Application of Differential of Factual Quadratic Function in Approximate Calculations

Researcher: Associate Professor Qudratullah Danyar

Department of Mathematics, Faculty of Education, Takhar University

Abstract

The concept of differential is one of the essential parts of derivatives in advanced math and it is widely used in many general rules of nature including physics and chemistry. We can use differential to obtain the amount of the changes of a quantity compared to other specific quantity if these quantities are related together by a function. Using differentials, we can also perform some approximate calculations very easily. The purpose of the article is to explain the applications of factual double function differential in approximate calculations, which has more applications than factual single functions, and its differentials are more attractive and efficient. This article is a literature review, and every effort has been made to collect data from authentic references. The findings show that factual quadratic function differential in approximate calculations has many applications such as approximate calculation of areas of geometric shapes, construction of buildings, identifying the amount of acceptable errors in economic calculations in case of changes of dimensions of surfaces area and workload of buildings, and calculation of parameters of electric cycle in electric power.

Keywords:

Partial differential of factual quadratic function

General differential factual quadratic function

Sensitivity to change

Preface

The concept of derative, in its present form, was first invented by Newton in 1666 and it was indepentdly invented by Leibniz a few years later. Differential is one the important parts of derivative and it has a wide range of applications, and it has covered many pratical areas of human life. For example, they are used in construction of buildings and scycrapers, advanced technical devices and natural sciences. As I mentioned before, there are many practical and applicable aspects of derivatives and facutal quadratic functions in the current age. İn this article, the writer dicusses a small part of the application of differential of factual quadratic functon, and various applied and analytical issues are explained in this article.

Partial Differential

We understand the definiton of derivative and differential of facutal single function and their calculation, and we understnad that the differental of each function is equal to product of its derivative at its differential of independent function. The concept and definition of differential of factual single-function hold true for differental of factual quadratic function.

We consider factual quadratic function $z = f(x, y)$. If $dy = \Delta y$, $dx = \Delta x$ is defined in this function when variable X with constant Y changes , Z is only the independent function of X. the partial differential of the mentioned function relative to X is follows.

$$dz_x = \frac{\partial z}{\partial x} dx \qquad (1)$$

Likewise, if the variable Y changes with constantY, the partial differental of the mentioned function relative to Y will be equal to:

$$dz_y = \frac{\partial z}{\partial y} dy \qquad (2)$$

General Differental of Facutal Quadratic functions

General differental of facutal quadratic functions is equal to total differential of the function. For example, General differental of facutal quadratic function Z results form the relations of (1) and (2).

$$dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy \qquad (3)$$

Proof: If $dy = \Delta y$, $dx = \Delta x$ shows the increase of variables of x and y respectively, then we can have the following:

$$\Delta z = f(x + \Delta x, y + \Delta y) - f(x, y) \qquad (4)$$

In the above relation, Δz increase of function is f(x, y) = z. if the given function with the first order partial derivatives is constantly in one area, and then we can write the following:

$$dz = \frac{\partial f}{\partial x} \Delta x + \frac{\partial f}{\partial y} \Delta y + \varepsilon_1 \Delta x + \varepsilon_2 \Delta y$$

$$= \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \varepsilon_1 dx + \varepsilon_2 dy \qquad (5)$$

In the relation (5), when Δx and Δy approaches zero, ε_1 and ε_2 will be equal to zero. Therefore, the differential of function of Z will be as follows:

$$dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy \qquad (5)$$

The differential of function z and dz is considered the main part of Δz (8:490).

Approximation by General First order Differential of Factual Quadratic Functions

We understnad that $dy \neq \Delta y$ from dy has been used a good approximation for Δy. This $\Delta z \neq dz$ is also possible in factual quadratic function, and we can write:

If the quantities of $\Delta x = dx$ and $\Delta y = dy$ are very small, dz is quantitatively close to the amount of Δz and it can be used as the approximation of Δz. For the clarification of the issue, the following relations are used.

$$\Delta z = f(x + \Delta x, y + \Delta y) - f(x, y) \approx dz$$

$$\Delta z = f(x + dx, y + dy) - f(x, y) \approx dz$$

The following is concluded from above relations.

$$f(x+dx, y+dy) \approx f(x,y)+dz \qquad (6)$$

The above relation shows the way of calculation of the amount of function $f(x+dx, y+dy)$ when dx and dy are both small. Thereby, we can calculate the changes of the function using this relation (6:495).

