A Non-Newtonian Model of Two Phase Hepatic Blood Flow in Capillary during Jaundice

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Abstract:

Our research deals with two phase hepatic flow in jaundice keeping in the view nature of circulatory system in human body. P. N. Pandey and V. Upadhyay have already considered the blood flow as two phased. One of which is that of red blood cells ad other is plasma. They have also applied Power –Law non Newtonian model and Herchel-Bulkley model in bio fluid mechanical set up. We have collected a clinical data in case of Jaundice patients for hematocrit v/s blood pressure drop. The graphical presentation for particular parametric value is much closer to clinical observation.

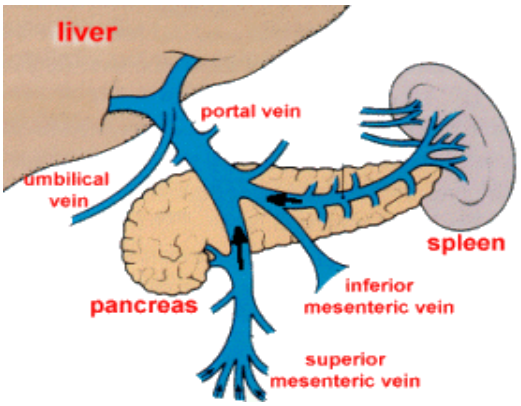
Keywords: ***Two phase Model, Power law, Harchel-Bulkley model, hemoglobin and blood pressure***.

I. Introduction

The application of mathematics to problems arising in the life sciences is a rapidly growing area yielding quantitative understanding of questions about such things as the spread of infectious diseases, population growth and interaction, organ (e.g. heart) function, cell signaling, nutrient supply, and more. This course will introduce students to the fascinating world of modeling biological systems. A variety of biological problems will be considered, in the context of which students will be exposed to a variety of mathematical techniques. No previous exposure to biology is necessary.

1.1 The Portal Circulations- The liver is unusual in that it has a double blood supply; the right and left hepatic arteries carry oxygenated blood to the liver, and the portal vein carries venous blood from the GI tract to the liver.[1]

1.2 The venous blood from the GI tract drains into the superior and inferior mesenteric veins; these two vessels are then joined by the splenic vein just posterior to the neck of the pancreas to form the portal vein. This then splits to form the right and left branches, each supplying about half of the liver [3]. Figure 1.1



1.2 Structure and function of liver- A human liver normally weighs 1.44 – 1.66 kg (3.2–3.7 lb). The liver plays an active role in process of digestion through the production of bill. Bill is a mixture of water, bile salts.

The bile duct and is released in to the large masses of fat. The large of fat in to smaller piece that have more surface area and are the therefore easier for the body to digest. [4].

1.3 Hepatic Circulatory System- The hepatic portal vein supplies about 75% of the blood the liver requires, with the other 25% supplied by the hepatic artery. Blood from the hepatic artery is oxygenated but nutrient-poor compared to that supplied by the hepatic portal vein. Blood from either source passes into cavities between the hepatocytes of the liver called sinusoids, which feature a fenestrated, discontinuous endothelium allowing for the effecient transfer and processing of nutrients in the liver.[5]. Since blood received from the hepatic portal vein may be contaminated with pathogens such as bacteria, the liver is rich in specialized immune cells called Kupffer cells that detect and destroy foreign organisms. Following processing, blood collects in a central vein that drains into the hepatic vein and finally the inferior vena cava. [6].

1.4 Blood Composition- Plasma is the largest component of blood, making up about 55% of its overall content. [7]. It's mainly made of water and surrounds the blood cells, carrying them around the body. Plasma helps maintain blood pressure and regulates body temperature.

It contains a complex mix of substances used by the body to perform important functions. These substances include minerals, salts, hormones and proteins.

