COMPARISON OF POWER FUNCTION DISTRIBUTION BY MEDIAN RANKS AND LEAST SEQUARE REGRESSION METHOD USING OPTIMALLY CONSTRUCTED GROUPED DATA

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Abstract

The objective of this paper is to estimate the parameters and also construct an Optimal Grouped sample in the absence of prior knowledge or guess values of parameters. In this heuristic algorithm, the Median Ranks Regression Method (MRR) is used to find out Estimates the parameters and also Least Square Regression (LSR) estimation of parameters. Compare these two methods by using Average Estimate (AE), Variance (VAR), Mean Absolute Deviation (MAD), Mean Square Error (MSE), Relative Absolute Bias (RAB) and Relative Efficiency (RE) for both the parameters under grouped sample based on 1000 simulations to assess the performance of the estimators.

Keywords: Power Function Parameters, Optimally Grouped sample, Median Ranks Regression, Least Square Regression.

1.1 Introduction

Rider (1964) the name Power Function Distribution has been used. Johnson (1970) given that the moments of the power function distribution are simply the negative moments of the Pareto distribution. Ahsanullah and Kabir et al (1975) discussed the Estimation of the location and scale parameters of a Power function distribution. According to Dallas et al (1976), if Y is power function distribution then Y-1 is the Pareto distribution model. Cohen and Whitten et al (1980) used the estimation in the three parameter lognormal distribution. Rosaiah et al (1991) studied the problem of asymptotically optimal grouping of sample into equiclass grouped sample for maximum likelihood estimation in two parameter gamma distribution. Vasudevarao et al (1994) considered the problem of asymptotically optimal grouping for maximum likelihood estimation in a two parameter Weibull distribution in the case of equispaced group. They also studied the same for maximum likelihood estimation of Weibull shape parameter when scale parameter is known in the case of unequispaced grouped samples. Meniconi and Barry et al(1996) explore the performance of Power function distribution on electrical components and illustrated that power function distribution is most suitable distribution on electrical component data as compared to log-normal,Weibull and exponential models. Theoretically, Kleiber et al (2003) studied power function distribution has an inverse relationship with the standard Pareto distribution, and it is also a special case of Pearson type I distribution. Saran & Pandey et al (2004) estimate the parameters of Power Function Distribution and they also characterize this distribution. Balakrishna et al (2004) and Kantam et al (2005) constructed the Optimum group limits for un-eqi-spaced grouped sample using M. L. Estimation in Scaled Log-Logistic distribution. Reliability Hotwire- The e-Magazine et al (2007) had mentioned the Correlation Coefficient tool in 'How our weibull distribution be good'.CH. Rama Mohan et al (2011) Studied Least Square Estimation of the Weibul parameters from an optimally constructed grouped sample. Rahman, Roy & Baizid et al (2012) applied the Bayesian estimation method to estimate the parameters of Power Function Distribution. Zarrin et al (2013) applied power function distribution to assess component failure of semi-conductor device data by using both the maximum likelihood and Bayesian estimation methods. A Comparison of Maximum Likelihood and Median Rank Regression for Weibull Estimation proposed by Ulrike Genschel William et al (2010). Accelerated Life Test Modeling Using Median Rank Regression (2016) discussed by Austin J. Rhodes. Vijaya lakshmi et al (2018), studied the Estimation of Location (μ) and Scale (λ) for Two-Parameter Rayleigh Distribution by Median Rank Regression Method. Vijaya lakshmi, O. V. Raja Sekharam and G. V. S. R. Anjaneyulu (2018) proposed Estimation of Scale (θ) and Shape (α) Parameters of Power Function Distribution by Least Square Method Using Optimally Constructed Grouped Data. Vijaya lakshmi and Anjaneyulu studied Estimation of Location (μ) and Scale (λ) for Two Parameter Half Logistic Pareto Distribution (HLPD) by Median Rank Regression Method.

