STUDY OF FLOW CHARATERISTICS OF TWO FLUID CASSON MODEL IN AN ARTERY UNDER VARIOUS CONDITIONS

[1]Dr. Pankaj mathur,[2]Dr. Surekha Jain
[1]Associate Professor, [2]Assistant professor
Department of mathematics
Government College, Tonk (Raj.)

Abstract:Blood flow through stenosed inclined artery under the influence of body acceleration taking blood as Two-layered blood flow using Casson fluid in core region and Newtonian fluid in peripheral region is investigated. Velocity, Wall shear stress, plug velocity under the influence of body acceleration in an inclined stenosed artery calculated and presented graphically. Comparative study of variation of velocity w.r.t. radial distance for different values of inclination angle artery for two-fluid Bingham plastic modeland two fluid Casson model is also presented in figure.

I.INTRODUCTION

The human body is adapted to changes in velocity profiles and velocity distributions of blood to a certain extent. But when the changes in the velocity are of a very large magnitude and extend over a long period of time, there is evidence that such velocity changes may bring about serious physiological effects, which may sometimes lead tofatality. It is therefore desirable to set a standard for long and short terms exposures of human beings to such acceleration. Misra and Pal [1999] developed a mathematical model for the study of the pulsatile flow of blood under externally imposed body acceleration, by treating blood as a Non-Newtonian fluid, using a biviscosity model.Arthur et.al [2015] analyzed Casson Fluid Flow over a Vertical Porous Surface with Chemical Reaction in the Presence of Magnetic Field. Priyadharshini and Ponalagusamy [2019] the magnetohydrodynamics effects on flow parameters of blood carrying magnetic nanoparticles flowing through a stenosed artery under the influence of periodic body acceleration are investigated. Blood is assumed to behave as a Casson fluid. The flow of blood through a multi-stenosed artery under the influence of external applied magnetic field is investigated by Bali and Awasthi [2012]. They have considered artery as a circular tube and blood as a Cassonfluid.Nagarani and Sarojamma [2008] studied the pulsatile flow of blood through a stenosed artery under the influence of external periodic body acceleration by modeling blood as a Casson fluid. The non-linear coupled equations governing the flow are solved using perturbation analysis assuming that the Womersley frequency parameter is small which is valid for physiological situations in small blood vessels. A mathematical model is developed to study the steady flow of Casson fluid through an inclined tube of non-uniform cross section with multiple stenoses by Sreenadh S. et. al [2011]. The effects of magnetohydrodynamics on the blood flow when blood is represented as a Casson fluid, along with magnetic particles in a horizontal cylinder is studied by Ali et.al [2017] Two dimensional flow of a non-Newtonian fluid through an asymmetric stenosed artery is analysed under the influence of body acceleration with an external magnetic field on the flow field investigted Shaw et.al [2010] The flow is assumed to be unsteady, laminar, two-dimensional, asymmetric and of pulsatile nature and blood as casson fluid, influenced by externally imposed periodic body acceleration. Sankar et.al [2011] analysed the pulsatile flow of blood through narrow arteries with mild asymmetric stenosis. Blood is modeled as a two-fluid model with the suspension of all the erythrocytes in the core region being treated as Casson fluid and the plasma in the peripheral layer being assumed as Newtonian fluid. Perturbation method is used to solve the resulting coupled implicit system of non-linear partial differential equations.

In this paper we studied the blood flow through stenosed inclined artery under the influence of body acceleration taking blood as Two-layered blood flow using Casson fluid in core region and Newtonian fluid in peripheral region. Flow is assumed pulsatile ,laminar and fully developed.

II. FORMULATION OF THE PROBLEM AND GOVERNING EQUATION:

The governing equation (Navier Stoke's) the two-fluid Casson model in the non dimensional form are

$$\alpha_c^2 \frac{\partial u_c}{\partial t} = 4[1 + esint + Acos(\omega t + \varphi) - \frac{2}{r}\frac{\partial}{\partial r}(r\tau_c) + \frac{sin(al)}{D} \quad ,0 \le r \le R_1(z) \tag{2.1}$$

$$\alpha_N^2 \frac{\partial u_N}{\partial t} = 4[1 + esint + Acos(\omega t + \varphi) - \frac{2}{r}\frac{\partial}{\partial r}(r\tau_N) + \frac{sin(al)}{D} \quad ,R_1(z) \le r \le R(z) \tag{2.2}$$

where $\propto_c^2$ is Womersely parameter in core region,$\propto_N^2$ is Womersely parameter in peripheral region, al is small angle of inclination