All white blood cells have nuclei, which distinguishes them from the other blood cells, the anucleated red blood cells (RBCs) and platelets. [8]. Types of white blood cells can be classified in standard ways. Two pairs of broadest categories classify them eitherby structure (granulocytes or agranulocytes) or by cell division lineage (myeloid cells or lymphoid cells). [9]

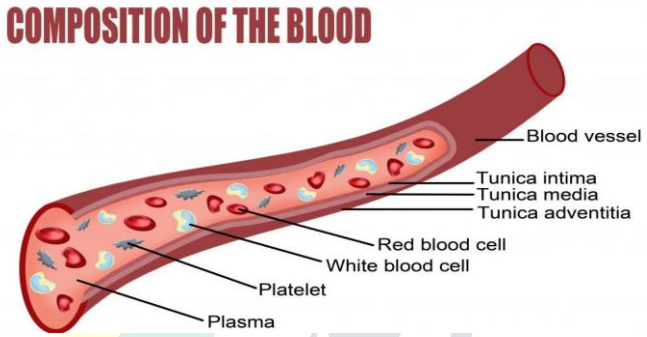


Figure 1.2 &1.3

1.5 Structure and function of blood- The total volume of the blood of the human body to be about 5-6 liters which is distributed in heart, aorta, main arteries, arterioles, capillaries etc.

Since,

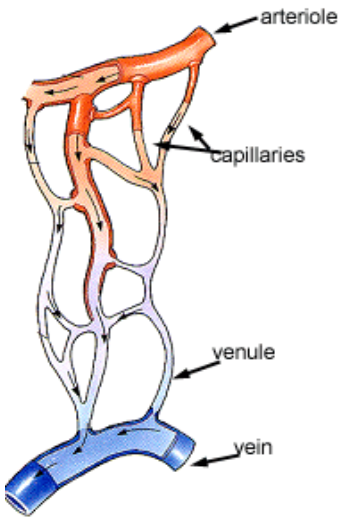
140 ml of blood required = 2800km lengths of blood vessels

5 l of blood require a lengths = (5000/140) × 2800km = 100000km

(The total length of blood vessels)

The approximate distribution of the blood in human body is –

Arteries	80 %	Arterioles	1 %
Capillaries	5 %	Systemic venous system	54 %
Heart cavities	12 %	pulmonary circuit	18 %



1.6 Anatomy and Physiology of Capillaries- Capillaries are small, normally around 3-4µm, but some capillaries can be 30-40 µm in diameter. The largest capillaries are found in the liver.[10]

Capillaries connect **arterioles** to **venules**. They allow the exchange of nutrients and wastes between the blood and the tissue cells, together with the interstitital fluid. This exchange occurs by passive diffusion and by pinocytosis which means 'cell drinking'. Pinocytosis is used for proteins, and some lipids. Also, importantly, white blood cells can move through intercellular junctions, into the surrounding tissue to repair damage, and fight infections. This route is also used by metastasising cancerous cells.[11] Figure 1.4

1.7 Fact of Jaundice

- Jaundice is caused by a buildup of bilirubin, a waste material, in the blood.
- An inflamed liver or obstructed bile duct can lead to jaundice, as well as other underlying conditions. [12]
- Symptoms include a yellow tinge to the skin and whites of the eyes, dark urine, and itchiness.
- Diagnosis of jaundice can involve a range of tests.
- Jaundice is treated by managing the underlying cause.
- On the basis of causes Jaundice can be classified into three types. Pre-hepatic Jaundice (Breakdown of RBC), Hepatic Jaundice (Liver damage), Post hepatic Jaundice (Bile duct).[13]

2. Mathematical Model

2.1 Equation of continuity –The continuity equation state that the amount of blood flows through one cardiac chamber is the same as the blood flow through the other chambers of the orifices. It is based on the conservation of mass. Where mass flow in must flow out. The 25 % flow in sub system, approximately 25% flow out [14].

2.2 The equation of Power Law in two phase Non-Newtonian blood flow-

$$\tau = \eta e^{n}$$

Where η is viscosity cofficient. This is found to hold good in the broad blood vessels where there is low hematocrit [10]. The strain rate is whole body between 5 and 200 sec-1.[15]

$$0.68 \leq n \leq 0.80$$

The yield stress is given by the formula $\tau_0^{1/2} = A\frac{m-M_m}{100}$, Where A = $(0.008 +\text{-}0.002 \text{dyme/cm}^2)^{1/3}$

2.3 Boundary Conditions are as follows- The velocity of blood flow on the axis of capillaries at r=0 will be maximum and finite, say V0 = maximum velocity 2. The velocity of blood flow on the wall of blood vessels at r=R, where, R is the radius of capillary, will be zero. This condition is well known as no-slip condition.