The literature mentioned above, reveals that much attention seems to have been paid for inference based on grouped data from two parameter Power Function distribution. In this, when we have no prior Knowledge about the unknown parameters that we used to construct an asymptotically Optimal Groped Data, which can be used to estimation of parameters using Median Ranks Regression Method. The optimal group limits of a grouped sample from two parameter Power Function distribution constructed which are presented at the in the chapter as Table 4.7. Here we developed a practical procedure to construct an optimally grouped sample even when there is no prior knowledge or guess values of the parameters are given in section 4.2. In section 4.3 we made an attempt to study some problems of point estimation from grouped data based on Power Function distribution. The Median Ranks Regression method was used to estimate the parameters from such an optimally constructed grouped sample in two parameter Power Function distribution using the optimal group limits constructed and The Average Estimate (AE), Variance (VAR), Mean Square Error (MSE), Relative Absolute Bias (RAB) and Relative Error (RE) of the Scale parameter (θ) and Scale (α) are calculated for assessing the performance of the estimated parameters.

Let y1, y2, …,yn be a raw sample of size 'n' dawn from two-parameter Power Function distribution with unknown scale(θ) and shape(α) parameters. The Probability density (p.d.f.) and cumulative distribution function (c.d.f.) of Power Function distribution are respectively given by

$$f(y;\theta,\alpha) = \frac{\alpha y^{\alpha-1}}{\theta^{\alpha}}, 0 < y < \theta,\ \alpha > 0,\ \theta > 0 \qquad (1.1.1)$$

$$F(y;\theta,\alpha) = \left(\frac{y}{\beta}\right)^{\alpha}, 0 < y < \theta,\ \alpha > 0, \theta > 0 \qquad (1.1.2)$$

1.2. OPTIMALLY CONSTRUCTED GROUPED SAMPLE BY USING MRR

In this section, we develop a practical procedure to construct an optimally grouped sample in the case when there is no a priori knowledge or guess values of the parameters. In this procedure, we will prefix the number of test units to be failed in each group of the optimal grouped sample and then we record some arbitrary time point after failure of the number of the test units that are to be failed in that group, but before starting the failure of a test unit in the next group. Suppose N is the number of test units put under a life-testing experiment which assumes the Power model (4.1.1) and suppose the experimenter wishes to obtain the grouped life-time data with k classes. Then Table 4.7 can be used to compute the expected number of test units to be failed in the time interval (ti-1, ti) and is given by fi = Npi ; For i= 1, 2, …, k (1.2.1)

Where pi = $\frac{1}{\theta^{\alpha}}[(x_{i-1})^{\alpha} - (x_i)^{\alpha}]$

and

$F(Xi) = \frac{i-0.3}{N+0.4}$; i = 1,2,3,…,k

1. fi is expected number of failures in the ith interval

2. xi' s are optimal group limits obtained from the above procedure

3. k is number of groups

4. N is total frequency

1.3. OPTIMALLY CONSTRUCTED GROUPED SAMPLE BY LSR

In this section, we develop a practical procedure to construct an optimally grouped sample in the case when there is no a priori knowledge or guess values of the parameters. In this procedure, we will prefix the number of test units to be failed in each group of the optimal grouped sample and then we record some arbitrary time point after failure of the number of the test units that are to be failed in that group, but before starting the failure of a test unit in the next group. Suppose N is the number of test units put under a life-testing experiment which assumes the Power model (1.1.1) and suppose the experimenter wishes to obtain the grouped life-time data with k classes. Then Table 1.5 and 1.6 can be used to compute the expected number of test units to be failed in the time interval (ti-1, ti) and is given by

fi = Npi ; For i= 1, 2, …, k (1.3.1)

Where pi = F(Xi-1) – F(Xi)

$= \frac{1}{\theta^{\alpha}}[(x_{i-1})^{\alpha} - (x_i)^{\alpha}]$

$F(Xi) = \frac{i}{N+1}$, i= 1,2,3,…,N (1.3.2)

fi 's may be rounded to the nearest integers so that N=f1+ f2+…..+f k. Thus, the optimal group limits may be used to compute the expected optimum number of test units to be failed in ith interval (ti-1, ti), for i=1,2,…, k. Here, it may be noted that the experimenter has to observe the random time instants y1, y2, …, yk-1 so as the optimum pre-fixed number of units fi to be failed in the time interval (yi-1, ti) for i=1, 2,…, k taking to=0 and tk = ∞. In other words, record a random time instant after failure of first f1 test units, but before the failure of (f1+1)th test unit and to record a random time instant after failure of first f1+ f2 test units, but before the failure of (f1+ f2 +1)th test unit and so on. Further, it may be noted that it is difficult to record all exact failure times of the individual units but, it is not so difficult to note a random time instant between the failure times of two consecutive test units.

