Mathematical modeling and Design modification of a Cooler used for Air Cooling and Refrigeration

[1]Bhupendra Sahare, [2]Chhavikant Sahu

[1]M.Tech Student, Department of Mechanical Engineering, Government Engineering College Jagdalpur, Chhattisgarh, India.,
[2]Assistant Professor, Department of Mechanical Engineering, Government Engineering College Jagdalpur, Chhattisgarh, India.

Abstract: This paper deals with a simple mathematical model and design modification for an evaporative cooling system. In this paper, we make a refrigeration box and fitted to the bottom of the existing model of the evaporative cooler. The purpose of this paper is to develop a mathematical model and this mathematical model is to be validated by the experiments and modified in the cooler to provide adequate cooling comfort, reduce environmental impact, and reduce energy consumption in buildings.

Key words **- Evaporative cooling, Mathematical modeling, Air cooler, Refrigeration box, etc.**

I. INTRODUCTION

Evaporative air generally referred as desert coolers is very popular nowadays used for space cooling offers an economical alternative to conventional air conditioning systems in a hot and dry climate. Compared to air conditioning units, evaporative air Coolers also operate without ozone harming hydro-chlorofluorocarbon (HCFCs) used by refrigeration-based systems. An evaporative air cooler is a device that cools air through the simple water evaporation. It is especially well suited for climates where the air is hot and humidity is low. In India, the northern regions are good locations for the implementation of this equipment if sufficient water is available. In these climates, the installation and operating cost of an evaporative air cooler can be much lower than that of air conditioning units. Although the utilization of desert cooler for space air conditioning is an effective alternate to compressors-based air conditioning system but still the poor effectiveness of the evaporative cooling-based desert cooler in humid climatic conditions is one of the reasons for the decline in popularity of evaporative cooling-based device. Still it is suitable in many parts of the country due to favorable climatic conditions. Evaporative cooling can be categorized as direct and indirect cooling system. The mathematical model is helpful in the evaluation of exit and mean temperature of water, mean exit temperature, inlet exit temperature and temperature inside the refrigeration box.

II. MATHEMATICAL MODELING

1. Mathematical Model

Following earlier mathematical model proposed by Sodha and Somwanshi (2012), considering a evaporating pad (Fig.1), with water flowing from the top to a bottom tray in z direction; the air flow is normal to the pad in x direction. The variation of the temperature of water in the direction of flow of air has been neglected. However, the dependence of the temperature of water on z (the direction of flow of water viz. the vertical) has been taken into consideration.

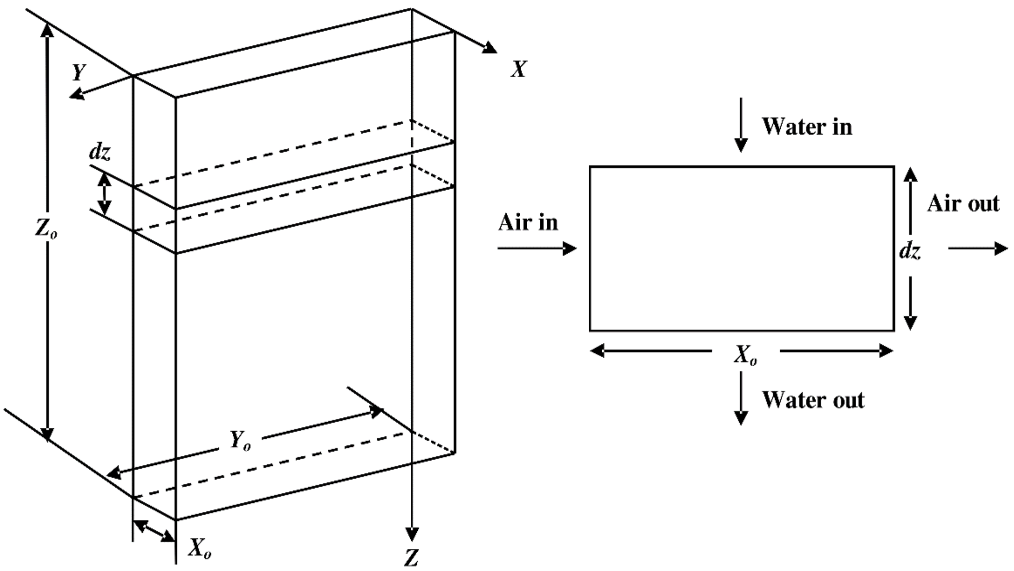

Fig. 1 *Pad profile and elemental volume*

Following earlier work e.g. Wu et al. (2009a), the temperature of the air inside the pad is given by

$$\frac{T_a - T_w}{T_{ai} - T_w} = \exp(-\alpha x), \qquad (1)$$

where $\alpha = h_c F_p / \rho_a v_a$ .The term T_{ai} is just the temperature of the inlet atmospheric air,