2.4 The Non-Newtonian Herschel-Bulkley equation:

$$\tau = \eta e^{n} + \tau_0 \, (\tau \geq \tau'_0)$$
$$e = 0 \qquad (\tau' < \tau'_0)$$

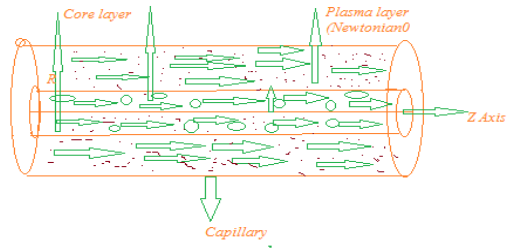
Where τ'_0 is yield stress.

2.5 The non–Newtonian casson equation –

$$\tau^{\frac{1}{2}} = \eta_m^{\frac{1}{2}} e^{\frac{1}{2}} + \tau_0^{\frac{1}{2}}, (\tau \geq \tau_0) \qquad where \;\; e = 0(\tau < \tau_0)$$

We apply power law and Herschel Bulkley model for Non-Newtonian blood flow.

2.6 Model of Blood Flow in Capillaries –



So the viscosity of core layer is given by

$$\eta_m = X\eta_c + (1 - X)\,\eta_p.$$
$$\tau^{ij} = -pg^{ij} + \eta_m\,(e^{ij})^n$$
$$= -pg^{ij} + \tau'^{ij}$$

Where τ^{ij} is stress tensor and τ'^{ij} is shear stress tensor.

The relative velocity of plasma layer with respect to core layer.

3. Methodology

The total flow –flux of blood through tube of the Capillary is Q defined by:

$$\mathbb{Q} = \int_0^R 2\pi r V\, dr$$

Using v_p *and* v, Now

$$\mathbb{Q} = \int_0^{v_p} 2\pi r v_p\, dr + \int_{v_p}^R 2\pi r v\, dr$$

$$v_p = \frac{n}{n+1}\left(\frac{p}{2\eta_m}\right)^{1/n} (R - r_p)^{\frac{1}{n}+1}$$

And

$$v = \frac{n}{n+1}\left(\frac{p}{2\eta_m}\right)^{1/n}\left\{(R - r_p)^{\frac{1}{n}+1} - (r - r_p)^{\frac{1}{n}+1}\right\}$$

So,

$$Q = 2\pi \int_0^{v_p} r\,\frac{n}{n+1}\left(\frac{p}{2\eta_m}\right)^{\frac{1}{n}} (R - r_p)^{\frac{1}{n}+1} dr \; +$$

$$2\pi \int_{v_p}^R r\,\frac{n}{n+1}\left(\frac{p}{2\eta_m}\right)^{1/n}\left\{(R - r_p)^{\frac{1}{n}+1} - (r - r_p)^{\frac{1}{n}+1}\right\} dr$$

Integration and solve these equation, let R = 1 and r_p = 1/3.

We get,

$$\mathbb{Q} = \pi\left(\frac{\mathcal{P}}{3\eta_m}\right)^{\frac{1}{n}} \frac{2}{27}\left\{\frac{26n^2+33n+9}{6n^2+5n+1}\right\}$$

$$\frac{27\mathbb{Q}}{2\pi} = \left(\frac{\mathcal{P}}{3\eta_m}\right)^{\frac{1}{n}}\left\{\frac{26n^2+33n+9}{6n^2+5n+1}\right\}$$

Both sides take power n $\left(\frac{27Q}{2\pi}\right)^n = \frac{P}{3\eta_m} A^n$ Where $A^n = \left\{\frac{26n^2+33n+9}{6n^2+5n+1}\right\}$

$$P = 3\eta_m\left(\frac{27Q}{2\pi A}\right)^n$$

Since, $P = \frac{\partial p}{\partial z}$ $\frac{\partial p}{\partial z} = 3\eta_m\left(\frac{27Q}{2\pi A}\right)^n$

