Design Optimization for Electrical Submersible Pumps Handling Viscous Fluids

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Abstract

Pumping of viscous fluid like heavy crude oil using Electrical Submersible Pump (ESP) resulted in significant pressure losses, which effect on the ESP performance and efficiency, this situation increases the operating cost, this paper showing the results of optimization of ESP stage geometrical design parameters, for inlet to outlet impeller radii ratio, impeller inlet & outlet angles, diffuser inlet & outlet angles. This is performed using Surrogate method as an easy and reliable method, in this concern in order to enhance ESP performance while pump viscous fluid especially like heavy crude oil, to increase ESP efficiency to reduce operating cost. In this study, a mathematical model was developed for prediction pump performance, by calculation pump theoretical head, then calculation friction losses across a pumping stage to get the actual pump head curve. From the obtained pump head versus flow rate, the output power is calculated and brake power from the shaft torque is calculated by using fluid velocity components, then to prove the reliability of the mathematical model, a correlation to be performed for it against actual pump, then a viscosity correction for pump performance to get pump performance while handling viscous fluid.

Introduction

in this study a mathematical model and its correlation is created, then optimization for pump geometrical parameters is performed using Surrogate method through MATLAB program, first step to build is to do the design of experiment to get data sample to feed the model and check the validity of surrogate model was performed.

On model creation, the results can be applied to deduce the specific influence of each of the pump geometrical parameter on pump efficiency, by conducting screening process using Morris method and get the relation between most effective parameter on the pumpefficiency.

The optimum geometrical parameters for three different values of viscosities were conducted using generic algorism to get the optimum of the studied parameters, which can be applied for further

Literature Review

There is different studies conducted to study of the pump performance while handling viscous fluid as illustrated by table 1, this studies can be classified into three categories:

1. Experimental study to study the actual effect of viscous fluid on the pump performance.
2. Numerical studies use CFD simulation to study the effect of pumping viscous fluid on the pump performance.
3. Optimization studies using mathematical methods of optimization to optimize pump performance during handling either water or viscous fluid.

From this previous work we can conduct of the following:

1. Viscous fluid have negative impact on the pump performance this proven by experimental and numerical investigations .
2. There is mathematical model conducted to predict the performance of ESP, this model validity is proven by experimental investigation.
3. Mathematical model of optimization is used to optimize pump performance, also comparisons between mathematical models optimization results and CFD optimization results shows acceptable match.

Study	Year	Content
Datong Sun and Mauricio Prado	2003	Single-Phase Model for ESP's Head Performance
Jianjun Zhu	2016	CFD simulation and experimental study of oil viscosity effect on multi-stage electrical submersible pump (ESP) performance
Jianjun Zhu	2018	A numerical study on ow patterns inside an electrical submersible pump(ESP) and comparison with visualization experiments
M. H. ShojaeeFard, F. A. Boyaghchi and M. B. Ehghaghi	2004	Experimental Study and Three-Dimensional Numerical Flow Simulation in a Centrifugal Pump when Handling Viscous Fluids
BING Hao	2012	Prediction method of impeller performance and analysis of loss mechanism for mixed-flow pump
F.E. Trevisan, M.G. Prado	2010	Experimental Investigation on the Viscous Effect on Two-Phase Flow Patterns and Hydraulic Performance of Electrical Submersible Pumps
G. M. Paternost, A. C. Bannwart and V. Estevam	2015	Experimental Study of a Centrifugal Pump Handling Viscous Fluid and Two-Phase Flow
H. M. Banjar	2013	Experimental Study of Liquid Viscosity Effect on Two-Phase Stage Performance of Electrical Submersible Pumps
Javier Ibarra	2014	Experimental Inter-stage Study of an Electrical Submersible Pump Handling Viscous Fluid in Multiphase Conditions
A. Joe Ajay	2017	DESIGN AND OPTIMIZATION OF SUBMERSIBLE PUMP IMPELLER

Table 1 – Previous Work

Mathematical model

To build the pump mathematical model for this study, Datong Sun and Mauricio Prado model [1]was used. This model is approximate one-dimensional along ESP channel. The frictionless pressure ODE given by Cooper (1966) for an inducer [2].

Mass Balance Equation

The following equation is yields by the derivation of the one-dimensional mass balance equation [3] in an impeller or diffuser.

$$\frac{\partial}{\partial t}(\rho_l)+\frac{1}{r\sin\beta}\frac{\partial}{\partial s}(r\rho_l W\sin\beta)=0,$$

Where β is the blade angle, s is the streamline coordinate, which is the distance between the entrance to any location along the channel, and t is time, all the geometrical parameters are illustrated as shown in Fig.1.

For incompressible steady-state liquid flow along the ESP channel, relative velocity W is equal,

$$W=\frac{Q_l}{2\pi r H\sin\beta},$$

Where Q is the liquid flow rate and H is the channel height.

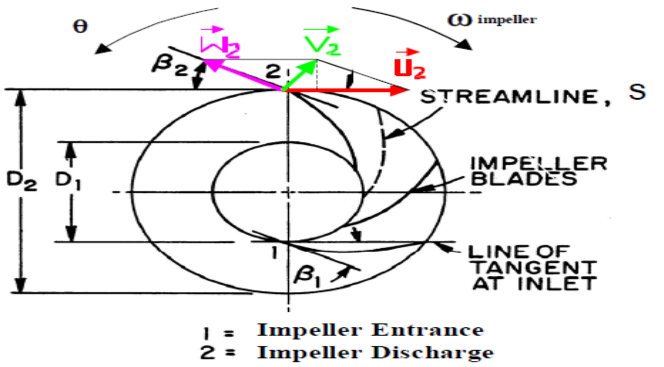


Figure 1 – Sketch of a radial impeller's geometry

Pump Head Equation

The head created by each pump stage includes two parts: first is head created by impeller second is the created by diffuser. First equations assuming frictionless scenario will be presented first. Details of the derivation of the frictionless pressure and head equation can be found in Sun (2002) [3]. Later in this section, the final form of the model, including friction, will be presented.

Frictionless Pressure and Head Equation

Head created by Impeller can be expressed as,

$$H_{impeller}=\frac{V_2^{\,2}-V_1^{\,2}}{2g}+\frac{U_2^{\,2}-U_1^{\,2}}{2g}+\frac{W_1^{\,2}-W_2^{\,2}}{2g},$$

Where U is the peripheral velocity, which is expressed as,

U = ω r

The velocity component along a radial impeller channel is illustrated by Fig. 1.

Since the head created by the diffuser is equal zero for frictionless case, so pump head for a stage is equal to the impeller head,

By using velocity triangle relationships, Euler head can be expressed as,

$$H_e = \frac{U_2^{\,2} - U_1^{\,2}}{g} - \frac{W_2 U_2 \cos\beta_2 - W_1 U_1 \cos\beta_1}{g}.$$

Finally, Euler head H_e can be expressed as [4],

$$H_e = \frac{\omega^2}{g}\left(r_2^{\,2} - r_1^{\,2}\right) - \frac{Q_l \omega}{2\pi\, g\, H}\left(\frac{1}{\tan\beta_2} - \frac{1}{\tan\beta_1}\right).$$

Pressure and Head Equation Including Friction Losses

For case when fluid friction is considered, the friction loss term can be superimposed onto the pressure frictionless ODE equation. The pressure distribution ODE at the radial position r along an ESP channel then becomes:

$$dp = \rho_l \omega^2 r dr - \rho_l \frac{dW^2}{2} + \left(\frac{dp}{dr}\right)_f dr - \rho_l g_s\, ds,$$

Where $\left(\frac{dp}{dr}\right)_f$ is fluid friction pressure radial gradient, which can be related to a pressure gradient along the channel $\left(\frac{dp}{ds}\right)_f$ length position s as,

$$\left(\frac{dp}{dr}\right)_f = \left(\frac{dp}{ds}\right)_f \frac{ds}{dr},$$

the relationship between s and r can be expressed as,

$$\frac{ds}{dr} = \frac{j}{\sin\beta_h \cos\gamma},$$

and j=1 for impeller and j=-1 for diffuser. If the channel has a hydraulic diameter d_H and the fluid relative velocity W to the channel, the term $\left(\frac{dp}{ds}\right)_f$ is given by,

$$\left(\frac{dp}{ds}\right)_f = -f \frac{\rho}{d_H} \frac{W^2}{2},$$

Where f is a friction factor.

Calculation of Friction Factor

To calculate the friction factor, the hydraulic diameter it is required to calculate first which is related to cross-section geometry. An ESP channel with near rectangular cross-section with channel width a and channel height b, illustrated by Fig.2. Both parameters can be obtained from the geometric relationship,

$$a = \frac{2\pi r}{n} \sin\beta,$$

$$b = H,$$

Where n is the number of impeller blades or diffuser blades. The hydraulic diameter, d_H, is expressed as following,

$$d_H = \frac{2ab}{a+b}.$$

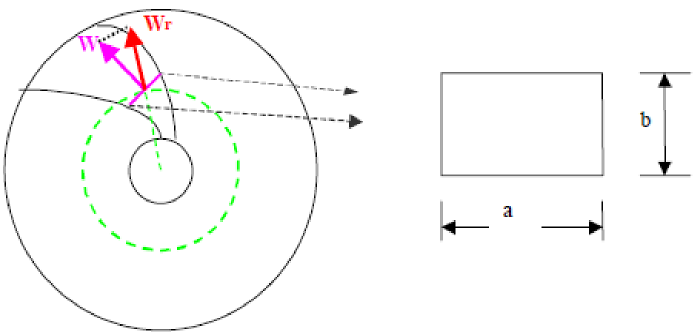


Figure 2 – The shape of A Channel Cross Section

Reynolds Number

The friction factor depends the flow regime inside the channel either is laminar or turbulent. The determination of the flow regime depends on the value of Reynolds number N_{Re}, which is function of the relative velocity W along ESP channels as,

$$N_{\mathrm{Re}} = \frac{d_H W \rho_l}{\mu_l},$$

Where μ is liquid viscosity.