From Eq.(1) the exit and the mean temperature of air is given by, $T_{ae} = T_w \{1 - \exp(-\alpha x_o)\} + T_{ai} \exp(-\alpha x_o)$ (2)

$$\bar{T}_a = T_w + \{T_{ai} - T_w\}\{1 - \exp(-\alpha x_o)\} / \alpha x_o \,, \quad (3)$$

where the bar indicates average over *x*.

Considering an element of pad of thickness *dz* (fig.1), the energy balance of water may be expressed as,

$$\dot{m}_w c_w \frac{dT_w}{dz} dz + \dot{Q}_L dz + \dot{Q}_S dz = 0 \,; \quad (4)$$

Heat transfer per unit area, associated with the mass transfer is given by Tiwari, (2002).

$$\dot{q}_L = C_n h_c (P_w - \gamma P_a). \quad (5)$$

C_n is a constant and it is given by,

$$C_n = \frac{M_w L}{RT \rho_a c_{pa} Le^{2/3}}$$

$$\rho_a = \frac{P_a M_a}{RT}$$

$$C_n = \frac{L}{c_{pa}} \frac{M_w}{M_a} \frac{1}{P_T Le^{2/3}}$$ here $P_a = P_T$

$$Le = \frac{\alpha'}{D_{ab}}$$

Remembering that the evaporating surface in the element $x_o y_o dz$ is $F_p x_o y_o dz$.

$$\dot{Q}_L = \dot{q}_L F_p x_o y_o$$

$$= C_n h_c (P_w - \gamma P_a) F_p x_o y_o \,; \quad (6)$$

further,

$$\dot{Q}_S = h_c (T_w - T_a) F_p x_o y_o . \quad (7)$$

The saturation vapor pressure of water can be represented to a very good approximation by,

$$P = R_1 T^2 + R_2 T + R_3 \,, \quad (8)$$

where $R_1 = 6.36 Nm^{2o} C^{-2}$, $R_2 = -112.8 Nm^{2o} C^{-1}$ and $R_3 = 1890 Nm^{-2}$.

From Eqs.(3),(4),(5),(6),(7) and (8) one obtains,

$$\frac{dT_w}{dz} = -AT_w{}^2 + BT_w + C \,, \quad (9)$$

where,

$$N_1 = \frac{\dot{m}_w c_w}{h_c F_p x_o y_o} \,,$$

$$K_1 = \{1 - \exp(-\alpha x_o)\} / \alpha x_o \,,$$

$$A = R_1 C_n / N_1 \,,$$

$$B = (R_2 C_n - K_1) / N_1$$ and

$$C = [C_n R_1 \gamma T_{ai}{}^2 + T_{ai}(K_1 - \gamma C_n R_2) + C_n R_3 (\gamma - 1)] / N_1$$

Integrating Eq.(9) one obtains,

$$\frac{T_w - (B/2A) - C_1}{T_w - (B/2A) + C_1} = \beta \exp(-2AC_1 z) \,,$$

where,

$$\beta = \frac{\left[T_{wi} - (B/2A) - C_1\right]}{\left[T_{wi} - (B/2A) + C_1\right]} \,,$$

and

$$C_1 = \sqrt{(B/2A)^2 + C/A} \,,$$

where T_{wi} is the temperature at the top of the evaporative pad ($z = o$)