METHODOLOGY

1.3. MEDIAN RANKS REGRESSION ESTIMATION OF THE PARAMETERS FROM THE OPTIMALLY CONSTRUCTED EQUISPACED GROUPED SAMPLE

We know that t1, t2, …, tk-1the group limits of te optimally constructed grouped Sample using the procedure explained in the above section, are the observed values of the true asymptotic optimal group limits x1,x2, …, xk-1 where as their estimated values are given by

$$\hat{y}_i = \theta\,(x_i)^{\frac{1}{\alpha}} \qquad (1.3.1)$$

where $\hat{\beta}$ and $\hat{\gamma}$ are obtained by using the principle of Median Ranks Regression method (MRR) is extensively used in reliability engineering and mathematics problems. According to the Median Ranks Regression method (MRR) linear relation between the two parameters taking the natural logarithm of above equation as follows

$$\log t_i = \log(\theta) + \left(\frac{1}{\alpha}\right)\log(x_i) \text{ for i= 1, 2, …, k-1} \qquad (1.3.2)$$

After simplification, we get

$Y_i = \log t_i$, $A = \log(\theta)$, $B = \frac{1}{\alpha}$, $X_i = \log(x_i)$

Thus, equation (1.3.2) is a linear equation and is expressed as

$$Y_i = A + BX_i$$

To compute a and d by simple linear regression we proceed as follows

Let $$S(A,B) = \sum_{i=1}^{k-1}(y_i - A - Bx_i)^2 \qquad (4.3.3)$$

Differentiating (1.3.3) w.r.t to A and B then equate to zero, we obtain the following two normal equations

$$\sum_{i=1}^{n} y_i = nA + B\sum_{i=1}^{n} x_i \qquad (1.3.4)$$

$$\sum_{i=0}^{n} x_i y_i = A\sum_{i=1}^{n} x_i + B\sum_{i=0}^{n} {x_i}^2 \qquad (1.3.5)$$

Solving the above two equations for A and B, we obtain the Median Ranks Regression estimates (MRRE) of A and B as:

$$A = \bar{y} - B\,\bar{x}$$

$$B = \frac{\sum_{i=1}^{k-1} x_i y_i - \frac{\left(\sum_{i=1}^{k-1} x_i y_i\right)}{k-1}}{\sum_{i=1}^{k-1} {x_i}^2 - \frac{\left(\sum_{i=1}^{k-1} x_i\right)^2}{k-1}}$$

$$A = \frac{\sum_{i=1}^{k-1}\log t_i}{k-1} - B\,\frac{\sum_{i=1}^{k-1}\log x_i}{k-1} \qquad (4.3.6)$$

$$B = \frac{\sum_{i=1}^{k-1}\log(x_i)(\log t_i) - \frac{\left(\sum_{i=1}^{k-1}\log x_i\right)\left(\sum_{i=1}^{k-1}\log t_i\right)}{k-1}}{\sum_{i=1}^{k-1}(\log(x_i))^2 - \frac{\left(\sum_{i=1}^{k-1}\log x_i\right)^2}{k-1}} \qquad (1.3.7)$$

where $A = \log(\theta)$ and $B = \frac{1}{\alpha}$

Therefore $$\hat{\theta} = \text{Antilog}\left\{\frac{\sum_{i=1}^{k-1}\log t_i}{k-1} - B\,\frac{\sum_{i=1}^{k-1}\log x_i}{k-1}\right\} \qquad (1.3.8)$$

and $$\hat{\alpha} = \frac{\sum_{i=1}^{k-1}(\log(x_i))^2 - \frac{\left(\sum_{i=1}^{k-1}\log x_i\right)^2}{k-1}}{\sum_{i=1}^{k-1}\log(x_i)(\log t_i) - \frac{\left(\sum_{i=1}^{k-1}\log x_i\right)\left(\sum_{i=1}^{k-1}\log t_i\right)}{k-1}} \quad (1.3.9)$$

The rationale for applying least square method is that for a given k, xi's, are fixed values and are can be borrowed from Table 1.5 where as ti's are random values and are obtained as observations from the experiment. . It may be noted that the least square estimates, $\hat{\theta}$ and $\hat{\alpha}$ obtained from the equations (1.3.8) and (1.3.9)