Integration both side and limit take initial to final

$$\Delta\mathcal{P} = 3\eta_m \left(\frac{27\mathbb{Q}}{2\pi A}\right)^n \Delta\mathcal{Z}$$

Some useful value for Capillary

- 1 mmHg at 0^0C = **133.322** Pa (Pa = Pascal = n/m^2)[16]
- Hematocrit = 3 time of hemoglobin / Density of blood (1060 kg/m^3)
- $\eta_m = 0.0034\, pas$ [17]
- $\eta_p = 0.0013\, pas$ [18]
- $\mathbb{Q} = 800 - 1000\, ml/\min = 900\, ml/min$ (Average) $= 1.5 \times 10^{-5}$ [18]
  [1000ml/min = $1.666\times 10^{-8} m^3/s$]
- $l = 20 - 30\, \mu m$ (largest in liver)

  $= 25\, \mu m\, (Average) = 2.5 \times 10^{-5}\, m\, (1\mu m = 10^{-6} m)$
- $\mathcal{R} = 3.5 \times 10^{-6}$
- $\mathcal{B.P.D.} = \frac{2}{3}\left(\frac{\frac{S+D}{2}+D}{3}\right) - \left(\frac{\frac{S+D}{2}+D}{3}\right)$

4. Analysis

4.1 Numerically analysis of Model for Hepatic capillaries :-

1. Blood flow in two phase – One is Newtonian, other is Non- Newtonian (power law) -

Power law equation of continuity: $\frac{1}{\sqrt{g\sqrt{(gv)^i}}} = 0$ ..... (4.1.1)

Equation of motion: $\rho_m \frac{\partial v^i}{\partial t} + \rho_m v^j v^i_j = T^{ij}_j$ ..... (4.1.2)

Where T^{ij} is Power law constitutive equation.

Blood density equation is: $\rho_m = X\rho_c + (1-X)\rho_p$ .....(4.1.3)

$\eta_m = X\eta_c + (1-X)\eta_p$ .....(4.1.4)

(4.1.4) Is the viscosity of mixture of blood.

$X = \frac{H}{100}$ Is volume ratio of blood cells.

In cylindrical form $x^1 = r\,, x^2 = \theta, x^3 = z$

Tonsorial form in cylindrical co-ordinates : $\{g_{ij}\} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

The conjugate metric tensor is as follows: $\{g^{ij}\} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{r^2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

The non-vanishing Christoffel's symbols of 2nd kind are as follows [19]

$$\left\{ {1 \atop 2,\ 2} \right\} = -r\ , \quad \left\{ {2 \atop 2,\ 1} \right\} = \left\{ {2 \atop 1\ ,2} \right\} = \frac{1}{r}$$

Relation between contravariant and component of velocity of blood flow-

$$\sqrt{g_{11}v^1} = v_r \Rightarrow v^1\,, \qquad \sqrt{g_{22}v^2} = v_\theta = rv^2\ , \qquad \sqrt{g_{33}v^3} = v_z \Rightarrow v^3$$

The **Component of:** $p_{,j}\, g^{ij}$ *are* $- \sqrt{g_{ij}}\, p_{,j}\, g^{ij}$

The components of shearing stress tensor:

$$T'^{ij} = \eta_m (e^{ij})^n = \eta_m \left(g^{jk} v_k^i + g^{jk} v_k^j\right)^n \; as\ follows:$$

$$= \begin{bmatrix} 0 & 0 & \eta_m (\frac{dv}{dr})^n \\ 0 & 1 & 0 \\ \eta_m (\frac{dv}{dr})^n & 0 & 0 \end{bmatrix}$$

The covariant derivative T'^{ij} is: $T'^{ij}_{i} = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x} \left(\sqrt{g}\, T'^{ij}\right) + \left\{ {i \atop j \quad k} \right\} T'^{kj}$

Equation of continuity; $\frac{\partial v}{\partial z} = 0$ …….(4.1.5)

Equation of Motion- $-\frac{\partial p}{\partial r} = 0$ …….(4.1.6)

θ *component* , $v_\theta = 0$ ……(4.1.7)

Z-component, $0 = -\frac{\partial p}{\partial z} + \frac{\eta_m}{r} \frac{\partial}{\partial r} [r \left(\frac{\partial v_z}{\partial r}\right)^n]$ ……..(4.1.8)

The blood flow is symmetric in capillary concerned $v_\theta = 0$.