Friction Factor for Straight Stationary Pipes with Circular Cross Sections

The friction factor for laminar flow in a circular, straight, stationary pipe is given by,

$$f_{\text{circular,straight,stationary}} = \frac{64}{N_{\mathrm{Re}}}.$$

The friction factor for turbulent flow in a circular, straight, stationary pipe is given by Churchill (1977) [5] as follows,

$$f_{circlar,straight,stationary} = 8\left[2.457 \ln \frac{1}{\left(\frac{7}{N_{\mathrm{Re}}}\right)^{0.9} + 0.27\left(\frac{\varepsilon}{d_H}\right)}\right]^{-2},$$

Where ε is the absolute roughness of the channel.

Friction factor effects

The conventional friction factor used in a straight, stationary pipe with a circular cross section is not applicable to ESP impeller and diffuser channels. Due to the shape of the ESP channel is curved rectangular cross-section, and the impeller rotates during operation. The flow characterization inside this geometry is different than fluid flow inside straight, stationary pipes with circular cross-sections. The presence of secondary flows inside the impeller and diffuser channels must be considered as pointed out by Schlichting (1955) [6] and Ito (1971) [7].

Cross Section Shape Effect

To calculate Reynolds number with considering the cross section shape effect The works of Shah (1978) [8] and Jones (1976) [9] going to be used to calculate the shape effect on the friction factor for laminar and turbulent flow, respectively.

Critical Reynolds number. The critical Reynolds Number for flow regime transition due to shape effect is:

$$(N_{\mathrm{Re}})_{\text{crit\_rectangular}} = 2300.$$

The "equivalent diameter" d_{eq}, which is defined by using work of Cornish (1928) [10] to calculate the friction factor under laminar flow.

$$d_{eq} = \left[\frac{2}{3} + \frac{11}{24} l\,(2 - l)\right] d_H,$$

Where l is the aspect ratio of the rectangular cross section for liquid defined as,

$$l = \frac{\min(a, b)}{\max(a, b)},$$

The corresponding equivalent Reynolds number N_{Re_eq} is:

$$N_{\mathrm{Re}\_eq} = \frac{d_{eq} W \rho_l}{\mu_l}.$$

Laminar Flow. For fluid flowing inside a rectangular cross-section, straight, stationary pipe under a laminar flow, the friction factor is presented by Shah (1978) as [8],

$$f_{\text{rectangular,straight,stationary}} = \frac{64}{N_{\mathrm{Re}\_eq}}.$$

The multiplication factor $F_{rectangular}$ for laminar flow inside a diffuser or an impeller with a rectangular cross section can finally be written as,

$$F_{rectangular} = \frac{f_{rectangular, straight, stationary}}{f_{circular, straight, stationary}} = \frac{1}{\frac{2}{3} + \frac{11}{24} l\,(2 - l\,)}.$$

Turbulent Flow

The effect of the rectangular cross-section shape on the friction factor for straight, stationary pipes in turbulent flow was studied by Jones (1976) [9].

$$f_{rectangular, straight, stationary} = 0.316 \times N_{Re\_eq}^{-0.25}.$$

One can then obtain the multiplication factor, $F_{rectangular}$ under turbulent flow for a diffuser or impeller with a rectangular cross section as:

$$F_{rectangular} = \frac{f_{rectangular, straight, stationary}}{f_{circular, straight, stationary}} = \frac{1}{\left[\frac{2}{3} + \frac{11}{24} l\,(2 - l\,)\right]^{0.25}}.$$

Pipe Curvature Effect

The Effect of pipe curvature on the friction factor for circular cross-section, stationary pipes has been studied by Ito (1959) [11]. The pipe curvature effect changes the criteria for determination of the flow regime and calculation of friction factor.

Critical Reynolds Number. The transition from laminar to turbulent flow occurs at value for Reynolds number called critical Reynolds number, $(N_{Re})_{crit\_curved\ curvature}$, which is a function of the channel radius of curvature, R_C, and the hydraulic radius,r_H, as follows,

$$(N_{Re})_{crit\_curved} = \begin{cases} 2\times10^{4}\times\left(\frac{r_H}{R_c}\right)^{0.32} & \text{if } \frac{R_c}{r_H} < 860, \\ 2300 & \text{if } \frac{R_c}{r_H} \geq 860 \end{cases}$$

Wherer_H is the hydraulic radius based on the hydraulic diameter given by,

$$r_H = \frac{d_H}{2}.$$

Laminar Flow Friction factor

If the Reynolds number value is less than or equal the critical Reynolds number, ($N_{Re} \leq (N_{Re})_{crit\_curved\ curvature}$), so the flow is laminar.

The laminar flow friction factor also depends on the ratio between the channel radius of curvature and the ESP channel hydraulic radius.

(a) Straight Pipe Approach

If the ratio between the radius of curvature, R_C, and hydraulic radius, r_H, is greater than equal 860, at this case the pipe can be considered straight and the curvature multiplication factor, F_{curved}, is,

$$F_{curved} = \frac{f_{circular, curved, stationary}}{f_{circular, straight, stationary}} = 1.$$

(b) Curvature Effect Approach

If the ratio between the radius of curvature R_c and hydraulic radius r_H is less than 860, effect of curvature must be considered. The friction factor for laminar flow in curved pipes was obtained in this study by fitting White's (1929) empirical curve [11] sketched in Ito (1959), as follows,

$$f_{\text{circular, curved, stationary}}\sqrt{\frac{R_c}{r_H}} = 1.5\left\{\frac{\left[N_{\text{Re}}\left(\frac{r_H}{R_c}\right)^{0.5}\right]}{53}\right\}^{-0.611}.$$

Finally, a multiplication factor for the curvature effect F_{curved} is obtained as,

$$F_{\text{curved}} = \frac{f_{\text{circular, curved, stationary}}}{f_{\text{circular, straight, stationary}}} = 0.266 N_{\text{Re}}^{0.389}\left(\frac{r_H}{R_c}\right)^{0.1945}.$$

Turbulent Flow

If the Reynolds number is greater than the critical Reynolds number ($N_{Re}>(N_{Re})_{crit\_curved}$), this means the flow is turbulent.

There are two equations for turbulent friction factor calculation are available [11].

(a) If $N_{Re}(r_H/R_C)^2 \geq 300$, then,

$$F_{\text{curved}} = \frac{f_{\text{circular, curved, stationary}}}{f_{\text{circular, straight, stationary}}} = \left(N_{\text{Re}}\left(\frac{r_H}{R_c}\right)^2\right)^{0.05}.$$

(b) If $300 > N_{Re}(r_H/R_C)^2 > 0.034$, then,

$$F_{\text{curved}} = \frac{f_{\text{circular, curved, stationary}}}{f_{\text{sircular, straight, stationary}}} = 0.092\left[N_{\text{Re}}\left(\frac{r_H}{R_c}\right)^2\right]^{0.25} + 0.962.$$

(c) If $N_{Re}(r_H/R_C)^2 \leq 0.034$, then,

$$F_{\text{curved}} = \frac{f_{\text{circular, curved, stationary}}}{f_{\text{circular, straight, stationary}}} = 1.$$

For a radial pump, the channel radius of curvature, R_C, can be calculated by the following formula[3]:

$$R_c = \frac{1}{\sin\beta}\frac{1}{-\frac{d\beta(r)}{dr} + \frac{1}{r\tan\beta}}.$$

Rotational Speed Effect

The effect of rotation on the friction factor for straight pipes with circular cross-section was studied by Ito (1971) [7]. He suggested that the flow regime and friction factor for rotational pipes function of the rotational Reynolds number $N_{Re\,\Omega}$ which is defined as,

$$N_{\text{Re}\,\Omega} = \frac{\omega d_H^{\,2}\rho_l}{\mu_l}.$$

If the rotational Reynolds number is less than 28, so the pipe can be considered stationary. If the rotational Reynolds number is equal or greater than 28, rotational speed effects must be considered.

Critical Reynolds Number

In order to distinguishing laminar and turbulent flow, a transition is occur at a critical Reynolds number which is function of the rotational Reynolds number.

$$(N_{Re})_{crit\_rotation} = \begin{cases} 1070(N_{Re\Omega})^{0.23} & \text{if } N_{Re\Omega} \geq 28 \\ 2300 & \text{if } N_{Re\Omega} < 28 \end{cases}$$

Laminar Flow

If $N_{Re} \leq (N_{Re})_{crit\_rotation}$, then the flow laminar.

The friction factor for a rotating pipe under laminar flow conditions depends on the dimensionless parameter $K_{laminar}$ defined as,

$K_{laminar} = N_{Re\,\Omega} N_{Re}$.

The following are the expressions of the rotating multiplication factor under laminar flow.