From Eq. (9) the exit and the mean (over *z*) temperature of water flowing through the cooler pad and mean exit air temperature is given by,

$$T_{We} = (B/2A) + C_1 \left\{ \frac{(1+\beta \exp(-2AC_1 z_o))}{(1-\beta \exp(-2AC_1 z_o))} \right\} \quad (10)$$

$$\langle T_W \rangle = \frac{B}{2A} + \frac{1}{Az_O} \ln \left\{ \frac{\exp(2AC_1 z_O) - \beta}{1-\beta} \right\} - C_1, \quad (11)$$

$$\langle T_a \rangle = \exp(-\alpha x_O) \{ T_{ao} - \langle T_W \rangle \} + \langle T_W \rangle . \quad (12)$$

C_n is a constant and it is given by,

$$C_n = \frac{M_w L}{RT \rho_a c_{pa} Le^{2/3}}$$

c_{pa} is the specific heat of moist air given by Morway and Gvozdenac [9],

$$c_{pa} = [(1.0029 + 5.4x10^{-5} T) + \xi(1.856 + 2.0x10^{-4} T)] kJ/kgK$$

$$\rho_a = \frac{P_a M_a}{RT}$$

$$C_n = \frac{L}{c_{pa}} \frac{M_w}{M_a} \frac{1}{P_T Le^{2/3}}$$ here $P_a = P_T$

$$Le = \frac{\alpha'}{D_{ab}}$$

α' is the thermal diffusivity and D_{ab} is the diffusion coefficient of water vapor in air and it is given by Boltz and Tuwe [10],

$$D_{ab} = -2.775x10^{-6} + 4.479x10^{-8} T + 1.656x10^{-10} T^2$$

here T is the temperature in Kelvin

2. *Box temperature*

Rate of heat given by refrigeration box (Load) will be,

$$\dot{Q}_h = U_x [T_h(t) - T_t(t)] \quad (13)$$

$\dot{Q}_h$ is the rate of heat transfer from refrigeration box to surroundings , T_h and T_t are the temperature inside box and surrounding tank water temperature, U_x is given by,

$U_x = U(2A_h + 4A_s)$, when rectangular box is completely surrounded by tank water (Experiment)

$U_x = U(2A_h + 3A_s)$, one of the vertical face is insulated and exposed to atmosphere, door of refrigerator box. (Fig.3)

Here, A_h and A_s are area of horizontal and vertical faces of box. U is the overall heat transfer coefficients between box and water,

The Energy balance of cooler tank is,

$$M_t c_w \frac{dT_t}{dt} = Q_h + Q_m - Q_e \quad (14)$$

Q_m is the rate of heat added by make-up water,

$$Q_m = m_m c_w (T_a - T_t) \quad (15)$$

m_m is the mass of make-up water added per sec. T_a is the initial make-up water temperature assumed equal to ambient $T_a = T_{ai}$

Q_e is the rate of evaporative heat transfer from tank water through cooler pads

$$Q_e = m_c c_w (T_{pi} - T_{we}) \quad (16)$$

m_c is the mass flow rate of tank water flowing into the cooler pads, T_{pi} is temperature of tank water in to the cooler pad (at top of cooler pad) and T_{we} is the temperature of water at exit of cooler pad. Neglecting pumping and pipe loses inlet water temperature in pad will be equal to temperature of tank water $T_t = T_{pi}$.

From Eqs. 2, 3 4 and5,

$$M_t c_w \frac{dT_t}{dt} = \dot{Q}_h + m_m c_w (T_a - T_t) - m_c c_w (T_{pi} - T_{we}) \quad (17)$$

$$\frac{dT_t}{dt} + K_1 T_t = K_2$$

$$K_1 = \frac{m_m c_w}{M_t c_w}$$

$$K_2 = \frac{\dot{Q}_h + m_m c_w T_a + m_c c_w T_{we} - m_c c_w T_{pi}}{M_t c_w}$$

$$T_t = \frac{K_2}{K_1}\{1 - \exp(-K_1 t)\} + T_{t0} \exp(-K_1 t) \quad (18)$$

By putting the value of T_t, Eqn. 1, the temperature inside the refrigeration box is given by,

$$T_h(t) = \frac{Q_h}{U_x} + T_t(t) \quad (19)$$

3. *Proposed design of cooler*

Cubical air cooler which is the most commonly used cooler can be modified to a cooler which serves dual purpose of cooling air as well can be utilized as mild refrigeration. As in the desert cooler the air is continuously cooled due to evaporation of water droplets trickling down the evaporative pads. Due to heat and mass transfer both air and water gets cooled. The cooled tank water is stored in the tank of cooler and not generally utilized. The new proposed design of cooler can be utilized as air-cooler as well as refrigerator. The proposed design is shown in figure below.