The blood flow - $\frac{\partial p}{\partial t} = \frac{\partial v_r}{\partial t} = \frac{\partial v_\theta}{\partial t} = \frac{\partial v_z}{\partial t} = 0$

Integrate the equation (4.1.5), we get $v_z = v(r)$, v not depend θ.

Integrate the equation (4.1.6) $p = p(z)$

Solve the equation $0 = -\frac{dp}{dz} + \frac{\eta_m}{r} \frac{d}{dr} [r \left(\frac{dv}{dr}\right)^n]$

Let pressure gradient; $\frac{dp}{dz} = p$ of blood flow in capillary.

$$0 = -\frac{dp}{dz} + \frac{\eta_m}{r} \frac{d}{dr} [r \left(\frac{dv}{dr}\right)^n]$$

$$p = \frac{\eta_m}{r} \frac{d}{dr} [r \left(\frac{dv}{dr}\right)^n]$$

$$\frac{d}{dr} \{r \left(\frac{dv}{dr}\right)^n\} = \frac{pr}{\eta_m}$$

Integrate the equation $r \left(\frac{dv}{dr}\right)^n = -\frac{pr^2}{2\eta_m} + A$

$r = 0, v = v_0$ are boundary conditions. $A = 0$

$$r \left(\frac{dv}{dr}\right)^n = -\frac{pr^2}{2\eta_m} \Rightarrow -\frac{dv}{dr} = \left(\frac{pr}{2\eta_m}\right)^{1/n}$$

Integrate the equation $v = -\left[\frac{p}{2\eta_m}\right]^{1/n} \frac{r^{\frac{1}{n}+1}}{\frac{n+1}{n}} + B$

Above equation under no slip boundary condition $= 0, r = R$, R = radius of vessel.

We get the equation- $B = \left[\frac{p}{2\eta_m}\right]^{1/n} \frac{r^{\frac{1}{n}+1}}{\frac{n+1}{n}}$

$$V = (\frac{p}{2\eta_m})^{\frac{1}{n}} \frac{n}{n+1} \{[R]^{\frac{1}{n}+1} - [r]^{\frac{1}{n}+1}\} \quad \text{……(4.1.9)}$$

The power law model of velocity of core layer –

$$v_m = [\frac{p}{2\eta_m}]^{\frac{1}{n}} \frac{n}{n+1}\left[R^{\frac{1}{n}+1} - r^{\frac{1}{n}+1}\right] + [\frac{p}{4\eta_p}(R^2-(R-\delta)^2) - [\frac{p}{2\eta_m}]^{\frac{1}{n}} \frac{n}{(n+1)}\left[R^{\frac{1}{n}+1} - (R-\delta)^{\frac{1}{n}+1}\right]$$
$$0 \le r \le (R-\delta)$$

The relative velocity of plasma layer along to core layer is Second kind equation.

2. Blood flow in two phase - Both are Newtonian

The blood is fluid when share rate increases are 200/s. The velocity of plasma $\eta_m \rightarrow \eta_p$ in the Newtonian model as follows –

$$v_p = \frac{p}{4\eta_p}(R^2 - r^2) \qquad R-\delta_0 \le r \le R \qquad \text{.....(4.2.1)}$$

The core layer velocity denoted by –

$$v_m = \frac{p}{4\eta_m}(R^2 - r^2) + \frac{p}{4\eta_m}[R^2 - R - \delta^2]\left(\frac{\eta_m}{\eta_p} - 1\right) \quad 0 \le r \le (R-\delta) \quad \text{.....(4.2.2)}$$

The Capillary radius R and δ = thickness of plasma layer.