If $K_{laminar} \leq 220$ and $\frac{N_{Re\Omega}}{N_{Re}} < 0.5$ then,

$$F_{rotation} = \frac{f_{circular,\ straight,\ rotation}}{f_{circular,\ straight,\ stationary}} = 1 .$$

(b) If $220 < K_{laminar} < 10^7$ and $< \frac{N_{Re\Omega}}{N_{Re}}$ 0.5 then,

$$F_{rotation} = \frac{f_{circular,\ straight,\ rotati}}{f_{circular,\ straight,\ station}}$$

$$0.0883 K_{laminar}^{\ 0.25} (1 + 11.2$$

(c) If $\frac{N_{Re\Omega}}{N_{Re}} \geq 0.5$ then,

$$F_{rotation} = \frac{f_{circular,\ straight,\ rotation}}{f_{circular,\ straight,\ stationary}} = \frac{0.0672 N_{Re\,\Omega}^{0.5}}{1 - 2.11 N_{Re\,\Omega}^{-0.5}}$$

Turbulent Flow

If $N_{Re} \leq (N_{Re})_{crit\_rotation}$,, then the flow is considered turbulent.

The friction factor for a rotating pipe under turbulent flow is depending on the dimensionless parameter $K_{turbulent}$ which is calculated by,

$$K_{turbulent} = \frac{(N_{Re\,\Omega})^2}{N_{Re}} .$$

The following are the equations to calculate the rotating multiplication factor for turbulent flow.

(a) If $K_{turbulent} < 1$ then,

$$F_{rotation} = \frac{f_{circular,\ straight,\ rotation}}{f_{circular,\ straight,\ stationary}} = 1 .$$

(b) If $1 \leq K_{turbulent} \leq 15$ then,

$$F_{rotation} = \frac{f_{circular,\ straight,\ rotation}}{f_{circular,\ straight,\ stationary}} = 0.942 + 0.058 K_{turbulent}^{\ 0.282}$$

(c) If $K_{turbulent} > 15$ then,

$$F_{rotation} = \frac{f_{circular,\ straight,\ rotation}}{f_{circular,\ straight,\ stationary}} = 0.942 K_{turbulent}^{\ 0.05}$$

Validation check for the model

After mathematical model was created using the previously stated equation it was required to check the validity of the model against actual measured data, so a comparison between the predicted pump performance and the pump performance from manufacturer catalog using data for the impeller and liquid properties as per table 2 and geometric data for the impeller and diffuser as per table 3.

This single-phase model can predict ESP performance under different fluid viscosities, the results of comparison is illustrated in fig.3.

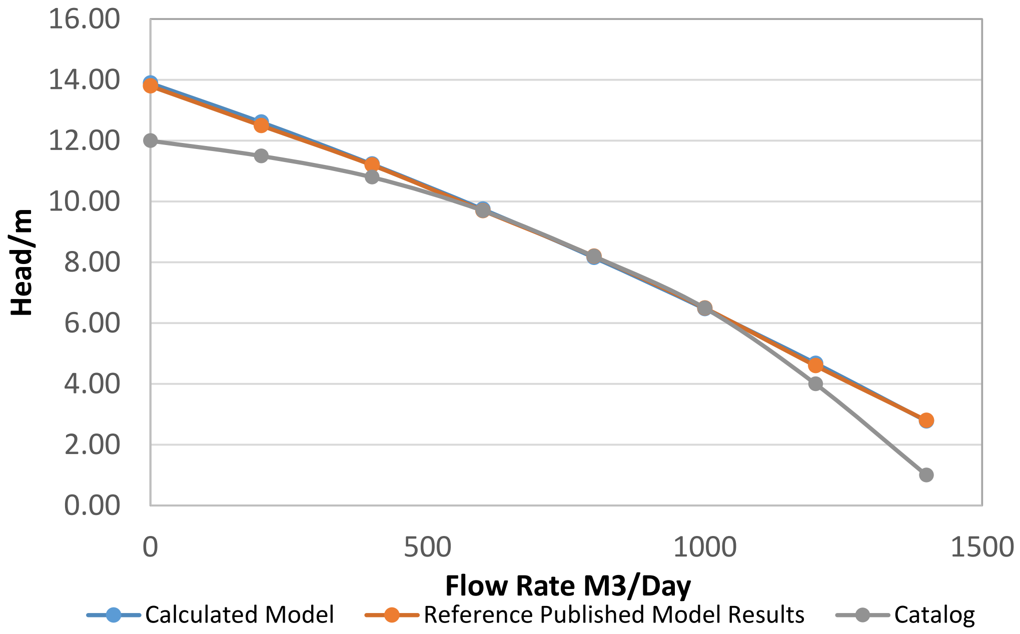


Figure 3 – Mathematical Model Validation Comparison

Brake Power Calculation

P$_{brake}$ = $M \cdot \omega$

Where:

M = torque [Nm]

w = angular velocity [rad/s]

$$P = \rho \cdot Q \cdot \left(r_2 \cdot c_2 \cdot \cos\alpha_2 - r_1 \cdot c_1 \cdot \cos\alpha_1 \right) \cdot \omega$$
$$= \rho \cdot Q \cdot \left(u_2 \cdot c_{u2} - u_1 \cdot c_{u1} \right) \cdot \omega$$
$$= \rho \cdot Q \cdot g \cdot H_t$$

In order to get a better understanding of the different velocities that represent the head we rewrite the Euler's pump equation.

$$H_t = \frac{u_2 \cdot c_{u2} - u_1 \cdot c_{u1}}{g}$$

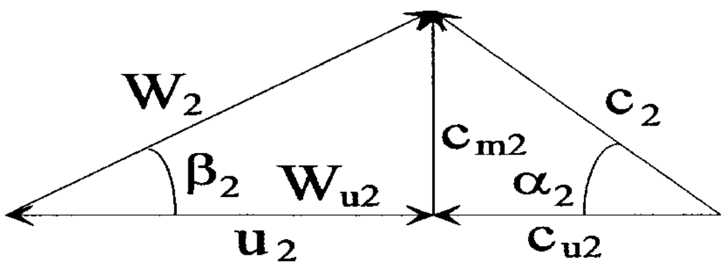


Figure 4 – Pump Velocity triangle Diagram

Pump Efficiency Calculation

To calculate pump eff, hydraulic pump output calculated first as following

$P_{hydraulic}= Q\ \Delta P$

Then: $\eta= P_{hydraulic} \backslash P_{brake}$

The results of calculating brake Power, fluid power and pump efficiency are illustrated at Fig.5.

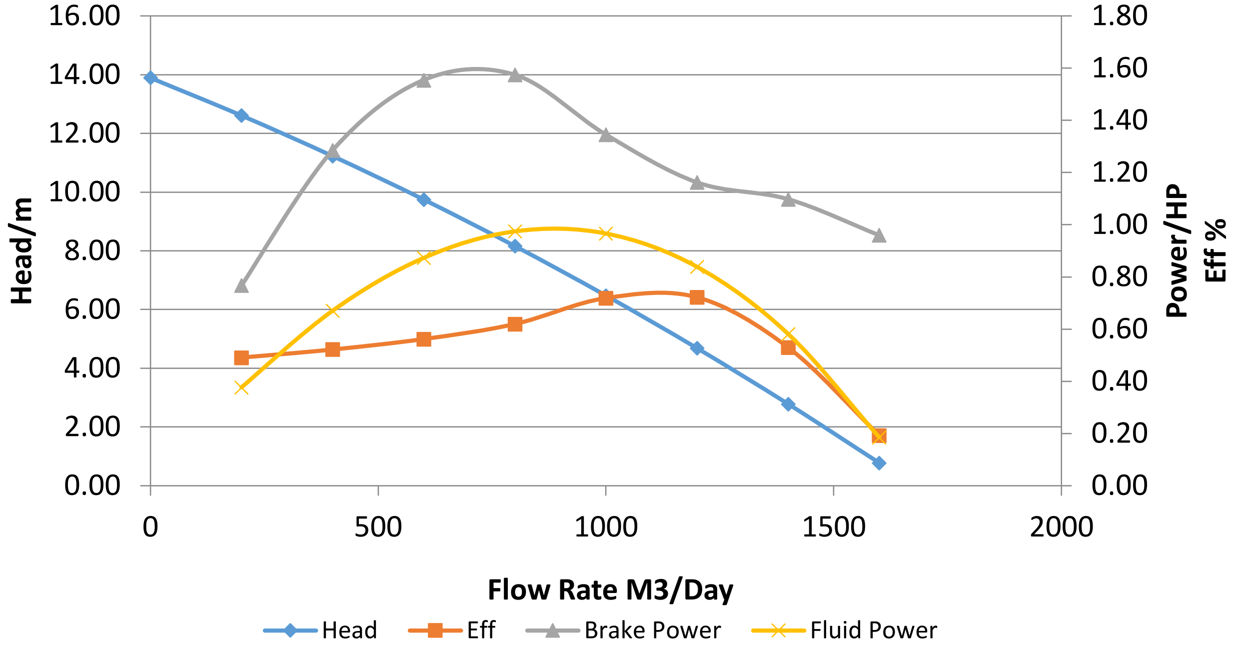


Figure 5 – Pump Performance Chart

Correction of Pump Curves for Viscosity

Viscous liquids cause more hydraulic losses in the pump, so that at greater viscosities, pumping head and pump efficiency decrease while required power increases. The pumping head and pump efficiency curves fall below the corresponding water performance curves, while the shut-off head point remains same, regardless of viscosity.

