$

$$\frac{d\Pi_3(t_1)}{dt_1} = f(t_1)G_3(t_1), \quad \ldots(3.47)$$

respectively, where

$$G_1(t_1) = \left(\frac{F_1\theta + F_2 + \theta^2 C_3}{\theta^2}\right)\left(e^{\theta t_1} - 1\right) + \left(\frac{H_1\theta + (\theta t_1 + 1)H_2 + \theta^2 C_3 - \theta t_1 F_2}{\theta^2}\right)$$
$$-\left(\left(\frac{H_1\theta + (\theta t_1 + 1)H_2 + \theta^2 C_3}{\theta^2}\right) + C_1 B\left(T - T_1^1\right) + C_2(1-B)\right)\frac{dT_1^1}{dt_1}$$

$$G_2(t_1) = \left(\frac{F_1\theta + F_2 + \theta^2 C_3}{\theta^2}\right)\left(e^{\theta t_1} - 1\right) + \left(\frac{H_1\theta + (\theta t_1 + 1)H_2 + \theta^2 C_3 - \theta t_1 F_2}{\theta^2}\right)$$
$$-\left(\left(\frac{H_1\theta + (\theta t_1 + 1)H_2 + \theta^2 C_3}{\theta^2}\right) + C_1 B\left(T - T_1^2\right) + C_2(1-B)\right)\frac{f(u)}{f(t_1)}\frac{dT_1^2}{dt_1}$$

$$G_3(t_1) = \left(\frac{F_1\theta + F_2 + \theta^2 C_3}{\theta^2}\right)\left(e^{\theta t_1} - 1\right) + \left(\frac{H_1\theta + (\theta t_1 + 1)H_2 + \theta^2 C_3 - \theta t_1 F_2}{\theta^2}\right)$$
$$-\left(\left(\frac{H_1\theta + (\theta t_1 + 1)H_2 + \theta^2 C_3}{\theta^2}\right) + C_1 B\left(T - T_1^3\right) + C_2(1-B)\right)\frac{f(T_1^3)}{f(t_1)}\frac{dT_1^3}{dt_1}$$

The second order derivatives for the cost functions in the three branches are respectively

$$\frac{d^2\Pi_1(t_1)}{dt_1^2} = f(u)\frac{dG_1(t_1)}{dt_1}, \quad \ldots(3.48)$$

$$\frac{d^2\Pi_2(t_1)}{dt_1^2} = f'(t_1)G_2(t_1) + f(t_1)\frac{dG_2(t_1)}{dt_1}, \quad \ldots(3.49)$$

$$\frac{d^2\Pi_3(t_1)}{dt_1^2} = f'(t_1)G_3(t_1) + f(t_1)\frac{dG_3(t_1)}{dt_1}, \quad \ldots(3.50)$$

Since f (t_1)>0, the first order condition for optimum in each of the branches relies on the functions $G_i(t_1)$, i = 1, 2, 3. For each of these functions, there may exist value(s) of t_1, say $t_{1(i)}^*$ such that $G_i(t_{1(i)}^*)$=0, or we may have $G_i(t_1)$>0 or $G_i(t_1)$ < 0 for all $t_1$values, in the intervals in which these functions are defined. Depending upon the situation which rises, we may have a unique optimum solution which is an interior point in the domain of definition of the consistent function or we may have optima which are at the boundary points. We found it impossible to auxiliary analytically scrutinize either the existence of $t_{1(i)}^*$ value(s) or the positive or negative character of $G_i(t_1)$ and so the comportment of $\Pi_i(t_1)$. However, inclusive dexterity gathered from computational work done with credible set of data specifies the existence of unique point $t_{1(i)}^*$ for one of the three branches, while the cost function corresponding to other two branches divulges monotonic comportment. Although this is the usual case, optimal solutions at the boundary points indisputably exist for extreme cases and are not to be omitted. In order to sightsee the question of optimality a bit additional, we assume that for some i = 1, 2, 3 there exists at least one point t_1^* such that $G_i(t_{1(i)}^*)$ = 0.

3.5 Algorithm for Optimal Policy

Based on the above analysis and algorithms, we state the algorithm which permits us to acquire the inclusive optimal policy for the inventory system and permit us to select the optimal system.

Step 1: Input all the parameters.

Step 2: Solve the model for the L_2 -system as follows:

Calculate the optimum cost for the L_2 -system

as $\Pi^* = \min\{\Pi_1(t_1), \Pi_2(t_2), \Pi_3(t_3)\}$

Step 3: Stop the process

3.6 Numerical Examples

As an illustration of traditional model, a numerical example is accessible. To perform the numerical analysis, data have been taken from the literature in appropriate units.