The constitutive equation for fluids in the motion in the core region (Casson fluid) and peripheral region (Newtonian fluid) in non dimensional form are given by

$$\frac{\partial u_c}{\partial r}=0 \; if \;\; \tau_c \le \theta, \;\; 0\le r\le R_p(z) \tag{2.3}$$

$$(\tau_c)^{\frac{1}{2}}=\left(-\frac{1}{2}\left(\frac{\partial u_c}{\partial r}\right)\right)^{\frac{1}{2}}+\theta^{\frac{1}{2}}, \qquad \tau_c\ge\theta, \qquad R_p(z)\le r\le R_1(z) \tag{2.4}$$

$$\tau_N=-\frac{1}{2}\left(\frac{\partial u_N}{\partial r}\right), \quad R_1(z)\le r\le R(z) \tag{2.5}$$

where u_c and u_N are velocities and τ_c and τ_N are shear stress of Casson fluid and Newtonian fluid respectively, θ is the yield stress, R_p is the plug radius in the core region.

Geometry of the stenosis in the peripheral region and core region in non-dimensional form becomes(Neeraja and Vidya [2012])

$$R(z)=\begin{cases}1-\delta_p\left(1+\cos\left(\frac{\pi z}{2z_0}\right)\right) & -2z_0\le z\le 2z_0\\ 1 & otherwise\end{cases} \tag{2.6}$$

$$R_1(z)=\begin{cases}\beta-\delta_c\left(1+\cos\left(\frac{\pi z}{2z_0}\right)\right) & -2z_0\le z\le 2z_0\\ \beta & otherwise\end{cases}$$

where $2\delta_p$ and $2\delta_c$ = max. projections of the stenosis in the peripheral and core region respectively, R and R_1 are radius of the stenosed artery with the peripheral region and core region respectively and $4z_0$ is the length of the stenotic region.

Boundary conditions in non-dimensional form becomes

τ_c is finite and $\frac{\partial u_C}{\partial r}=0$ at r = 0 (2.7)

$$u_N=u_s \;\; at \;\; r=R(z) \tag{2.8}$$

$\tau_C=\tau_N, u_c=u_N$ at $r=R_1(z)$ (2.9)

where u_s is the slip velocity.

III. RESULT AND DISCUSSIONS:

The results obtained in this study consist of the expression for velocity profile , expressions for plug radius and expression for wall shear stress using perturbation technique of two-fluid casson model and displayed graphically.

Figure 1 represents velocity distribution w.r.t z for different values of pressure gradient *e* and stenosis height δ.It is noticed that plug velocity increases with *e* and significantly decreases with increasing δ, effect of *e* at starting and end of the constriction profile is more than at the throat of the stenosis. Figure 2 illustrates plug velocity distribution for different values of θ.It shows plug velocity decreases with increasing value of θ.

The results for the wall shear stress of the present study are shown in Fig 3-5 for the different values of the body acceleration parameter *A*, inclination angle *al* and pressure gradient parameter *e* repectively. Figure 3 shows variation of wall shear stress w.r.t. time t in the presence and absence of body acceleration. It shows number of oscillations increases in the presence of body acceleration. This figure reveals that wall shear stress increases as the *A* increases when t Є [0,45°],[135°,225°],and[315°,360°],whereas the reverse trend is observed in rest of the time intervals. It also demonstrates that in the presence of body acceleration wall shear stress decreases in time interval [0,90°] and[180°,270°]and reverse trend is follows in the rest of the time intervals. In the absence of the body acceleration wall shear increases in time interval [0,90°],[270°,360°] and reverse trend is follows in the rest of the time intervals because in this condition only pressure gradient is sine function of time. Figure 4 represents variation of wall shear stress for different *al*. It shows that wall shear stress increases with increasing value of *al*. Figure 5 illustrates variation of wall shear stress w.r.t. time for different values of *e*. This figure reveals that wall shear stress increases as the *e* increases when t Є[0,180°],whereas the reverse trend is observed in rest of the time interval.

IV.CONCLUSION: Main object of the present study is to find out the velocity profile in different cases. This study is useful to determine low velocity region and shows the behavior of blood flow in different conditions.