Hydraulic Institute model

The hydraulic Institute model[12] involves two diagrams for correcting liquid viscosity. The first employs the capacity (pumping rate) Q_{wbep} at the b.e.p. of the water performance curves. This is an independent variable instead of a Reynolds number-like value.

For converting the original hand procedures to numeric one Z.Turzo, G.Takacs & J.Zsugadevelop the following equations to be used instead of old hand procedures[13]:

$Q^{*}= \exp((39.5276+26.5605*\ln(\gamma_{o})- \gamma)/51.6565)$

$\gamma= -7.5946+6.6504*\ln(H_{Wbep})+12.8429*\ln(Q_{wbep})$

$CQ=1.0-4.0327*10^{-3}*Q*-1.7240*10^{-4}*Q*^{2}$

$CH_{1}=1.0-3.6800*10^{-3}Q^{*}-4.3600*10^{-5}*Q^{*2}$

$CH_{2}=1.0-4.4723*10^{-3}Q^{*}-4.18100*10^{-5}*Q^{*2}$

$CH_{3}=1.0-7.00763*10^{-3}Q^{*}-1.4100*10^{-5}*Q^{*2}$

$CH_{4}=1.0-9.0100*10^{-3}Q^{*}-1.3100*10^{-5}*Q^{*2}$

$CH_{\eta}=1.0-3.3075*10^{-2}Q^{*}-2.8875*10^{-4}*Q^{*2}$

The results of correction of pump performance curves for oil at viscosity equal to 300 (cSt), the correction includes pump head curve and pump efficiency, the results are illustrated at Fig.6. to compare between pump performance with water and 300 (cSt) viscous fluid.

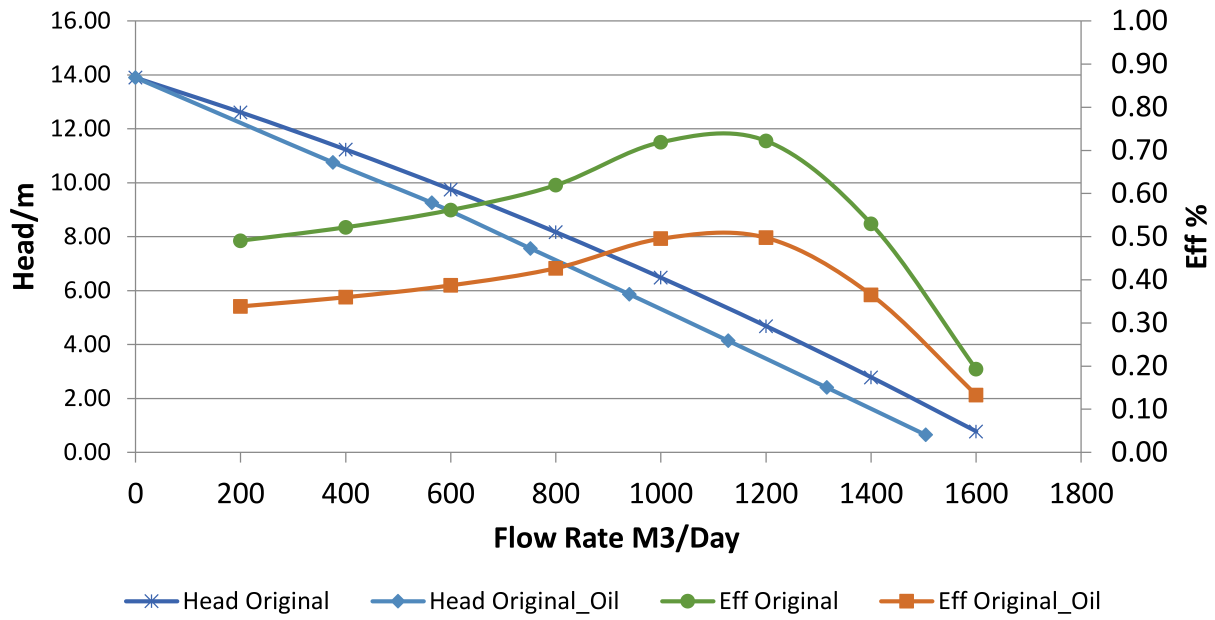


Figure 6 – Corrected Pump Performance for 300 cp Viscous Fluid Against Water

Optimization

Surrogate based optimization technique is an attractive technique, when high-fidelity data is calculated from expensive analysis codes such as CFD simulation. Surrogate modelling is used to greatly improve the design efficiency and be very helpful in finding global optima. The term "surrogate model" has the same meaning as "response surface model", "metamodel", "approximation model", "emulator" etc. The surrogate model is supposed to be cheap, smooth, easy to optimize and yet reasonably accurate so that it can produce a good prediction of the function's optimum.

Surrogate based optimization workflow

In surrogate based optimization studies, the surrogate model can be regarded as an approximation model to estimate the objective function values. Surrogate model is built from sampled data obtained randomly probing in the design space.

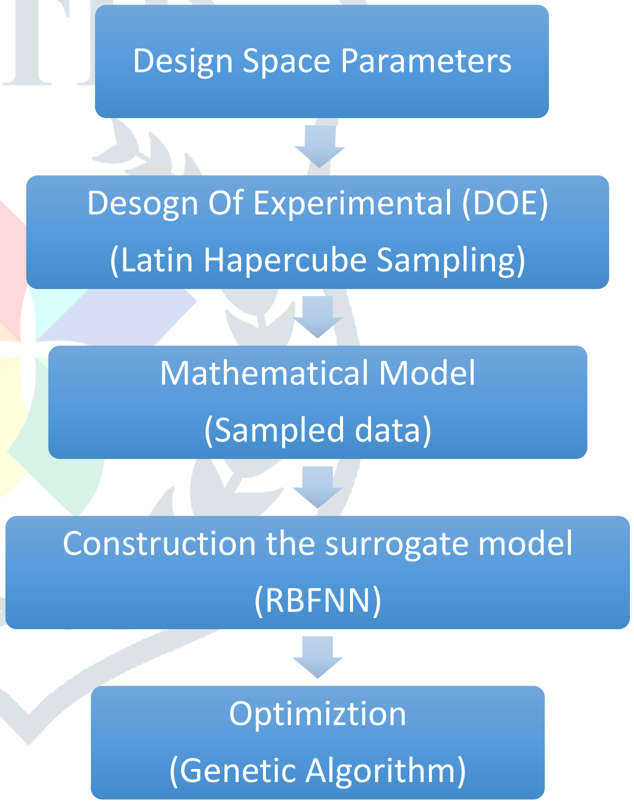


Figure 7 Surrogate based optimization work flow.

Design of numerical experiment

Thus, the design of experiment (DoE) is constructed to get these training data required in establishing surrogate model. Data was collected by mathematical model for the sampled points.

Figure (7) depicts the process which is employed to obtain the optimal design.

DoE techniques have a large influence on the accuracy of the surrogate model. To develop effective surrogate models, it is necessary to distribute the sample points throughout the design space in uniform fashion. DoE was conducted by MATLAB 2017a. There are several built in function available in MATLAB 2017a to distribute the sample points. Among of these techniques are full factorial design (fullfactin MATLAB) and Latin hypercube design (LHS) (lhsdesign in MATLAB) [14].

Genetic algorithms

Genetic algorithms (GA) were invented to mimic some of the processes observed in natural evolution. Genetic Algorithm technique is an adaptive heuristic search algorithm which is using the concept of evolutionary ideas of natural selection and genetics.

The application of the GA involves the following tasks [15]:

1. Create a random initial population.
2. Create a sequence of new population by:
   - Computing fitness value for the current population, and score each member according to its fitness value.
   - Scale the raw fitness scores to convert them into a more usable range of values.
   - Based on their fitness value the parents are selected.
   - Some individuals that have lower fitness value in the current population are selected to pass to the next population, this individuals are called elite.
   - The children are produced from the parents. The Children are produced by one of two ways, first by making random changes to a single parent (mutation), second by combining the vector entries of a pair of Parents (crossover).
   - The current population is replaced by the children to form the next generation.

3. When one of the stopping criteria is met the algorithm is stopped.

Generic Algorithm Setting

In this study, the optimization problem is unconstrained. Table 4 lists the settings used to obtain the optimum design for maximum pump efficiency.

The genetic algorithm stops iterations when average change in the fitness value becomes less than 10^{-6}.

Results and discussion

The main object of this study is to find the optimum of geometrical parameters for radial ESP stage to get the maximum pump efficiency while pump of viscous fluid, so it was first required to define the geometrical parameters of the radial ESP stage which is r1_Impeller\ r2_Impeller, β1_Impeller, β2_Impeller, β1_Diffuser, β2_Diffuser, ɣ1_Diffuser&ɣ2_Diffuser, then set the limits of each one as illustrated by table 5.

After definition of the selected geometrical design parameters of the ESP it was required to define the properties of the fluid used for this study, so for this study the mineral oil used by Javire Ibarra[16] and its viscosity illustrated by fig 8 was selected.