1.4. LEAST SQUARES ESTIMATION OF THE PARAMETERS FROM THE OPTIMALLY CONSTRUCTED GROUPED SAMPLE

We know that t1, t2, …, tk-1the group limits of te optimally constructed grouped Sample using the procedure explained in the above section, are the observed values of the true asymptotic optimal group limits x1,x2, …, xk-1 where as their estimated values are given by

$$\tilde{t_i} = \theta\,(x_i)^{\frac{1}{\alpha}} \quad (1.4.1)$$

Where $\tilde{\theta}$ and $\tilde{\boldsymbol{\alpha}}$ are obtained by using the principle of least square method (LSM) is extensively used in reliability engineering and mathematics problems. According to the least square method (LSM) linear relation between the two parameters taking the natural logarithm of above equation as follows

Therefore $$\tilde{\theta} = \text{Antilog}\left\{\frac{\sum_{i=1}^{k-1}\log t_i}{k-1} - B\,\frac{\sum_{i=1}^{k-1}\log x_i}{k-1}\right\} \quad (1.4.1)$$

and $$\tilde{\boldsymbol{\alpha}} = \frac{\sum_{i=1}^{k-1}(\log(x_i))^2 - \frac{\left(\sum_{i=1}^{k-1}\log x_i\right)^2}{k-1}}{\sum_{i=1}^{k-1}\log(x_i)(\log t_i) - \frac{\left(\sum_{i=1}^{k-1}\log x_i\right)\left(\sum_{i=1}^{k-1}\log t_i\right)}{k-1}} \quad (1.4.2)$$

The rationale for applying least square method is that for a given k, xi's, are fixed values and are can be borrowed from Table 1.6 where as ti's are random values and are obtained as observations from the experiment. . It may be noted that the least square estimates, $\tilde{\theta}$ and $\tilde{\boldsymbol{\alpha}}$ obtained from the equations (1.4.1) and (1.4.2).

Performance Indices: Goodness of Fit Analysis:

1.4. Comparison of MRR estimators by LSR

we compare with the corresponding MRR and LSR estimators obtained from grouped sample as well as asymptotically optimal grouped sample based on variance.

If $\omega\hat{}_{lm}$ is Median Ranks Method estimate of $\widehat{\omega}_m$, m=1, 2 where $\omega_m$is a general notation that can be replaced by $\omega_1 = \theta, \omega_2 = \alpha$ based on sample l,(l=1,2,…,r) then The Average Estimate (AE), Variance (VAR), Standard Deviation (STD), Mean Square Error (MSE) and Relative Absolute Bias (RAB) are given respectively by

$$\text{Average Estimate }(\widehat{\omega}_m) = \frac{\sum_{i=1}^{r}\widehat{\omega}_{lm}}{r}$$

$$\text{Variance }(\widehat{\omega}_m) = \frac{\sum_{i=1}^{r}(\widehat{\omega}_{lm} - \overline{\widehat{\omega}_{lm}})^2}{r}$$

$$\text{Standard Deviation }(\widehat{\omega}_m) = \sqrt{\frac{\sum_{i=1}^{r}(\widehat{\omega}_{lm} - \overline{\widehat{\omega}_{lm}})^2}{r}}$$

Mean Absolute Deviation = $\frac{\sum_{i=1}^{r} Med(|\widehat{\omega}_{lm} - \overline{\widehat{\omega}_{lm}}|)}{r}$

Mean Square Error $(\widehat{\omega}_m)$ = $\frac{\sum_{i=1}^{r}(\widehat{\omega}_{lm} - \omega_m)^2}{r}$

Relative Absolute Bias $(\widehat{\omega}_m)$ = $\frac{\sum_{i=1}^{r}|(\widehat{\omega}_{lm} - \omega_m)|}{r\omega_m}$

If $\omega\hat{}_{lm}$ is Median Ranks Method estimate of $\widetilde{\omega_m}$, m=1, 2 where ω_m is a general notation that can be replaced by $\omega_1 = \theta$, $\omega_2 = \alpha$ based on sample l,(l=1,2,…,r) then The Average Estimate (AE), Variance (VAR), Standard Deviation (STD), Mean Square Error (MSE) and Relative Absolute Bias (RAB) are given respectively by

Average Estimate $(\widetilde{\omega_m}) = \frac{\sum_{i=1}^{r}\widetilde{\omega}_{lm}}{r}$

Variance $(\widetilde{\omega_m})$ = $\frac{\sum_{i=1}^{r}(\widetilde{\omega}_{lm} - \overline{\widetilde{\omega}\hat{}_{lm}})^2}{r}$