Figure 6 shows the comparison of velocity distribution of two-layered blood flow taking Bingham-plastic fluid (calculated by us taking blood as Bingham- Plastic fluid in core region) in core region with those of two-layered blood flow taking Casson fluid in core region w.r.t. r for different value of inclination angle. It shows in both conditions velocity increases with increasing value of inclination angle. But velocity of Casson fluid is lower than velocity of Bingham-plastic fluid.

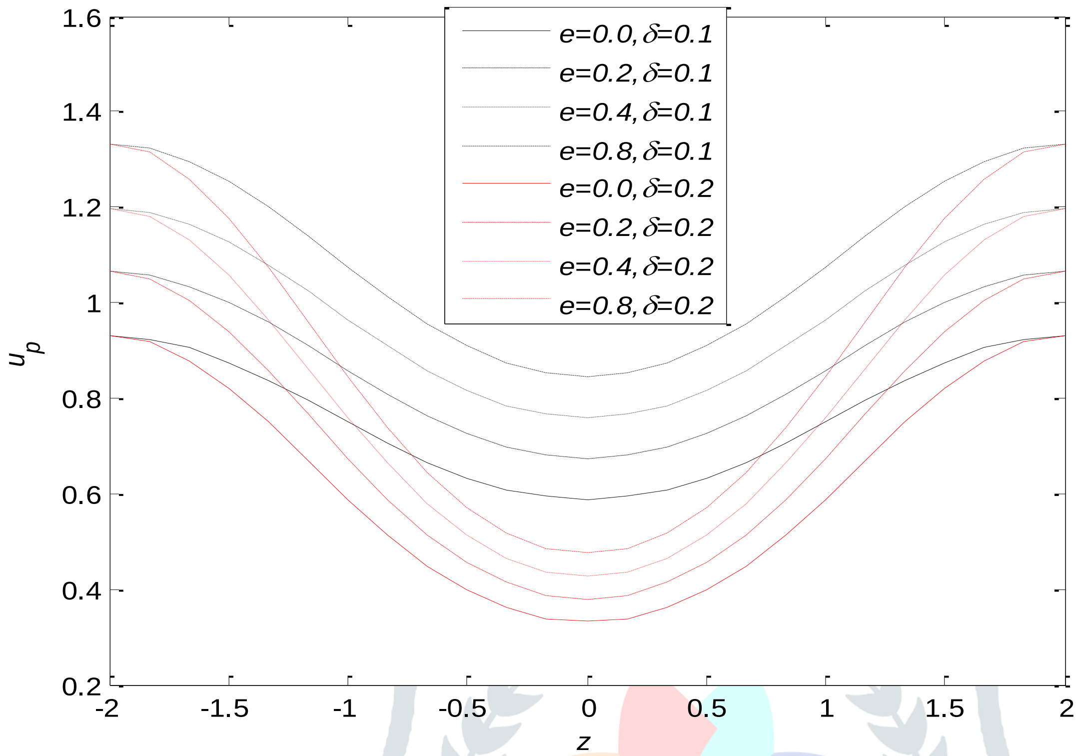

Figure 1 : Variation of plug velocity profile w.r.t. axial distance z for different values of pressure gradient *e* and stenosis height δ