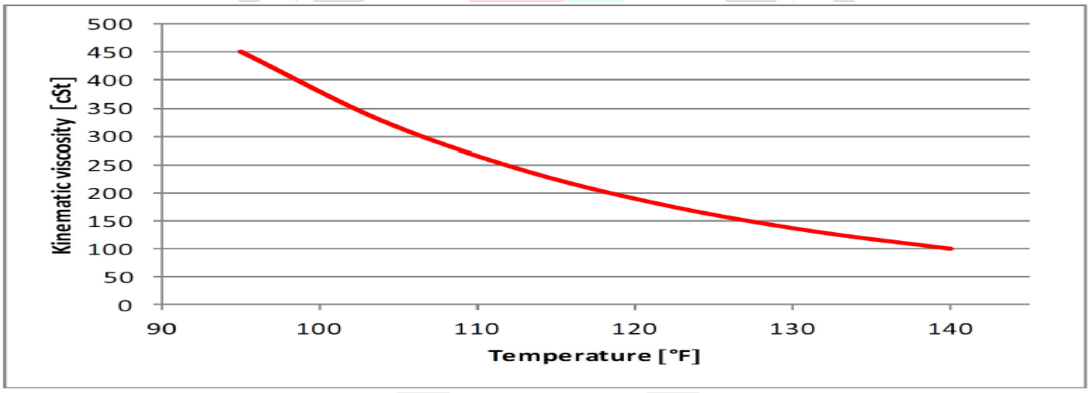


Figure 8—Kinematic viscosity (cSt) of mineral oil as a function of temperature (°F).

After setting the limits of each parameter and select the fluid properties, a design of experiment (DoE) sample data distributed using Latin hypercube sampling (LHS). It divide any individual variable range into a large number of equal sized bins and generates equal sized random subsamples among these bins. MATLAB 2017a provides a set of built in function (lhsdesign) to perform the Latin hypercube sampling plane, thus this method is used in the present study, the results of DoE are illustrated by table 6 shown below and the distribution.

The results of the DoE output are used as input for the mathematical model to calculate the pump efficiency at best efficiency point of the pump.

Since the running temperature of the ESP varied according to the setting depth of the ESP and the geothermal gradient of the well area so three different values of the viscosity at different temperature was selected to present a wide range of temperatures that can

The mathematical model was solved at this three different viscosity values and seven different best efficiency point flow rates to cover a wide range of operating rates to meet the variety of the wells and its most common production rates, this scenarios selected create twenty one different case for simulation those cases are illustrated by table 7.

The results of this scenarios and the sample date distribution for 800 m3/day and 100 (cSt) viscosity is illustrated at fig 9.

Screening of the design parameters

To study the relative importance of the seven design parameters, screening process was conducted using Morris method [17]. This method provide qualitative sensitivity measures of input parameter, i.e. it rank the input parameters in order of importance.

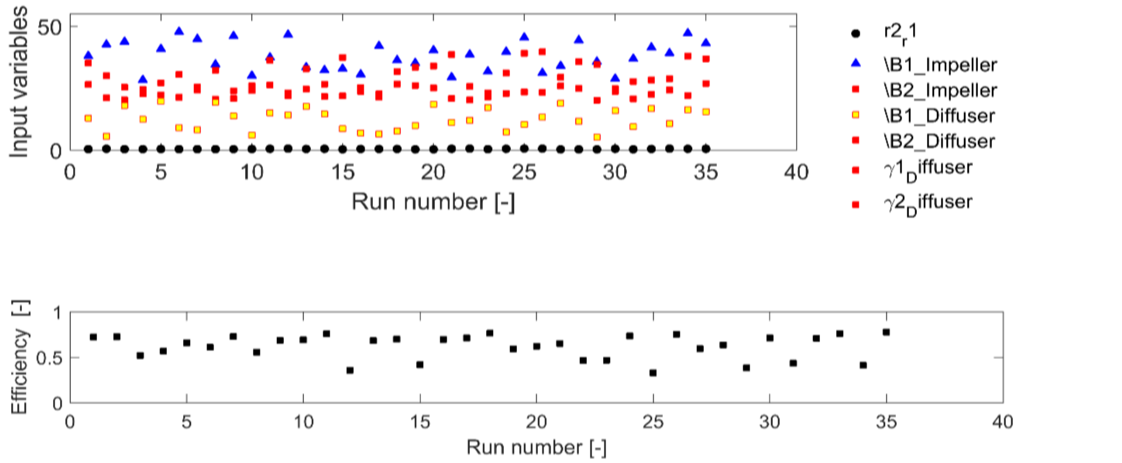


Figure 9- Distributions of sampled data.

The screening study has been used to define the effect weight of each one of the input parameters on the pump efficiency, the results of screening study shows that the most effective parameters are the ratio between impeller entrance and impeller discharge (r1_Impeller\ r2_Impeller) and β2_Impeller, also from the results β1_Impeller, β1_Diffuser, β2_Diffuser, ɣ1_Diffuser & ɣ2_Diffuser have almost same standard deviation and same effecting weight on the pump efficiency, additionally all the input parameters has a positive effect on the pump efficiency exceptβ1_Impeller,β2_Impeller, β1_Diffuser&ɣ1_Diffuser has a negative effect on the pump efficiency as illustrated by fig.10.

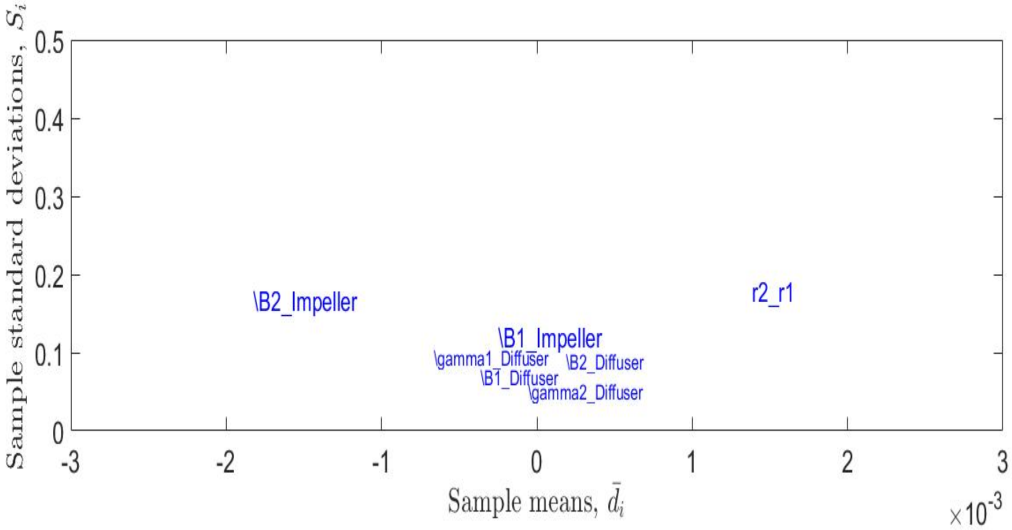


Figure 10- Screening of sampled data.

Effect of Impeller Entrance on the Pump Efficiency

As result of screening study it was required to study the effect of the ratio of (r1_Impeller\ r2_Impeller), since ESP had had standard outside diameters to fit the well bore hole, so the most effective and controllable part of this ratio is impeller entrance, so this study focused on the effect of (r1_Impeller\ r2_Impeller) at different rates and viscosities as illustrated by table 7.

The results are illustrated by fig 11, the results shows that for each b.e.p values used as the (r1_Impeller\ r2_Impeller) increased the pump efficiency also increasing till the optimum value of (r1_Impeller\ r2_Impeller) which is corresponding to the highest value for pump efficiency, then efficiency start decreasing with (r1_Impeller\ r2_Impeller) increasing, this results shows clearly that for each b.e.p rate there is optimum impeller radius accrued, at this point lowest friction losses and brake torque exist.

Shape optimization using generic algorithm

Single objective optimization was performed using genetic algorithm available at MATLAB 2017a.

Table 9 gives the optimum values for ESP geometrical design parameters at various values for b.e.p rates and viscosities. To understand the effect of this design parameter on the flow field pattern and performance, those parameters was used as input for the mathematical model to compare its results on the pump performance and efficiency against original design parameters.

Optimized Pump Performance Results

In order to check the results of optimization that done using surrogate method the outcome
results of the optimization was used as input for the mathematical model in order to get the improvement of the design at viscosities equal 100, 300 and 500 (cSt).

Results illustrated at fig. 15, 16& 17 shows clearly the improvement for pump efficiency after optimization, pump efficiency increased at b.e.p by 20%, 10% and 8.5% at viscosity 100, 300 and 500 (cSt) respectively.

The improvement is not limited to the pump efficiency only but also head per stage at b.e.p also improved by 43.8%, 38.4% and 43.9% at viscosity 100, 300 and 500 (cSt) respectively.

Conclusion

Most of the electrical submersible pumps available at the market currently are designed using water as fluid base during design phase, however while they are used to handle viscous fluids they are suffered from significant reduction at the head per stage and over all pump efficiency, using viscous fluid as base fluid during the design phase, will leads to adapt the geometrical parameters to get best efficiency while handling of viscous fluid like heavy oil, this well resulted in significant reduction for production cost of heavy oil and will open new potentials and bring more reserves to the economic limit and add more oil production.