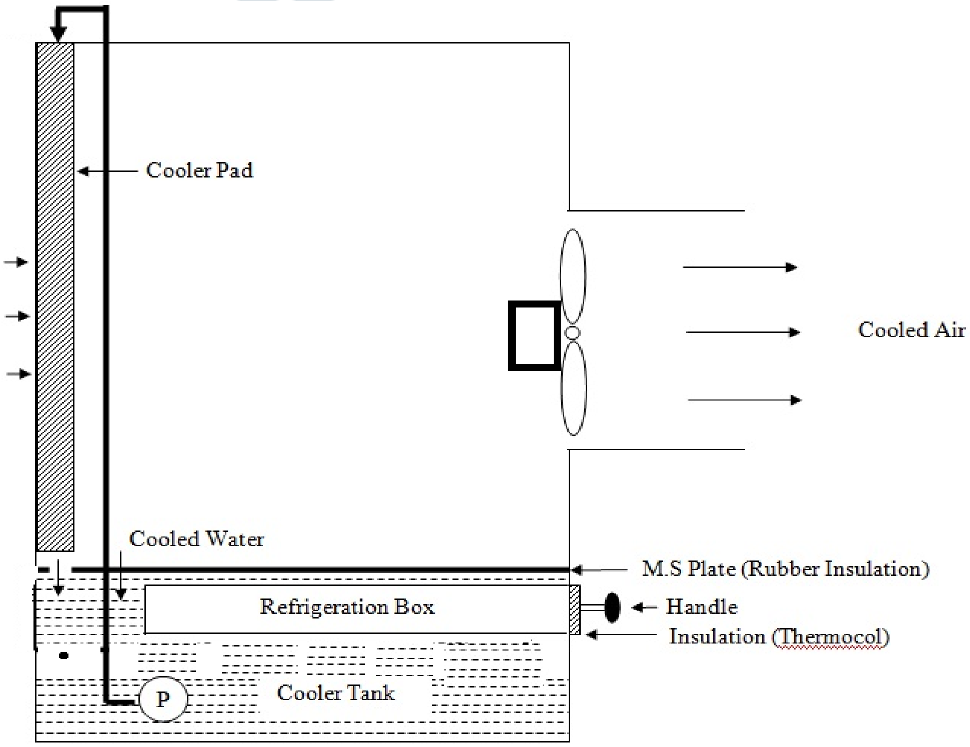

Fig. 2 *The proposed design of cooler*

The design is the modification of the existing design of cooler a box made of the mild steel is attached inside the cooler tank which is surrounded by tank water from five different faces. The front face is exposed to the atmosphere and a handle is provided to load and unload the box.

4. *Refrigeration Box*

The box we used in this project is made up of G.I. Sheet which is in rectangle shape. Which is to be kept inside the cooler tank for that purpose it has been fully insulated, so that we kept it inside the water? Inside the box we have fitted two bulbs are fitted of 15w & 10w and also a sensor is fitted for taking the reading of temperature inside the box. The box is having the sizes (Length=30cm, Breadth=35cm, Height=10cm).

Fig. 3 *Refrigeration Box*

III. EXPERIMENTAL VALIDATION

To validate the mathematical model experiment is performed in controlled atmosphere inside a room. The experiment cooler has tank capacity of 90L is cubical in shape with cellulose pad material is used as evaporative media and its surface 370 $m^2 m^{-3}$. An exhaust fan (100W) has been fixed in the front face to induce the hot air coming into the cooler pad. A pump (40W) is placed into the tank of the cooler to allow the water to flow from top to bottom into the cooler pad as the water flows into the pad comes in contact of air there will be a heat and mass transfer between the water and air as a result of this both air and water gets cooled. Cooled water is stored in the tank of cooler. The proposed method is to utilize the stored cool tank water. A refrigeration box made of mild steel plate is clamped inside the tank of the desert cooler as shown in figure 3. To simulate the refrigeration load electric bulb (5W) is placed inside the refrigeration box. Temperature sensors are fixed inside the box to measure the transient temperatures of enclosure in the box. To measure the temperature of tank water temperature sensor is placed inside the tank of the cooler.