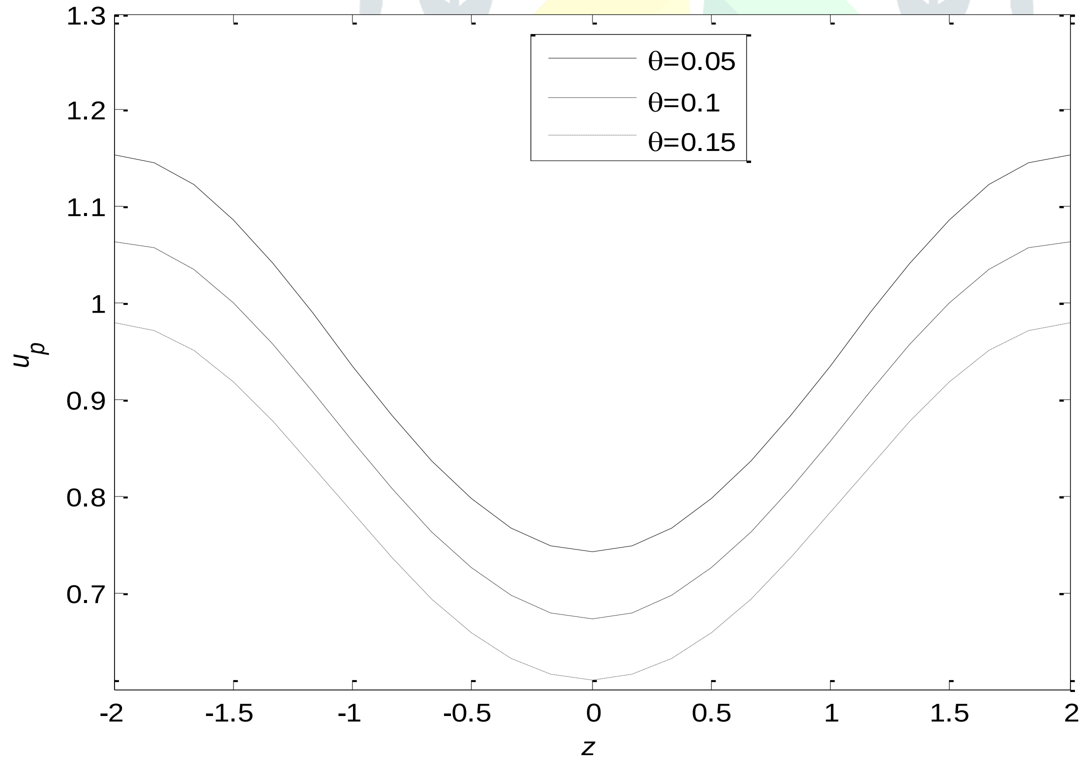

Figure 2: Variation of plug velocity profile w.r.t. axial distance z for different values of yield stress θ

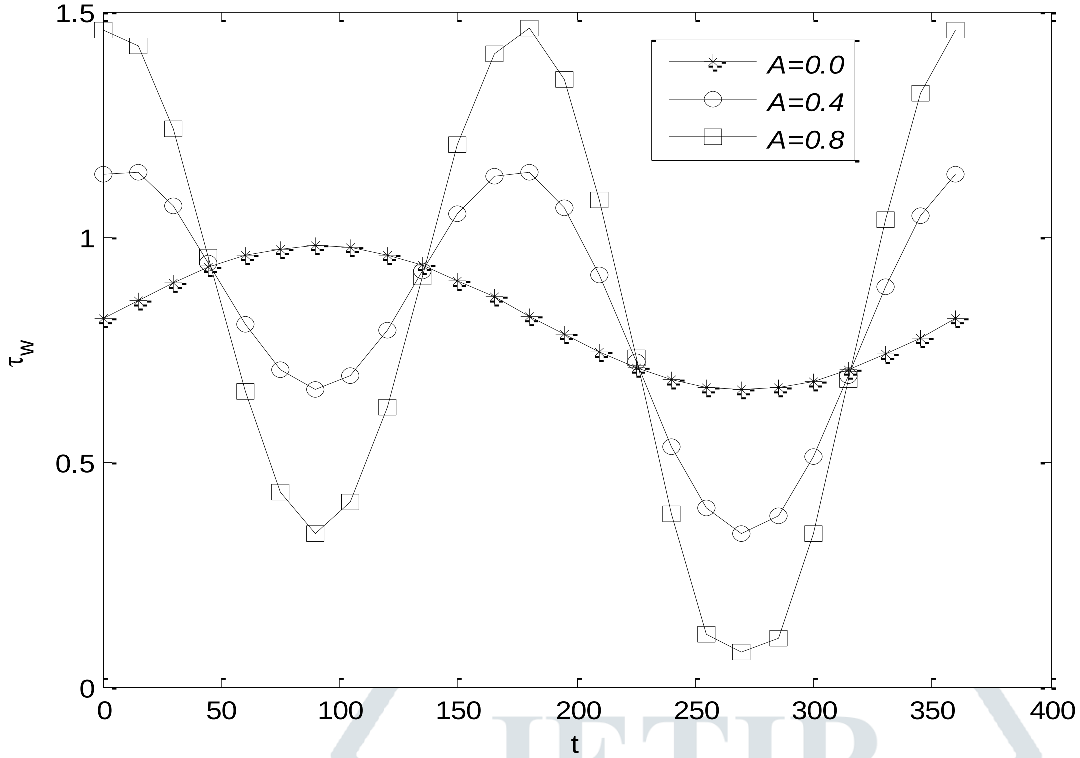

Figure 3: Variation of wall shear stress w.r.t. time t for different values of body acceleration parameter *A*