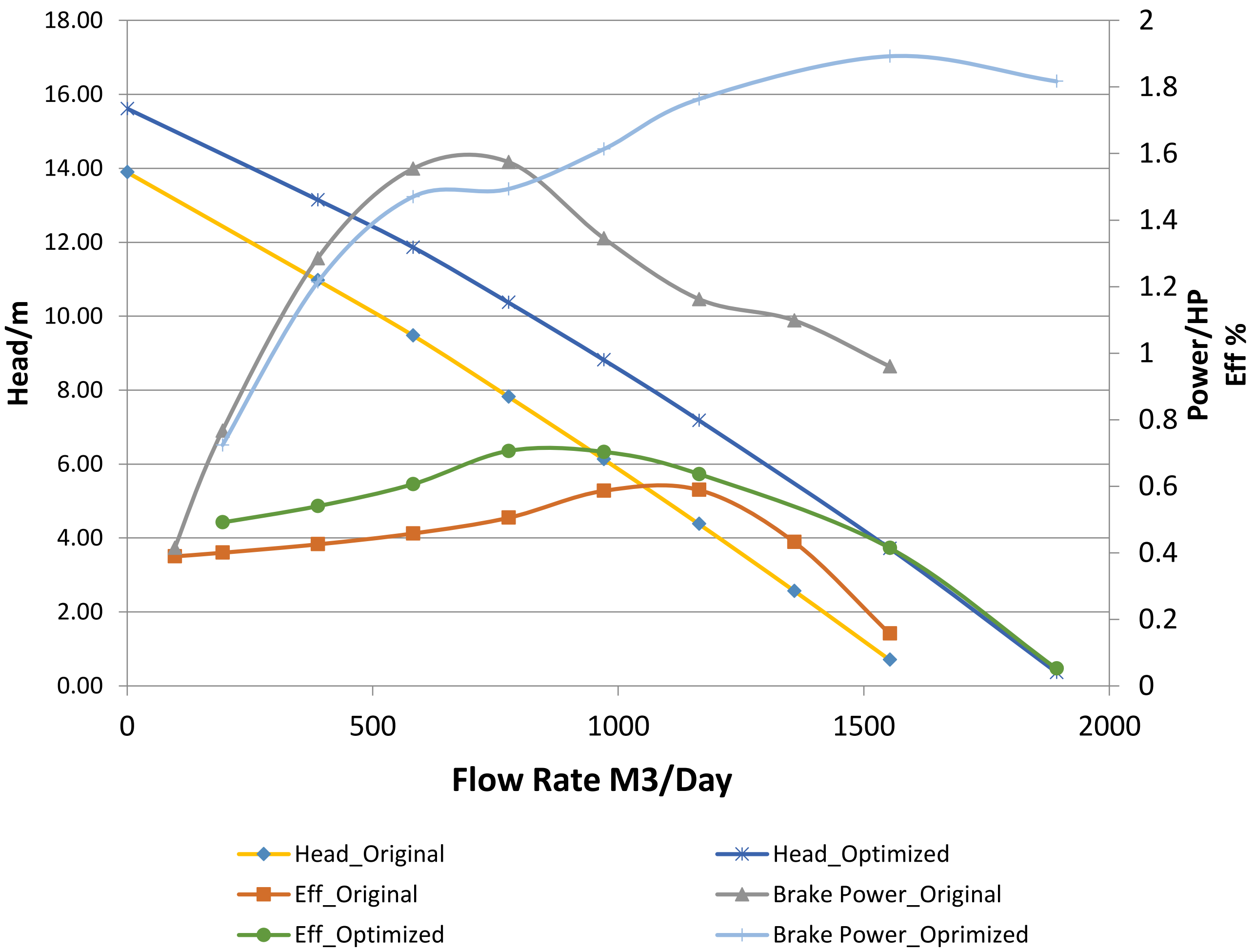


Fig 15- Comparison between original and optimized pump Performance @ 100(Cst)

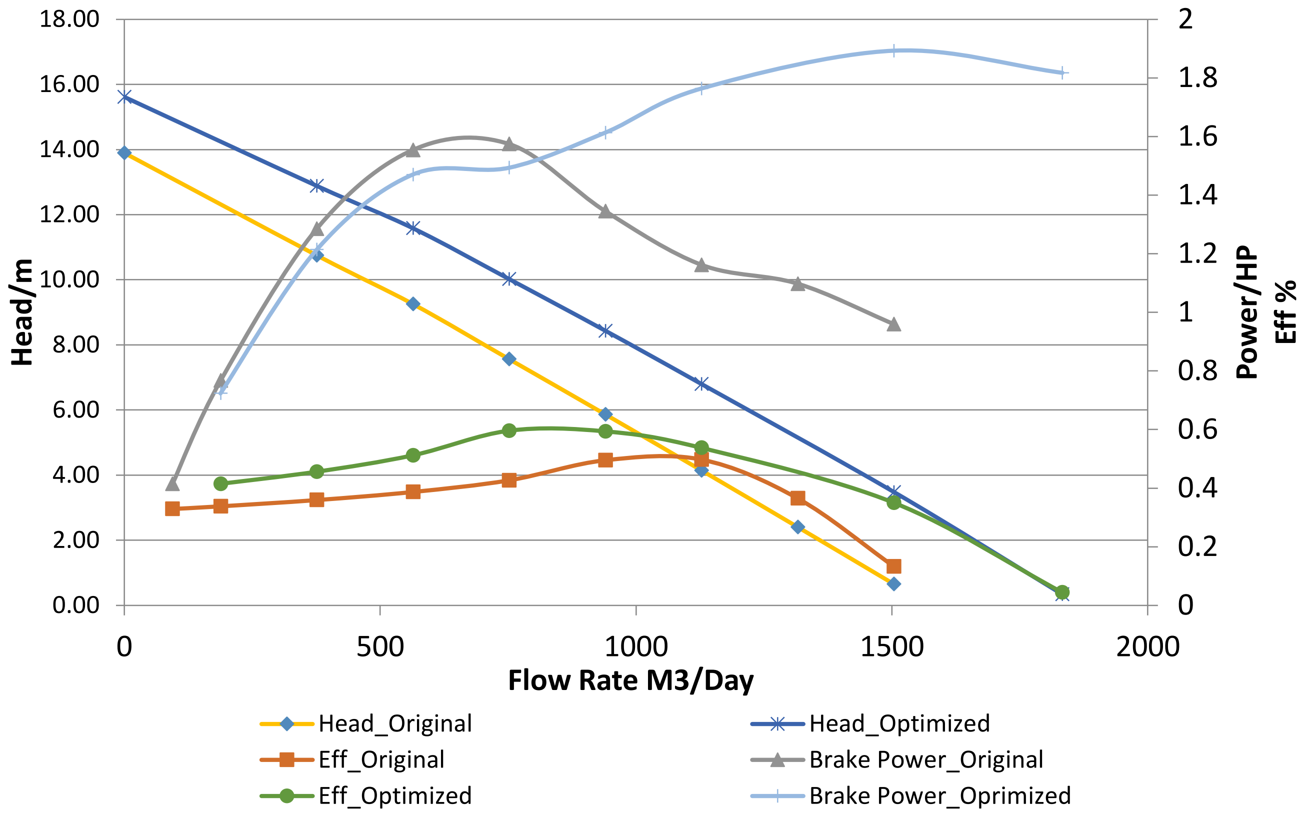


Fig 16- Comparison between original and optimized pump Performance @ 300(Cst)