The flow flux of blood in capillary is-

$$\frac{\pi p R^4}{8\eta_p}[1-(1-\frac{\delta}{R})^4(1-\frac{\eta_p}{\eta_m})] \qquad \text{.....(4.2.3)}$$

Viscosity is η, and then flow flux of blood is

$$Q = \frac{\pi p R^4}{8\eta_p} \qquad \text{.....(4.2.4)}$$

The equation VI & VII the viscosity as follows:

$$\eta = \eta_p[1-(1-\frac{\delta}{R})^4(1-\frac{\eta_p}{\eta_m})]^{-1} \quad \text{If } \frac{\delta}{R} \ll 1 \qquad \text{.....(4.2.5)}$$

5. Result And Discussion

Name ...F... Age...59... Sex...F...

Date	*Hemoglobin* $\mathcal{H}_B$	*Blood Pressure*	*Hematocrit* $\mathcal{H}_C$	*B. P. Drop*	*B.P.D. in Pascal*
19-July-15	*8.5*	*163/60*	*0.02405*	*-19.5*	*2540.45*
24-July-15	*8.9*	*159/66*	*0.02518*	*-19.8*	*2644.17*
29-July-15	*9.6*	*154/70*	*0.02716*	*-20.2*	*2696.03*
2-Aug-15	*9.2*	*148/73*	*0.02603*	*-20.3*	*2718.16*
7-Aug-15	*10.1*	*137/77*	*0.02858*	*-20.4*	*2725.63*

Table (a)

Take $\mathcal{H}_C = 0.02716$ and $\mathcal{B.P.}$ *drop* $= 2696.037$

and $$P(\mathcal{Z}) = \frac{\partial \mathcal{P}}{\partial \mathcal{Z}} = \frac{2696.037}{2.5\times10^{-5}} = 1.078\times10^8 = 107841480$$

We know that, $$\eta_m = \eta_c\chi + \eta_p(1-\chi) \qquad \text{where } \chi = \mathcal{H}_C/100$$

$$0.0034 = \eta_c\frac{\mathcal{H}_c}{100} + 0.0013\left(1-\frac{\mathcal{H}_c}{100}\right)$$

$$0.0034 = \eta_c\frac{0.02716}{100} + 0.0013\left(1-\frac{0.02716}{100}\right)$$

$$0.0034 = \eta_c 0.0002716 + 0.001299$$

$$\boldsymbol{\eta_c = 7.73325\ pas}$$

Again,

$$\eta_m = \eta_c\chi + \eta_p(1-\chi)$$

$$\eta_m = 7.73325\frac{\mathcal{H}_c}{100} + 0.0013(1-\frac{\mathcal{H}_c}{100})$$

$$\boldsymbol{\eta_m = 0.077319\,\mathcal{H}_c + 0.0013}$$

Now,

$$\frac{27\mathbb{Q}}{2\pi} = \left(\frac{\mathcal{P}}{3\eta_m}\right)^{\frac{1}{n}}\left\{\frac{26n^2+33n+9}{6n^2+5n+1}\right\}$$

$$\frac{27\times 0.000015}{2\times 3.142857} = \left(\frac{107841480}{3\times 0.0034}\right)^{\frac{1}{n}}\left\{\frac{26n^2+33n+9}{6n^2+5n+1}\right\}$$

$$n = -2.133$$

So,

$$\Delta\boldsymbol{\mathcal{P}} = \mathbf{3\eta_m}\left(\frac{\mathbf{27}\mathbb{Q}}{\mathbf{2\pi A}}\right)^{n}\Delta\mathcal{Z}$$