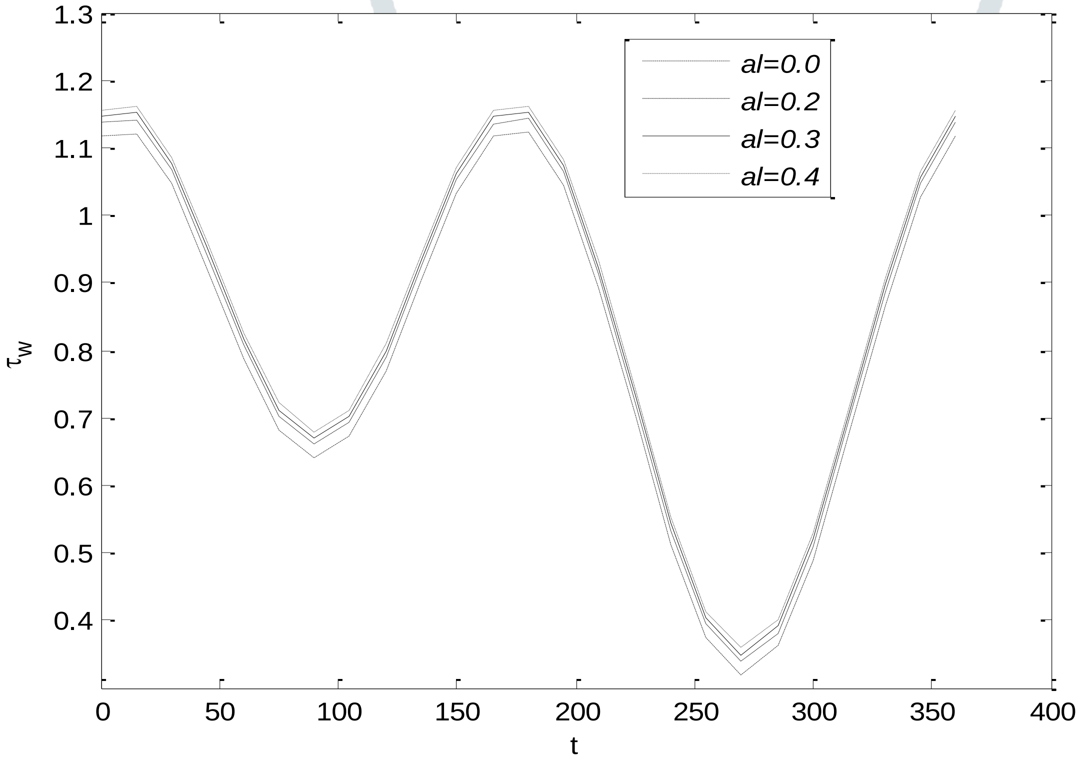

Figure 4 : Variation of wall shear stress w.r.t. time t for different values of inclination angle *al*