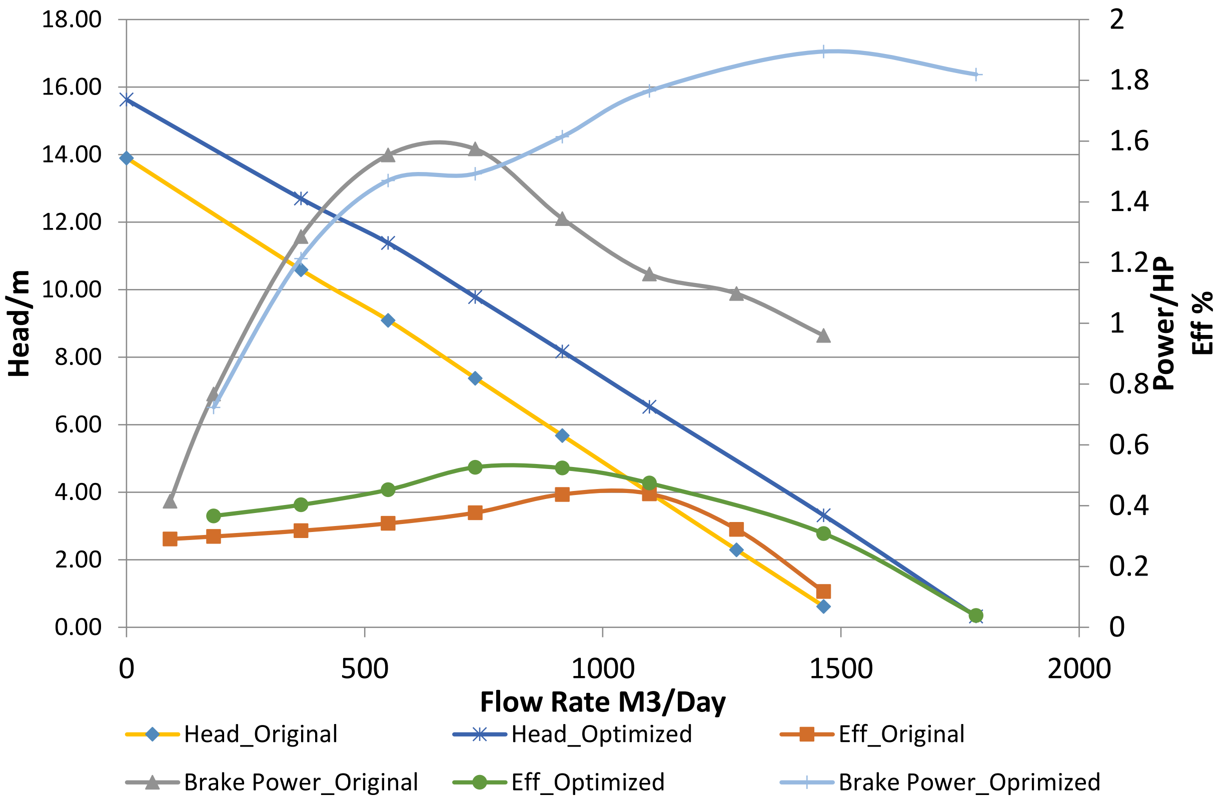
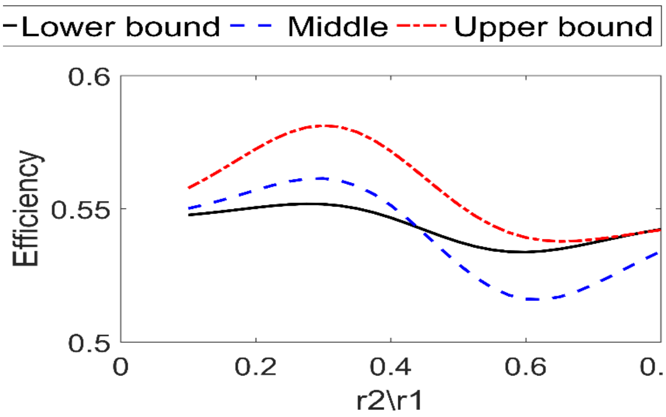
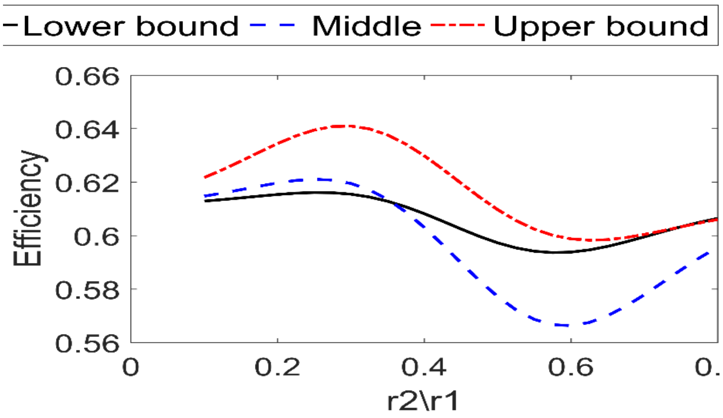
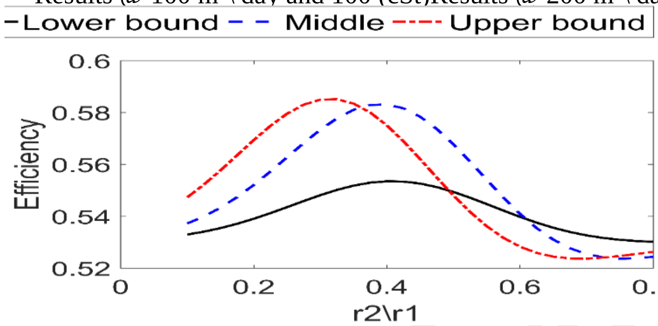


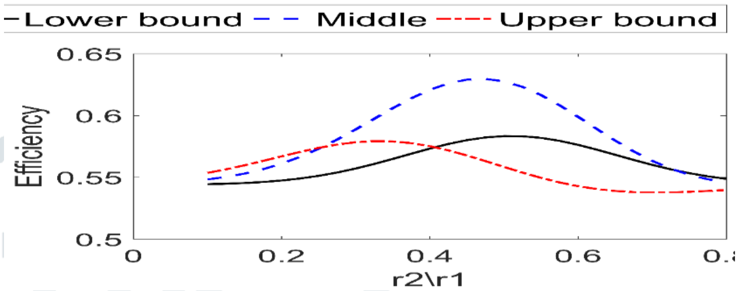
Fig 17- Comparison between original and optimized pump Performance @ 500(Cst)



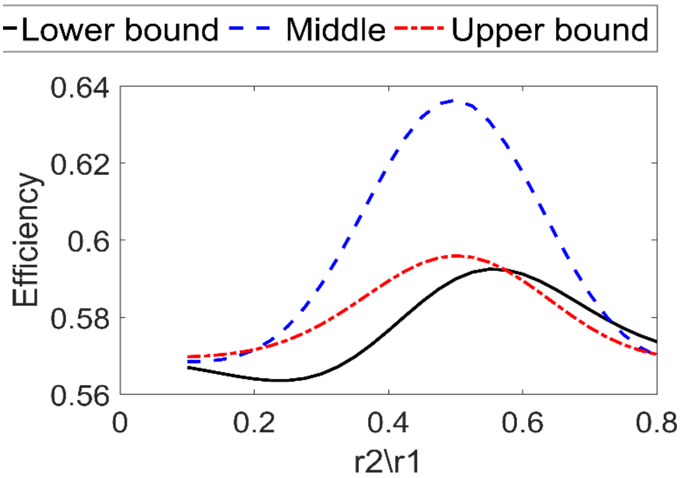


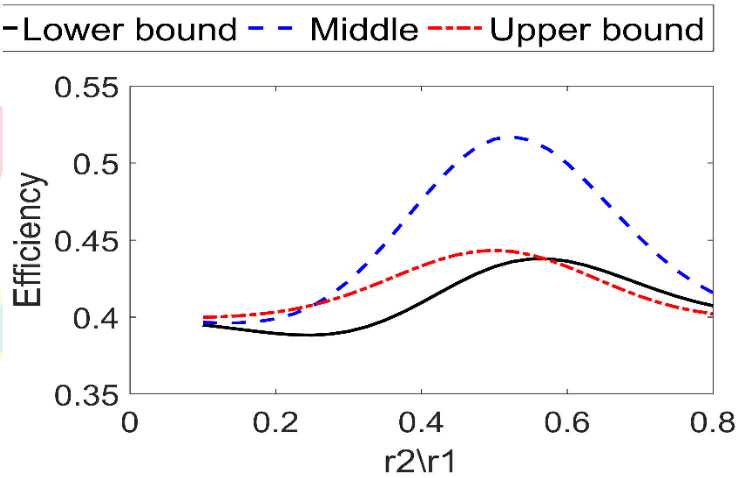
Results @ 100 m^3\ day and 100 (cSt)Results @ 200 m^3\ day and 100 (cSt)



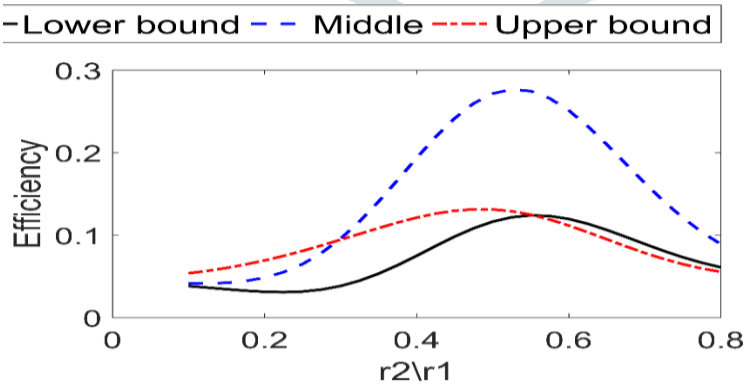


Results @ 400 m^3\ day and 100 (cSt)Results @ 600 m^3\ day and 100 (cSt)





Results @ 800 m^3\ day and 100 (cSt)Results @ 1000 m^3\ day and 100 (cSt)



Results @ 1200 m^3\ day and 100 (cSt)

Figure 11- Effect of Impeller Entrance on the Pump Efficiency

Nomenclature

a =Channel width, m
b =Channel height for impeller or diffuser, m
CH = Head correction factor
Cη = Efficiency correction factor