Example 3.1: In this example, we consider f (t) =Ae^{bt}

To illustrate the algorithm, we consider the following parameter values:

F_1 = 0.3 per unit per day, $F_2 = 0.1$ per unit per day, $H_1 = 0.2$ per unit per day, $H_2 = 0.11$ per unit per day, C_1 = 1.5 per unit per day, C_2 = 2.0 per unit per day, $C_3 = 1.1$ per unit per day, A = 30, b = 4.5, u = 0.5, B = 0.6, W = 50, T = 20 days, $\theta = 0.01$

We get,

For **Case: 1**, $t_1^* = 1.55524$, $T_1^* = 1.72804$, $S^* = 1868.53$, $\Pi_1(t_1)$ =49553.8

Case: 2, $t_1^* = 1.36244$, $T_1^* = 10.75$, $S^* = 3151.04$, $\Pi_2(t_1)$ =23658.6

Case: 3, $t_1^* = 0.56783$, $T_1^* = 0.669331$, $S^* = 136.126$, $\Pi_1(t_1)$ =54247.3

The optimum cost for the L_2-system is Π^* =23658.6. The optimal values of the decision variables are

t_1^*=1.36244, T_1^*=10.75, S^* = 3151.04

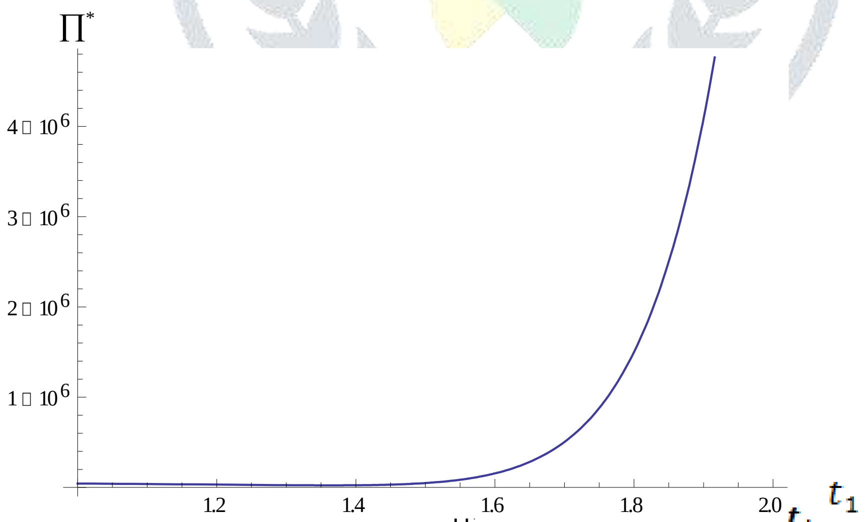

Figure 3.4 Behaviour of the total average profit functions Π with respect to time t_1.

3.7 Sensitivity Analysis

Table 1 shows sensitivity of the optimal solution of Example 1 with respect to the parameters of the model. The analysis is conceded out by changing the values of each of the parameters used in Example 1 by -50%,-40%,-30%,-20%,-10%,+10%, +20%,+30%,+40% and +50%, taking one parameter at a time, while fixing others at their original values. The percentage change in the values of t_1^*, S^*, T_1^* and Π^* is taken as a ration of sensitivity. From Table 1, the following implications can be drawn:

Table 3.1 Sensitivity of the optimal solution with respect to changes in values of the model parameters.

Changing parameters	Initial values	% change in parameter Value	% change in optimal values			
			t_1^*	T_1^*	S^*	$\prod^*$
A	30	+50	1.36363	10.6974	4726.56	35492.4
		+40	1.36346	10.7049	4411.45	33125.5
		+30	1.36326	10.7136	4096.35	30758.6
		+20	1.36303	10.7237	3781.24	28391.6
		+10	1.36276	10.7356	3466.14	26025.2
		-10	1.36204	10.7675	2835.93	21292.2
		-20	1.36154	10.7894	2520.83	18926
		-30	1.3609	10.8176	2205.73	16560.2
		-40	1.36005	10.8551	1890.62	14194.9
		-50	1.35884	10.9076	1575.52	11830.4
b	4.5	+50	1.136	41.2211	9640.85	1601090
		+40	1.17036	31.6195	7705.8	561159
		+30	1.20909	24.1235	6160.04	194350
		+20	1.25313	18.37	4925.26	72147.1
		+10	1.3037	14.0134	3938.91	34396.3
		-10	1.43163	8.32691	2521.77	20425.7
		-20	1.51449	6.5428	2019.17	18652
		-30	1.61583	5.24206	1618.17	16808.1
		-40	1.74307	4.30753	1298.24	14730.6
		-50	1.90851	3.65296	1043.49	12574.8
u	0.5	+50	1.60895	4.04078	9485.81	114585
		+40	1.55985	4.86012	7609.75	84271.4
		+30	1.51066	5.88879	6104.59	60936.9
		+20	1.46138	7.1741	4897.02	43513.5
		+10	1.41198	8.77238	3928.24	31244.6
		-10	1.31271	13.1838	2527.55	20555.9
		-20	1.26276	16.1612	2027.38	22010.5
		-30	1.21252	19.7795	1626.15	28380.8
		-40	1.16192	24.1435	1304.3	40325
		-50	1.11087	29.3625	1046.13	58812.1
F_1	0.3	+50	1.36029	10.75	3151.04	24186.1
		+40	1.36072	10.75	3151.04	24080.6
		+30	1.36115	10.75	3151.04	23975.1
		+20	1.36158	10.75	3151.04	23869.6
		+10	1.36201	10.75	3151.04	23764.1
		-10	1.36287	10.75	3151.04	23553.1
		-20	1.3633	10.75	3151.04	23447.6