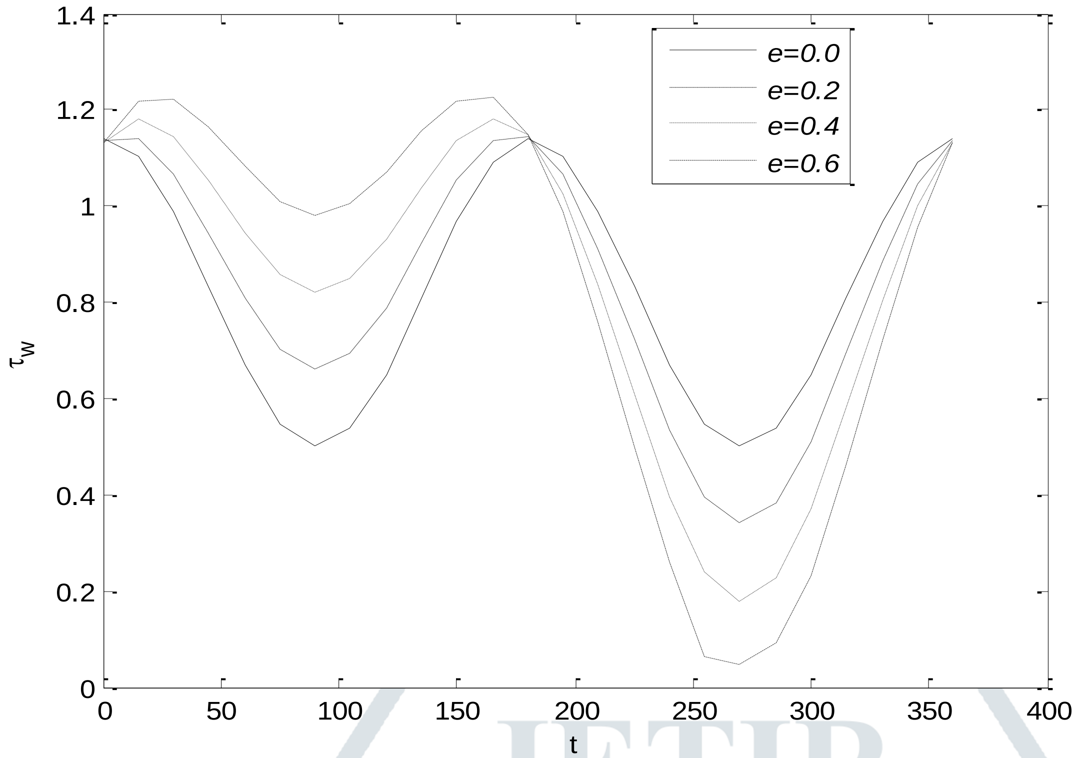

Figure 5 : Variation of wall shear stress w.r.t. time t for different values of pressure gradient parameter *e*

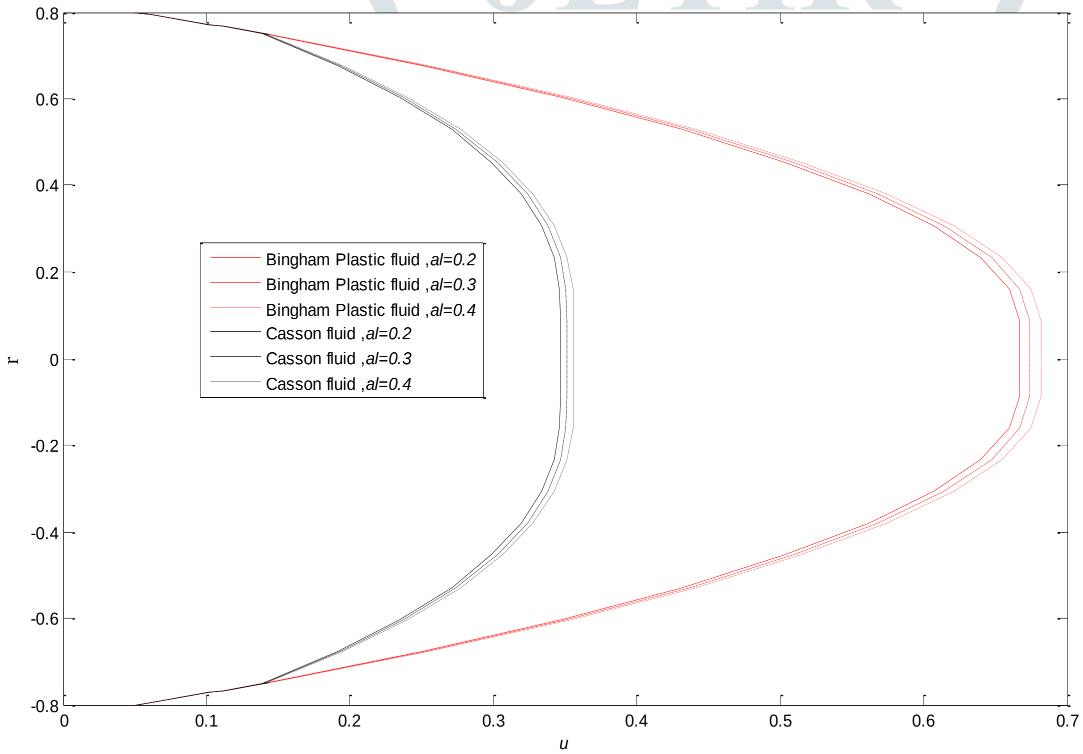

Figure 6 : Comparative study of variation of velocity w.r.t. radial distance r for different values of inclination angle *al* for two-fluid Bingham plastic model and two fluid Casson model

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