To determine the temperature and humidity of the inlet air, temperature sensors (RTD) and digital hygrometer are fixed at three different points inside room. The temperature of exit air was measured by two temperature sensors (RTD) kept in exit air duct. Inlet air velocity was measured by electronic digital anemometer. Tank water temperature, exit air temperature and preservation temperature (inside box) with load is recorded at the time interval of 5mins. Relevant cooler parameters used for numerical computations are shown in Table-1.

Table-1 Numerical parameters used for computations

$x_0 = 0.1m$	$c_w = 4200 J / kgK$	$U = 11.3 J / kgK$	$v_a = 1.5 m / s$	$F_p = 370 m^2 / m^3$
$y_0 = 0.45m$	$c_a = 1.0 J / kgK$	$M_t = 82.6 kg$	$A_h = 0.1050 m^2$	M_w =18
$z_0 = 0.5m$	$\dot{m}_c = 0.116 kg / s$	$\dot{m}_m = 10 kg / h$	$A_s = 0.03 m^2$	M_a =29
P_T =1.013x 10^5 N/m^2	L' =2200KJ/kg			

Table-2 Theoretical and experimental temperature inside box, tank water and exit air with Load ($T_a = 30.6^o C, \gamma = 38\%,\ v_a = 1.5 m/s$)

Time (m)	Box temperature (ºC)		Tank water temperature (ºC)		Exit air temperature (ºC)	
	Ex.	Th.	Ex.	Th.	Ex.	Th.
0	29.8	29.8	30.5	30.5	30.6	30.6
5	28.4	27.6	27.5	26.3	26.2	26.2
10	26.5	25.2	24.4	23.9	25.7	25.8
15	24.3	23.8	22.4	22.5	25.4	25.6
20	23.2	23.0	21.6	21.7	24.9	25.5
25	22.8	22.5	21.5	21.2	24.7	25.4
30	22.6	22.2	20.6	20.9	24.8	25.3
35	21.8	22.0	20.5	20.8	24.6	25.3
40	21.6	22.0	20.5	20.7	24.6	25.2
45	21.5	21.9	20.4	20.6	24.7	25.2
50	21.4	21.8	20.5	20.5	24.6	25.2
r		0.98		0.97		0.96

1. Graph plotted between Theoretical and Experimental value of Box temperature

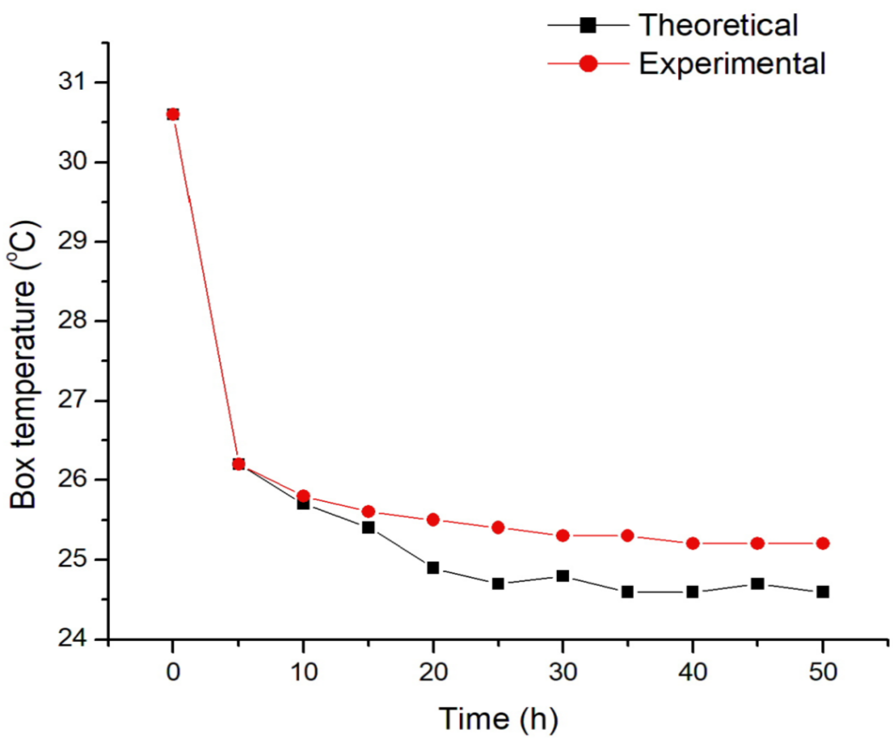

Fig. 1 *Variations in Theoretical and Experimental Box temperature value*

The relationship between the theoretical values and experimental values is presented by a coefficient called as coefficient of correlation (r). The experimental and theoretical values are said to be in a strong correlation, if the value of r is close to 1. The coefficient of correlation can be evaluated with the help of following expression as given by [12],