Standard Deviation $(\widetilde{\omega_m})$ = $\sqrt{\frac{\sum_{i=1}^{r}(\widetilde{\omega}_{lm} - \overline{\widetilde{\omega}\hat{}_{lm}})^2}{r}}$

Mean Absolute Deviation = $\frac{\sum_{i=1}^{r} Med(|\widetilde{\omega}_{lm} - \overline{\widetilde{\omega}_{lm}}|)}{r}$

Mean Square Error $(\widetilde{\omega}_{lm})$ = $\frac{\sum_{i=1}^{r}\widetilde{\omega}_{lm} - \omega_m)^2}{r}$

Relative Absolute Bias$(\widetilde{\omega}_{lm})$ = $\frac{\sum_{i=1}^{r}|\widetilde{\omega}_{lm} - \omega_m)|}{r\omega_m}$

CONCLUSION:

1. Variances of the estimators are decreasing as number of groups increases.

2. The estimates obtained from optimal grouped sample with equispaced efficient than the

   unequispaced sample when number of sample increaases .

3. When compared with small sample, the estimators in large sample are more efficient.

   Random Generated values of Power Function Distribution.

4. When we compare MRR and LSR. MRR is most efficient than LSR

AN ILLUTRATION:

A random sample of 200 observations is generated from a two-parameter Power function distribution with the θ = 4, α = 3 using R Software and the ordered sample is given below:

Table 5.1

0.5643	0.68283	0.6911	0.97532	1.03033	1.27493	1.34438	1.35793	1.37605	1.39571
1.42772	1.50664	1.56168	1.58275	1.60049	1.61385	1.68193	1.73975	1.75911	1.7654
1.83026	1.85858	1.86084	1.92251	1.94428	2.03693	2.03803	2.06997	2.10523	2.13578
2.19005	2.20756	2.23161	2.24989	2.26332	2.33308	2.34107	2.36156	2.3817	2.40646
2.4133	2.45333	2.50304	2.53859	2.54071	2.54162	2.54213	2.54796	2.56023	2.58833
2.59008	2.61792	2.63247	2.63397	2.65159	2.67836	2.67836	2.6893	2.75054	2.76331
2.76442	2.77519	2.80024	2.82289	2.83388	2.83857	2.84116	2.86014	2.868	2.87637
2.88053	2.88664	2.89171	2.90234	2.91327	2.95076	2.95896	2.96527	2.96534	2.98169
3.01012	3.03096	3.04479	3.06924	3.07366	3.10118	3.10186	3.13844	3.14714	3.15311
3.15971	3.16642	3.17645	3.19837	3.20965	3.21275	3.22231	3.22369	3.23461	3.24516
3.24584	3.2595	3.26726	3.2819	3.28449	3.28751	3.29124	3.29244	3.32526	3.32797
3.33478	3.34447	3.36305	3.36713	3.37726	3.38011	3.40641	3.40971	3.42271	3.47709
3.49269	3.49684	3.50285	3.50878	3.51047	3.51507	3.51686	3.52939	3.545	3.55111
3.55214	3.55719	3.56012	3.56186	3.56407	3.56683	3.57097	3.58855	3.59229	3.60441
3.6154	3.61878	3.63457	3.64284	3.64416	3.65282	3.65861	3.67522	3.67527	3.67835
3.68766	3.69621	3.70747	3.70814	3.71659	3.71664	3.72201	3.72356	3.72731	3.72993
3.73866	3.74076	3.75794	3.78971	3.79003	3.80214	3.80245	3.80866	3.81381	3.82305
3.83982	3.85074	3.85438	3.85736	3.85819	3.86068	3.86748	3.86774	3.87422	3.87664
3.87924	3.89823	3.899	3.90011	3.90276	3.90447	3.93014	3.93452	3.93813	3.94383
3.95693	3.9571	3.95997	3.963	3.96734	3.96892	3.98111	3.98165	3.98829	3.99695

The Average Estimate (AE), Variance (VAR), Standard Deviation (SD), Mean Square Error (MSE), Relative Absolute Bias (RAB) and Relative Error (RE) of the Scale parameter (θ) when Shape parameter (α) is known under complete data based on 1000 simulations. Population parameter values are θ= 4 & α = 3.