d_{eq} =Equivalent diameter, m
d_H =Hydraulic diameter, m
$\left(\frac{dp}{dr}\right)_f$ =Pressure gradient due to fluid friction, Pa/m
f = Friction factor
f_B = Blasius friction factor for smooth, straight pipes
F_{curved} =Curvature multiplication factor
$F_{rectangular}$ =Multiplication factor of rectangular effect
g =Gravitational acceleration, m/s^2
H =Channel height, m
j =Indicator for impeller or diffuser, j=1 for the impeller and j=-1 for the diffuser impeller and
j =-1 for the diffuser
$K_{laminar}$ =Dimensionless parameter for a rotating pipe under laminar flow conditions for liquid
$K_{turbulent}$ =Dimensionless parameter for a rotating pipe under turbulent flow conditions for liquid
l =Aspect ratio of the rectangular channel
n =Channel numbers
N_{Re} =Reynolds number
$(N_{Re})_{crit\_curved}$ =Critical Reynolds number for curvature effect
$(N_{Re})_{crit\_normal}$ =Critical Reynolds number foran or malpipe, namely, a straight stationary pipe with circular cross section
$(N_{Re})_{crit\_rectangula}$=Critical Reynolds number for rectangular effect
$(N_{Re})_{crit\_rotation}$ =Critical Reynolds number for rotational effect
N_{Re_eq} =Equivalent Reynolds number
N_{Re} =Reynolds Numbers for liquid
$N_{Re\,\Omega}$ =Rotational Reynolds number
Q_{bep} =Flow rate at the best efficiency point, m^3/s
H_{bep} =Pump head at the best efficiency point, m
H_w = Pumping head for water
H_O = Corrected Pumping head
H_{Wbep} = Water head at b.e.p.
P =Pressure, Pa
p_{Eye} =Impeller eye pressure of the stage intake, Pa
p_{next_Eye} =Impeller eye pressure of the next stage, Pa
Δp_{shock} =Shock loss, Pa
Q_l =Liquid flow rate, m^3/s
Q_w = Water capacity belonging to Hw
Q_{wbep} = Water capacity at b.e.p.
Q_O = Corrected capacity belonging to Ho
Q^* = Corrected capacity to determine head, capacity, and efficiency correction factor
r =Radial position of a point on the impeller, m
R_c =Radius of curvature along a channel, m
S = Distance from the entrance tip of impeller or diffuser to certain location on the streamline, m
U =Peripheral velocity, m/s
V =Absolute flow velocity, m/s
V_r =Radial absolute velocity of fluid, m/s
V_θ =Peripheral absolute velocity of fluid, m/s
V_z =Axial absolute velocity of fluid, m/s
W =Relative flow velocity between the fluids and the channel, m/s
x, y, z =Cartesian coordinates, m
x_c, y_c, z_c =Center coordinates of the approximate circular interval of the channel, m
z =Axial coordinate from pump intake to discharge, m

Greek

β =Blade angle, which is the angle between the outward blade tangent and the peripheral line opposing the rotating direction

β_1 =Entrance blade angle;

β_2 =Discharge blade angle

γ =Angle between the tangent of the blade and the plane perpendicular to theaxis

$\Delta p_{shock,base}$ =Shock loss at base rotational speed, Pa

Δp_{stage} =Pressure increment per stage, Pa

ε =Absolute roughness of the channel, m

θ =Tangential angle coordinate

μ_l =Liquid viscosity, Pa.s

ρ_l =Liquid Density, kg/m^3

Ω = Rotational

ω =Angular velocity of impeller or diffuser, rad/s

$\omega_{impeller}$ =Angular velocity of the rotating shaft or of the impeller, rad/s

η_w = Pump Efficiency for water

η_o = Corrected Pump Efficiency

γ_o = Kinematic viscosity of liquid pumped at pumping temperature, cst

Subscripts

1 =Entrance

2 =Discharge

1,2,3 =Any three points along the channel

bep =Best efficiency point

c =Center of a circle

curvature =channel curvature, "straight" or "curved"

effect ="rectangular", "curved", or "rotational"

eq =Equivalent

Eye =Impeller eye

F =Friction

H =Hydraulic

l =Liquid

laminar =Laminar flow

movement =Channel movement, "stationary" or "rotation"

next =Next

r =Radial

s =Streamline

shape =cross section shape, "rectangular" or "circular"

shock =Shock loss

turbulent =Turbulent flow

v =Vertical

z =Axial from pump intake to pump discharge

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Angular velocity	2915	rpm
Shaft outer radius	0.007	m
Impeller Entrance radius	0.029	m
Impeller Discharge radius	0.048	m
Liquid Density	1000	kg/m^3
Liquid Viscosity	1e-3	Pa.s
Channel Wall Roughness	1e-4	m

Table 2 – Input Data for the Impeller and Liquid Properties

Data	Impeller	Diffuser
$\beta_{h\_entrance}$	38°	10°
$\beta_{h\_discharge}$	23°	85°
$\gamma_{entrance}$	0°	30°
$\gamma_{discharge}$	0°	80°
Number of Channels	7	8
Channel Height	0.01 m	0.01 m

Table 3 – Input Geometric Data for the Impeller and Diffuser

Population size:	300
Elite count	2
Selection operation:	Tournament (tournament size equals 4)
Crossover fraction:	0.8 (default)
Crossover operation:	Intermediate crossover
Mutation operation:	Adaptive Feasible
Maximum number of generations	1000

Table 4- Genetic algorithm operators and parameters.

Parameter	Min. value	Max. Value
r1_Impeller\ r2_Impeller	0.2	0.6
β1_Impeller	28	48
β2_Impeller	20	27
β1_Diffuser	5	20
β2_Diffuser	75	90
γ1_Diffuser	20	40
γ2_Diffuser	70	90

Table 5 – Input Geometric Parameters Range

r2_Impeller\ r1_Impeller	β1_Impeller	β2_Impeller	β1_Diffuser	β2_Diffuser	γ1_Diffuser	γ2_Diffuser
0.34	38.00	26.50	12.93	85.07	35.14	78.29
0.48	42.57	21.10	5.64	75.21	30.00	89.14
0.30	43.71	20.30	18.07	78.21	25.43	73.71
0.33	28.29	24.50	12.50	80.36	22.57	72.57
0.39	40.86	22.30	19.79	76.93	27.14	73.14
0.29	47.71	21.30	9.07	75.64	30.57	80.00
0.37	44.86	25.50	8.21	84.21	24.29	71.43
0.35	34.57	20.50	19.36	85.50	32.29	89.71
0.32	46.00	23.90	13.79	80.79	20.86	86.29
0.45	30.00	24.10	6.07	79.07	26.00	70.29
0.47	37.43	26.30	15.07	77.79	36.29	74.86
0.56	46.57	23.10	14.21	89.36	22.00	76.00
0.40	33.43	24.70	17.64	88.50	32.86	72.00
0.54	32.29	21.70	14.64	76.50	26.57	85.71
0.24	32.86	21.90	8.64	81.64	37.43	77.14
0.38	30.57	25.30	6.93	84.64	23.71	88.57
0.42	42.00	22.70	6.50	78.64	21.43	81.71
0.41	36.29	26.70	7.79	82.50	31.71	86.86
0.21	35.14	26.10	9.93	86.36	33.43	85.14
0.23	40.29	25.10	18.50	85.93	34.00	88.00
0.49	29.43	20.90	11.21	82.07	38.57	78.86
0.58	38.57	25.70	12.07	82.93	20.29	80.57
0.31	31.71	21.50	17.21	87.64	23.14	82.86
0.50	39.71	22.90	7.36	89.79	31.14	75.43
0.59	45.43	23.50	10.36	81.21	39.14	79.43
0.57	31.14	23.30	13.36	76.07	39.71	84.00
0.27	34.00	25.90	18.93	77.36	29.43	70.86
0.22	44.29	24.90	11.64	79.50	35.71	82.29
0.25	35.71	20.10	5.21	88.93	34.57	77.71
0.53	28.86	23.70	15.93	83.79	24.86	84.57
0.26	36.86	20.70	9.50	83.36	27.71	76.57

0.43	41.43	22.50	16.79	88.07	28.29	87.43
0.55	39.14	24.30	10.79	87.21	28.86	81.14
0.51	47.14	22.10	16.36	86.79	38.00	74.29
0.46	43.14	26.90	15.50	79.93	36.86	83.43

Table 6 – DoE Results

Best Efficiency Point Flow Rate m^3/Day	Viscosity (cSt)		
100	100	300	500
200	100	300	500
400	100	300	500
600	100	300	500
800	100	300	500
1000	100	300	500
1200	100	300	500

Table 7 – Simulation Cases scenarios

Parameter	Min. value	Max. Value	Mean. value
r1_Impeller\r2_Impeller	0.2	0.6	0.4

Table 8 – Upper, middle and lower bound forr1_Impeller\r2_Impeller

Flow Rate@ b.e.p	r1/r2	β1_Impeller	β2_Impeller	β1_Diffuser	β2_Diffuser	γ1_Diffuser	γ2_Diffuser
Viscosity 100 (cSt)							
1200	0.54	37.9	25.2	12.7	81.2	29.9	81.6
1000	0.52	37.9	26.0	13.8	80.1	33.2	82.5
800	0.47	35.8	26.4	9.9	81.6	30.9	84.6
600	0.39	37.0	25.5	13.0	83.1	31.2	80.1
400	0.33	37.5	24.4	13.2	83.3	31.4	81.9
200	0.28	36.8	24.7	11.6	84.4	33.6	83.2
100	0.29	37.6	24.6	12.0	84.0	33.3	83.2
Viscosity 300 (cSt)							
1200	0.54	37.9	25.2	12.7	81.2	29.9	81.6
1000	0.53	37.5	25.4	13.2	81.7	31.6	82.5
800	0.47	35.8	26.4	9.9	81.5	30.9	84.6
600	0.42	35.8	25.5	13.3	83.4	31.4	78.9
400	0.40	34.1	24.1	13.2	83.1	29.1	78.6
200	0.28	36.9	24.7	11.6	84.4	33.6	83.2
100	0.29	37.6	24.6	12.0	84.0	33.3	83.2
Viscosity 500 (cSt)							
1200	0.53	37.9	25.2	12.7	81.2	29.9	81.6
1000	0.52	37.9	26.0	13.8	80.1	33.2	82.5
800	0.47	35.7	26.4	9.8	81.6	30.8	84.6
600	0.39	37.0	25.5	13.0	83.2	31.2	80.1
400	0.33	37.5	24.4	13.2	83.3	31.4	81.9
200	0.28	36.9	24.7	11.6	84.4	33.6	83.2
100	0.29	37.6	24.6	12.0	84.0	33.3	83.2

Table 9 - Optimal parameters.