$$r = \frac{N\sum X_{pre} X_{\exp} - (\sum X_{pre})(\sum X_{\exp})}{\sqrt{N\sum X_{\exp}{}^2 - (\sum X_{\exp})^2}\sqrt{N\sum X_{pre}{}^2 - (\sum X_{pre})^2}}$$

Here N is the number of observations.

IV. CONCLUSIONS

Equations to estimate the cooling efficiency for evaporative cooler used as refrigeration purpose has been evaluated and validated. Model accepted the thermal conductivity and thickness of pad material for cooling. It is seen that the value of coefficient of correlation "r" is in between 0.91 to 0.99 i.e. 91 - 99% accuracy. The information presented could be useful in the design of evaporative cooling for refrigeration purposes.

V. REFERENCE

[1] J. R. Watt, Evaporative Air Conditioning, The Industrial Press, New York, 1986

[2] M. L. Mathur and B. P. Jain, Performance of a portable cooler: desert cooler, Journal of Institute of Engineers (India), 59 (1979), 241–245.

[3] M. L. Mathur and B. P. Jain, Experimental study of performance of portable air washer type air cooler, Journal of Institute of Engineers (India), 63 (1982), 38–40.

[4] Sawhney RL, Singh SP, Bansal NK, Sodha MS. Optimization of an evaporative cooler for area cooling. Housing Science. 1987; 11(3); 225- 231.

[5] Singh SP, Sawhney RL, Bansal NK, Sodha MS. Sizing of an evaporative cooler for thermal comfort inside a room. Housing Science. 1987; 11(2): 141-148.

[6] Sodha MS, Singh SP, Sawhney RL. Evaluation of design patterns for direct evaporative coolers. Building and Environment. 1995; 30(2); 287-291.

[7] B. Riangvilaikul, S. Kumar "An experimental study of a unique dew point evaporative cooling system," Energy Build., 42, pp. 637-644 (2010).

[8] M.S. Sodha, A. Somwanshi, Variation of water temperature in direction of flow: effect in the performance of a desert cooler, J. Fundamental Renew. Appl. 2 (2012).

[9] J. A. Dowdy and N. S. Karabash, Experimental determination of heat and mass transfer coefficients in rigid impregnated cellulose evaporative media, ASHRAE Transactions, 93 (1987), 382–395

[10] J. R. Camargo and C. D. Ebinuma, A mathematical model for direct and indirect evaporative cooling air conditioning systems, in Proceedings of the 9th Brazilian Congress of Thermal Engineering and Sciences, Caxambu, Brazil, 2002.

[11] J. R. Camargo, C. D. Ebinuma, and J. L. Silveria, Experimental performance of a direct evaporative cooler operating during summer in Brazilian city, International Journal of Refrigeration, 28 (2005), 1124–1132.

[12] Y. J. Dai and K. Sumathy, Theoretical study on a cross-flow direct evaporative cooler using honeycomb paper as packing material, Applied Thermal Engineering, 22 (2002), 1417–1430.

[13] J. M. Wu, X. Huang, and H. Zhang, Numerical investigation on the heat and mass transfer in a direct evaporative cooler, Applied Thermal Engineering, 29 (2009), 195–201.

[14] J. M. Wu, X. Huang, and H. Zhang, Theoretical analysis on heat and mass transfer in a direct evaporative cooler, Applied Thermal Engineering, 29 (2009), 980–984.

[15] Sodha MS, Singh SP, Sawhney RL. Evaluation of design patterns for direct evaporative coolers. Building and Environment. 1995; 30(2); 287-291.

[16] D. Jain, G.N. Tiwari, Thermal aspects of open sun drying of various crops, Energy 28 (2003) 37–54.