		-30	1.36372	10.75	3151.04	23342.1
		-40	1.36415	10.75	3151.04	23236.6
		-50	1.36458	10.75	3151.04	23131.1
F_2	0.1	+50	1.36195	10.75	3151.04	23762.3
		+40	1.36205	10.75	3151.04	23741.6
		+30	1.36215	10.75	3151.04	23720.8
		+20	1.36225	10.75	3151.04	23700.1
		+10	1.36234	10.75	3151.04	23679.3
		-10	1.36254	10.75	3151.04	23627.8
		-20	1.36263	10.75	3151.04	23617.1
		-30	1.36273	10.75	3151.04	23596.3
		-40	136283	10.75	3151.04	23575.6
		-50	1.36293	10.75	3151.04	23554.8
H_1	0.2	+50	1.35324	10.75	3151.04	25010.5
		+40	1.35505	10.75	3151.04	24740.1
		+30	1.35688	10.75	3151.04	24469.7
		+20	1.35872	10.75	3151.04	24199.3
		+10	1.36058	10.75	3151.04	23929
		-10	1.36431	10.75	3151.04	23388.2
		-20	1.3662	10.75	3151.04	23117.8
		-30	1.3681	10.75	3151.04	22847.5
		-40	1.37001	10.75	3151.04	22577.1
		-50	1.37193	10.75	3151.04	22306.1
H_2	0.11	+50	1.33578	10.75	3151.04	26874.1
		+40	1.39055	10.75	3151.04	26231.4
		+30	1.34558	10.75	3151.04	25588.2
		+20	1.35087	10.75	3151.04	24945
		+10	1.35648	10.75	3151.04	24301.8
		-10	1.3688	10.75	3151.04	23015.4
		-20	1.37562	10.75	3151.04	22372.2
		-30	1.38298	10.75	3151.04	21728.9
		-40	1.39099	10.75	3151.04	21085.7
		-50	1.39976	10.75	3151.04	20442.5
C_1	1.5	+50	1.39274	10.75	3151.04	29138.2
		+40	1.38784	10.75	3151.04	28042.3
		+30	1.38245	10.75	3151.04	26946.4
		+20	1.3765	10.75	3151.04	25850.4
		+10	1.36988	10.75	3151.04	24754.5
		-10	1.354	10.75	3151.04	22562.7
		-20	1.34431	10.75	3151.04	21466.7
		-30	1.33301	10.75	3151.04	20370.8
		-40	1.31957	10.75	3151.04	19274.9

		-50	1.30319	10.75	3151.04	18179
C_2	2.0	+50	1.36619	10.75	3151.04	24711.7
		+40	1.36545	10.75	3151.04	24501.1
		+30	1.3647	10.75	3151.04	24290.5
		+20	1.36395	10.75	3151.04	24079.8
		+10	1.3632	10.75	3151.04	23869.2
		-10	1.36168	10.75	3151.04	23448
		-20	1.36091	10.75	3151.04	23237.3
		-30	1.36014	10.75	3151.04	23036.7
		-40	1.35937	10.75	3151.04	22816.1
		-50	1.3586	10.75	3151.04	22605.4
C_3	1.1	+50	1.36185	10.75	3151.04	23755.9
		+40	1.36197	10.75	3151.04	23736.5
		+30	1.36208	10.75	3151.04	23717
		+20	1.3622	10.75	3151.04	23697.5
		+10	1.36232	10.75	3151.04	23678.1
		-10	1.36256	10.75	3151.04	23639.1
		-20	1.36268	10.75	3151.04	23619.6
		-30	1.3628	10.75	3151.04	23600.2
		-40	1.36291	10.75	3151.04	23580.7
		-50	1.36303	10.75	3151.04	23561.2
B	0.6	+50	1.3887	10.75	3151.04	27558.5
		+40	1.38441	10.75	3151.04	26778.5
		+30	1.37972	10.75	3151.04	25998.5
		+20	1.37455	10.75	3151.04	25218.5
		+10	1.36883	10.75	3151.04	24438.6
		-10	1.35525	10.75	3151.04	22878.6
		-20	1.3470	10.75	3151.04	22098.6
		-30	1.33762	10.75	3151.04	21318.6
		-40	1.32657	10.75	3151.04	20538.7
		-50	1.31335	10.75	3151.04	19758.7
W	50	+50	1.36065	10.8288	3151.04	23657.5
		+40	1.36101	10.8131	3151.04	23657.5
		+30	1.36137	10.7973	3151.04	23657.5
		+20	1.36172	10.7815	3151.04	23657.5
		+10	1.36208	10.7658	3151.04	23657.5
		-10	1.3628	10.7342	3151.04	23657.5
		-20	1.36315	10.7184	3151.04	23657.5
		-30	1.36351	10.7026	3151.04	23657.5
		-40	1.36386	10.6869	3151.04	23657.5
		-50	1.36422	10.6711	3151.04	23657.5
		+50	1.36501	10.5339	3240.93	23922.6