Table 1.4

Power Function distribution using MRR

N	K	AE	VAR	MSE	RAB	SE
50	4	4.2687	0.0398	0.0722	0.008097	0.028213
	6	4.2359	0.0384	0.055649	0.264744	0.027713
	8	4.0296	0.0375	0.000876	0.25185	0.027386
	10	4.0224	0.0355	0.000502	0.2514	0.026646
100	4	4.1052	0.002767	0.011067	0.0263	0.00526
	6	4.1125	0.003164	0.012656	0.028125	0.005625
	8	4.1804	0.008136	0.032544	0.0451	0.00902
	10	4.007	1.23E-05	4.90E-05	0.00175	0.00035
200	4	3.9876	3.84E-05	0.000154	0.0031	0.00062
	6	4.0069	7.94E-06	4.76E-05	0.001725	0.000282
	8	3.9875	1.95E-05	0.000156	0.003125	0.000442
	10	3.9945	3.03E-06	3.03E-05	0.001375	0.000174

Table 1.5

Power Function distribution using LSR

N	K	AE	VAR	MSE	RAB	SE
50	4	3.3879	0.093667	0.374666	0.153025	0.030605
	6	3.3598	0.068309	0.409856	0.16005	0.036962
	8	3.0296	0.11771	0.941676	0.2426	0.04852
	10	3.0224	0.09557	0.955702	0.2444	0.04372
100	4	3.2087	0.156539	0.626156	0.153025	0.030605
	6	3.5359	0.035898	0.215389	0.16005	0.036962
	8	3.0359	0.116186	0.929489	0.2426	0.04852
	10	3.0244	0.09518	0.951795	0.2444	0.04372
200	4	3.1177	0.156539	0.778453	0.220575	0.044115
	6	3.3211	0.035898	0.460905	0.169725	0.027716
	8	3.0015	0.116186	0.997002	0.249625	0.035302
	10	3.0405	0.09518	0.92064	0.239875	0.030342

Asymptotic optimum group limits Y_i (i=1, 2, …, k-1) in the form $Y_i = \gamma \left(\frac{y}{\beta}\right)^{\gamma}$ ($t_0 = 0, t_\infty$) to estimate Power Function Scale (θ) = 4 and Shape (α) = 3 from a grouped sample are given by MRR and LSR in Table 1.6 and 1.7

Table-1.6

k	x1	x2	x3	x4	x5	x6	x7	x8	x9	x11	x12	x13	x14	
3	0.913	1.75												
4	0.893	2.013	3.256											
5	0.784	2.568	3.021	2.145										
6	0.658	1.356	2.453	2.0147	2.568									
7	0.587	2.013	2.018	2.897	3.2145	3.254								
8	0.237	0.856	1.256	1.078	2.147	2.014	3.586							
9	0.365	0.475	1.352	1.982	2.589	2.658	3.214	3.689						
10	0.214	0.586	1.025	1.874	1.874	2.247	2.985	3.487	3.986					
11	0.147	0.201	0.021	1.863	0.269	0.478	1.024	2.548	2.457	3.88				
12	0.25	0.5790.	0.745	1.269	3.248	1.385	1.458	2.004	2.879	3.254	3.568			
13	0.536	0.247	0.658	1.124	2.781	1.852	1.985	2.147	2.014	2.1478	3.247	3.894		
14	0.214	0.452	0.546	0.8552	1.258	1.358	1.478	1.547	2.982	2.698	2.0145	3.0214	3.925	

Table-1.6

k	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11	x12	x13
4	0.027	0.987	1.2385										
5	0.018	0.8546	1.2256	2.3178									
6	0.0025	0.846	1.1458	2.4245	2.496								
7	0.002	0.689	1.1356	2.589	2.5942	2.564							
8	0.001	0.556	1.0247	2.5966	2.684	2.478	2.704						
9	0.001	0.587	1.0369	2.3457	2.699	2.357	2.706	2.72					
10	0.001	0.423	1.0235	2.3947	2.6985	2.247	2.705	2.72	2.8406				
11	0.0002	0.489	1.154	2.3247	2.578	2.224	2.624	2.45	2.704	2.847			
12	0.0001	0.3745	1.2354	2.278	2.4783	2.145	2.547	2.26	2.589	2.8369	2.987		
13	0.0001	0.3698	1.0025	2.1457	2.2893	2.136	2.104	2.15	2.471	2.7854	2.996	3.015	
14	1.00E-05	0.2487	1.0014	2.1004	2.1478	2.129	2.235	2.16	2.3524	2.7451	2.274	3.334	3.457