RELATION BETWEEN CONVERGENCE AND TOPOLOGICAL SPACE

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Abstract: *This paper deals with the basic results of concerning various notation of convergence in topological space. He also defined the filters in topological space by defining a limit of the filter. Again, we observe that the generalize possibility of filter on some index set I and a map I→X. We conclude that the general situation (a map from x an index set to x) which will speak about ideal convergence.*

Keywords: *Filter, Convergence, Topological space, Net, Hausdorff space.*

INTRODUCTION:

In a topological space X, the closure of any subset S is the set of limits of convergent nets of elements of S. For a map f between the topological spaces X and Y, (a) f is continuous (b) If x is a net converging to X, then f(x) is a net converging to f(x) in Y.

Convergence *of nets:*

Definition*:* *We say that (D, ≤) is a directed set, if ≤ is a relation on D such that*

(1) $x \le y \wedge y \le z \Rightarrow x \le z$ for each x, y, z $\in$ Z ;

(2) $x \le x$ for each x $\in$ D;

(3) For each x, y $\in$ D there exist z $\in$ D with $x \le D$ and $y \le Z$.

In other words a directed set is a set with a relation which is reflexive, transitive and upward directed.

Definition*:* A subset A of set D directed by ≤ is confinal in D if for every d $\in$ D there exists an a $\in$ A such that $d \le a$.

A subset A of a directed set D is called residual if there is some $d_0 \in$ D such that $d \ge d_0$ implies $d \in A$.

Definition*:* *A net in a topological space X is a map from any non-empty directed set Σ to x. It is denoted by* $(x_\sigma)_{\sigma\in\Sigma}$

Definition*:* Let $(x_\sigma)_{\sigma\in\Sigma}$ be a net in a topological space x is said to be convergent to $x \in X$ if for each neighbourhood U of x there exists $\sigma_0 \in \Sigma$ such that $x_\sigma \in \cup$ for each If a net $(x_\sigma)_{\sigma\in\Sigma}$ converges to x, the point x is called a limit of this net. The set of all limits of a net is denoted by $\lim x_\sigma$.

Theorem*:* A point x belongs to $\overline{A}$ if and only if there exists a net consisting of elements of A which converges to x.

Theorem*:* A subset V of a topological space X is closed iff for each net $(x_\sigma)_{\sigma\in\Sigma}$ such that $x_\sigma \in V$ for each $r \in \Sigma$ every limit of $(x_\sigma)_{\sigma\in\Sigma}$ belongs to V as well.

Theorem*:* Let X, Y be topological spaces. A map f:X→Y is continuous iff whenever a net x_σ converges to x, the net f(x_σ) converges to f(x).

Several important notions, such as Hausdorffness and compactness can be characterizes with the help of nets.

Theorem: A topological space x Hausdorff $\Leftrightarrow$ every net in z has at most one limit.

We say that the net $(Y_e)e \in E$ is finer than the net $(x_d)d \in D$ or subset of if there exists a function ϕ of E to D with following properties:-

(1) For every $d_\circ \in D$ there exists an $e_0 \in E$ such that ϕ (e) ≥ $d_\circ$

(2) $X\phi_{(e)} = y_e$ for $e \in E$.

This definition can be formulated equivalently using the notion of co-final map.

Definition: A function f: P→D from a pre-ordered set to a directed set is cofinal if for each $d_\circ \in D$ there exists $P_\circ \in P$ such that f(P) ≥ $d_\circ$ whenever P ≥ $P_\circ$

Hence a net $\sigma^1 : \Sigma^1 \to X$ is a subnet of a net $\sigma : \Sigma \to X$ x if there exists a co-final map $f : \Sigma^1 \to \Sigma$ with $\sigma^1 = \sigma_0 f$

Theorem: Every net (x_r) in X has a universal subnet. Any universal net converges to each of its cluster points (i.e. , if it has a cluster point, it converges)

We can note that, for any map f:x→y the image of a universal net in X is again a universal net in Y.

Remark: Sometimes the notion of the limit of a net of closed subsets of a topological space is defined as follows:

If $(Ad)_{d \in D}$ is a net of subsets of X then

(1) The lower closed limit Li A_d of (A_d) consist of all such point x that each neighbourhood of x intersect A_d for all d in some residual subset of A.

(2) The upper closed limit L_s A_d of (A_d) consist of all such points x that each neighbourhood of x intersects A_d for all d in some co-final subset of A.

(3) If L_i A_d = L_s A_d then (A_d) is said to be kuratowski – Painleve Convergent.

Note that if we take $A_d = \{x_d\}$ then L_i (x_d) is precisely the set of all limits of (x_d) and L_s A_d is precisely the set of all cluster point of (x_d). What can be considered an advantage of this notation is that lim x_r one usually associates a point, where as L_i A_s is always a set.

Convergence of filters on $x_\sigma \to X$:

Another common possibility used when dealing with the convergence in a topological space x is to consider filters on X.

Definition: A filter on a set x is a subset $\mathcal{F}$ of p (x) such that

(1) $\phi \notin \mathcal{F}$

(2) $A, B, \in \mathcal{F} \Rightarrow A \cap B \in \mathcal{F}$;

(3) $A \in \mathcal{F} \wedge A \subset B \Rightarrow B \in \mathcal{F}$.

Remark: Let us note that it is possible to define a filter in a subfamily R of P (X) which has largest element . In this case, we need to reformulate the second part of condition (3) $A \subset B \in R$. Hence it is possible to define a filter in the family closed sets X, which is used in the definition of wallman compactification of a T_1 - space.

Example: Let X be a topological space. A neighbourhood filter N(x) of a point $x \in X$ is the set of all neighbourhood of x. (Neighbourhood of x is any sub-set V of X such there exists an open set U with $x \in \cup \subset V$.

Conclusion: We conclude that the convergence of nets describes completely the topology of X and also the convergence in a topological of X and also the convergence in a topological space x is to consider filter on Y.

Refrences

(1) James,I.M : "Topological and uniform spaces" springer verlag, Berlin 1987.

(2) Isbll, J.R. : Topological spaces Amer, Math. Soc. Survey,providence, 1964.

(3) Kelley,J.L. : General Topology ,Van Nostrand, New-York, 1955.

(4) Kothe, G : 'Topological vector spaces,springer verlag 1968.

(5) Jha, K. k. : Advance General Topology,Nav Bharat Prakashan 1977.