		+40	1.3645	10.5761	3222.69	23869.3
		+30	1.36399	10.6188	3204.58	23816.1
		+20	1.36348	10.662	3186.61	23763.3
θ	0.01	+10	1.36296	10.7057	3168.76	23710.8
		-10	1.36191	10.7948	3133.45	23606.7
		-20	1.36138	10.8402	3115.98	23555.3
		-30	1.36085	10.8861	3098.64	23504.2
		-40	1.36031	10.9327	3081.43	23455.6
		-50	1.35977	10.9798	3064.34	23403.4
		+50	1.4305	10.75	3151.04	62439.1
		+40	1.41909	10.75	3151.04	52634.1
		+30	1.40676	10.75	3151.04	43853.2
		+20	1.39335	10.75	3151.04	36097
T	20	+10	1.37866	10.75	3151.04	29365.5
		-10	1.34435	10.75	3151.04	18976.4
		-20	1.32392	10.75	3151.04	15318.9
		-30	1.30051	10.75	3151.04	12686
		-40	1.27315	10.75	3151.04	11077.9
		-50	1.24034	10.75	3151.04	10494.4

3.8 Observations

From Table 3.1, the following inferences can be drawn:

- The optimal values are highly sensitive to the changes in the values of parameters B, A, b, u, C_1 and T, moderately sensitive to the changes in the values of parameters F_1, H_1, H_2 and C_2, whereas sensitivity to the changes in values of parameters F_2, C_3, θ and W is low.
- On increasing the values of parameters A, θ and u, the values of t_1^*, S^* and Π^* increase whereas the value of T_1^* decreases. On increasing the values of parameters b, F_1, F_2, H_1, H_2, C_3, and W, the values of T_1^*, S^* and Π^* increase whereas the value of t_1^* decreases.
- Π^* Increases, whereas , T_1^*, S^* has no effect on increasing the values of F_1, F_2, H_1, H_2, C_1, C_2, C_3, B and T, sensitivity being higher for the change in the value of parameters F_1, H_1, H_2, C_1, C_2, B,T as compared with the change in the value of parameter F_2, C_3.
- Increase in the values of parameters , C_1 and C_2 results in increase in the values of t_1^* and Π^* . Their values are more sensitive to change in the value of parameter C_1 as compared with the change in the value of parameter C_2.
- t_1^* Decrease, whereas T_1^* increase on increasing the value of parameter W. For change in the value of W, S^* and Π^* have no effect.
- Increase in the values of parameters T and B result in increase in the values of t_1^* and Π^*. However, their values are highly sensitive to the change in the value of parameter T as compared with the change in the value of parameter B.

3.9 Conclusion

Inventory contrivance of products with finite epoch is quite substantial in many business administrations. Since deteriorating items entail special storage niceties, the vender may have a small warehouse providing such amenities and so the need for an excessive rented warehouse may ascend. In this paper, we study an inventory model for deteriorating items with variable holding cost following ramp-type demand, with pliability to operate as a two warehouse system depending on the model parameters. For models established for the two warehouse system, the optimal solution does depend on the form of the demand function. The optimal policy for the model was acquired. Sensitivity analysis endorses that the optimum total cost is highly sensitive to changes in the values of the demand parameters. Hence, demand parameters must be farsightedly evaluated. Along with demand parameters, other parameters that vividly affect the optimal values of decision variables are the holding cost, shortage cost and the cycle length. Another protuberant observation which transpires from the sensitivity analysis is the difference in the configuration of change in the optimum cost resulting from changes in the values of deterioration rates. It was perceived that cost persistent due to high deterioration rate at vender's OW can be recompensed by procuring a quantity that leads to storing more inventory at RW, thus tumbling the shortage cost. On the other hand, high rate of deterioration in RW leads to increase in the optimum total cost. Hence, storage amenities on condition that at RW should be a main stimulus while deciding upon the optimal strategies.

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