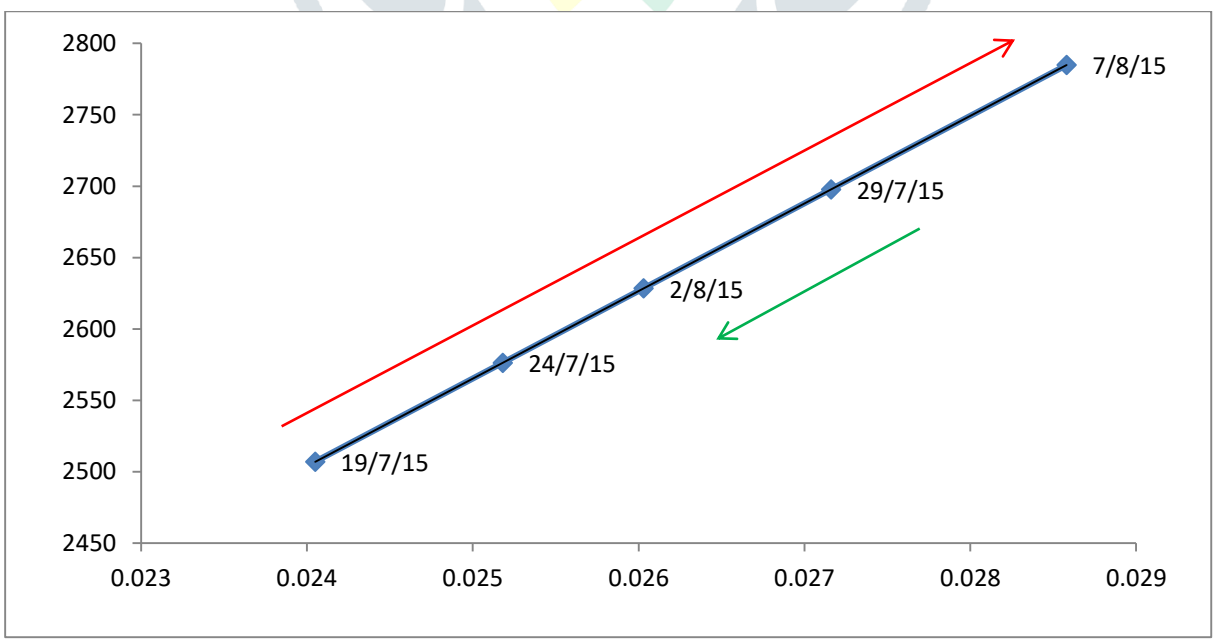
$$\Delta\mathcal{P} = \left(\frac{27\times 0.000015}{2\times 3.141\times 3.22704}\right)^{-2.133}\times 0.000025\,\{3\eta_m\}$$

$$\Delta\mathcal{P} = 264490.395\,\{3(0.077319\,\mathcal{H}_c + 0.0013)\}$$

$$\Delta\boldsymbol{\mathcal{P}} = \mathbf{61348.547}\,\mathcal{H}_c + \mathbf{1031.512}$$

$\mathcal{H}_C$	*0.02405*	*0.02518*	*0.02716*	*0.02603*	*0.02858*
Δp	*2506.94*	*2576.26*	*2697.73*	*2628.41*	*2784.85*

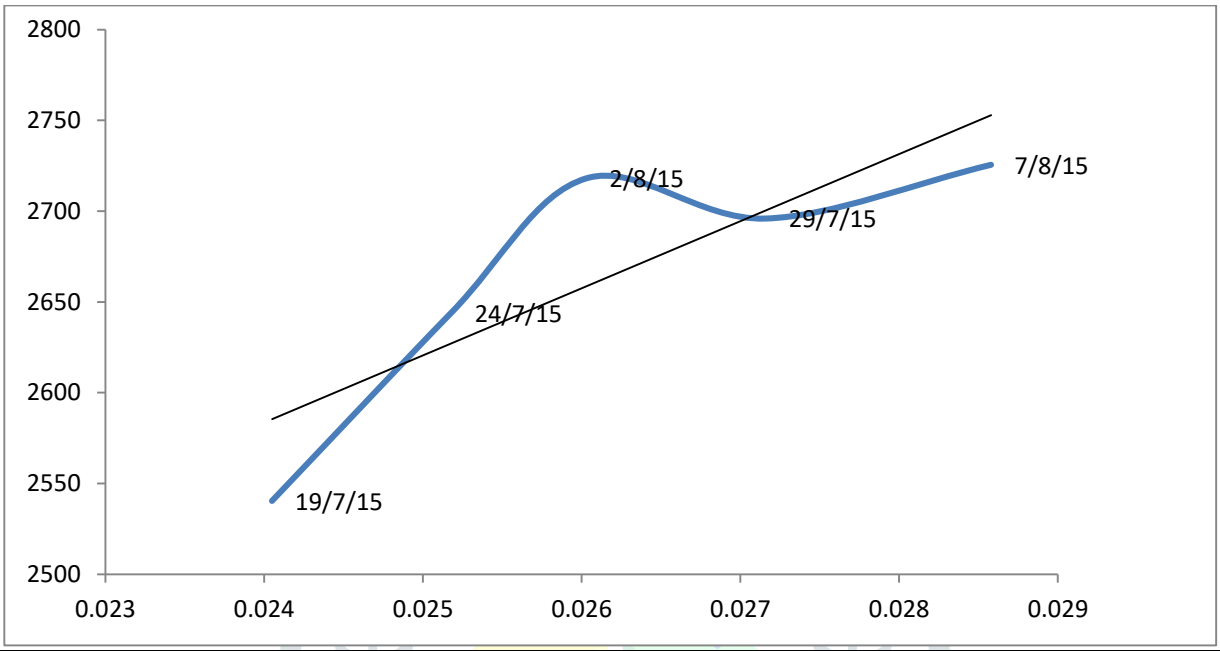
Table (b)



Graph (a)

$\mathcal{H}_C$	0.02405	0.02518	0.02716	0.02603	0.02858
B.P. D. Pascal	2540.450	2644.175	2696.037	2718.168	2725.634

Table (c)



Graph (b)

5 Conclusion

The graphs (a) and (b) show is date wise connection of H_C & $\Delta\mathcal{P}$ and H_C & B. P. D. July 19, 29 and Aug 8 graphs are below in trend line, so patient condition is normal. When July 24 & Aug 2 is increase of B.P.D so change of medicine. The trend line is increase.

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7 References

[1]. Pundir Sudhir k. and Pundir Rimple (2006) "Biomathematics", first edition, Pragati Prakashan meerut, ISBN: 81-7556-962-X. pp – 17.

[2]. Hristian Eipel (2010), "Regulation of Hepatic Blood Flow" Vol. 48, Issue 16, pp. 6046-6057

[3]. Linzy, Wang JH, Wang LY. Changes in intrahepatic portal hemodynamics in early stage hepatic abscesses. J Ultrasound Med 1996; 8: 595-598.

[4]. Tanaka S, Kitamura T, Fujita M.Color Doppler flow imaging of liver tumors. AJR 1990; 3: 509-574.

[5]. Miller MA, Balfe DM. Peripheral portal venous blood flow alterations by hepatic masses - evaluation with color and pulsed Doppler sonography. J US Med 1996; 10: 707-713.

[6]. Ralls PW. Color Doppler Sonography of the hepatic artery and portal venous system. AJR 1990; 3: 517-525.

[7]. Poniak MA, Baus KM. Hepatofugal arterial signal in main portal vein: an indicator of intravascular tumor spread. Radiology 1991; 180: 663-6.
[8]. Donsteves E. (1978) "In flamatory mechanism" pp. 293-324
[9]. Medhavi A., Srivastav R.K. (2012) "Two- Phase Arterial Blood Flow Through a Compsite Stenosis" JST, Vol. 4, Issue 7, pp. 83-95.
[10]. Jaundiceintroduction(2015),Retrievedfrom
[11]. http://www.nhs.uk/conditions/Jaundice/Pages/Introduction.aspx
[12]. se blood flow" Ph.D. Thesis, Central University, Allahabad, pp. 107-109.
[13]. Kapur J. N. & Gupta R. C. (1992) "Mathematical Model in Biology and Medicine" EWP New Delhi, pp. 354-55.
[14]. Fung. Y.C. (1994) "A first course in continuum mechanics: for physical and biological scientist and engineers" prentice hall, Inc. ASimon &Schuster company englewoodcliffs, new jersey 07632. ISBN: 0-13-061524-2, pp-21.
[15]. Glen Elert (2010) "Viscosity" The Physical Hypertext book. Hypertextbook.com, Retrievd: pp. 09-14.
[16]. Daniel R. Gustafson (1980) "Physics Health and Human Body" Wadsworth Publishing Company. pp. 518.
[17]. De U. C. and Ali Absos Shaikh (2008) "Tensor Calculas" Second Edition , Narosa Publication House, ISBs: 978-81-7319-884-7, pp. 82.
[18]. De U. C. and Ali Absos Shaikh (2008) "Tensor Calculas" Second Edition , Narosa Publication House, ISBs: 978-81-7319-884-7, pp. 84.