For calculation of approximate general differential of function, $z = f(x,y)$ the following is used.

$$dz = df = f'_x(x_0, y_0)\,dx + f'_y(x_0, y_0)dy = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy \qquad (7)$$

The Application of Differential of Quadratic Functions in approximate calculation of geometric shapes

Example 1

The map of a rectangular building with a length of 30 meters and a width of 15 meters is given, and its implementation is contracted with a company that charges $250 per square meters. During the implementation, for some reasons, 50cms is reduced from its length and 75cms added to its width. Now determine the difference between the previous and the current price of the project.

Solution

If we show the dimensions of the rectangular with x and y, the area is as follows:

$$s = xy$$

$$ds = \frac{\partial s}{\partial x}dx + \frac{\partial s}{\partial y}dy = ydx + xdy$$

We calculate the differential of function of s , we get the following one.

$$ds = (20)(-0.5) + (30)(0.75) = -10 + 22.5 = 12.5$$

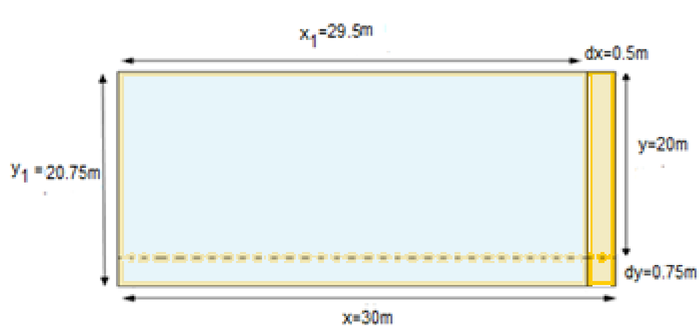
Now we can calculate the approximate area of the intended rectangular.

Approximate area of the rectangular $ds = s + ds = 600 + 12.5 = 612.5cm^2$

The price of the project at the beginning $= 600 \times 250 = 150000\$$

The price of the project after the implementation $= 612.5 \times 212 = 153000\$$

The difference in the price of the project $= 153000\$ - 150000\$ = 3000\$$



Shape (1) shows the map of a rectangular building (7:496).

Example 2

Please estimate the change in a Hypotenuse with a length of 15cms and 20cms when the length of its small side gets as big as $\frac{5}{8}cm$ and its large side gets as small as $\frac{5}{16}cm$

Solution

We use x, y and z to mark the smaller side, the larger side and the chord of the triangle respectively. The relation of the chord with two other sides are determined by Pythagorean Theorem (the square of the chord is equal to the total squares of two vertical sides). Therefore, we can write the following:

$$z = \sqrt{x^2 + y^2}$$

Considering the above relation as a factual quadratic function, we obtain its differential as below:

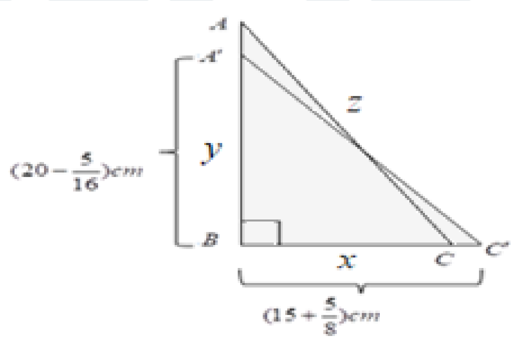
$$dz = \frac{x}{\sqrt{x^2 + y^2}}dx + \frac{y}{\sqrt{x^2 + y^2}}dy$$

If $y = 20$, $x = 15$ and $dy = -\frac{5}{16}$, $dx = \frac{5}{8}$ are considered, with their replacement in the relation dz, its amount is as follows:

$$dz = \frac{15}{\sqrt{15^2 + 20^2}}\left(\frac{5}{8}\right) + \frac{20}{\sqrt{15^2 + 20^2}}\left(-\frac{5}{16}\right) = \frac{15}{\sqrt{625}}\left(\frac{5}{8}\right) + \frac{20}{\sqrt{625}}\left(-\frac{5}{16}\right)$$

$$= \frac{75}{200} - \frac{100}{400} = \frac{3}{8} - \frac{1}{4} = \frac{1}{8}$$

Therefore, we can say that the chord of the mentioned triangular will get almost as big as $\frac{1}{8}cm$



Estimation of changes and rate of sensitivity relative to changes

Example 1

A company manufactures tin and cylindrical can with a height of 25 feet and radius of five feet. Please determine the sensitivity of volume of the can relative to small change of height and radius (3:179).

Solution

The volume of the cylinder is the factual quadratic function of r and h. The volume of the can is equal to:

$$v = \pi r^2 h$$

The volume changes, which results from small changes of $dr\ and\ dh$ in radius and height, is equal to:

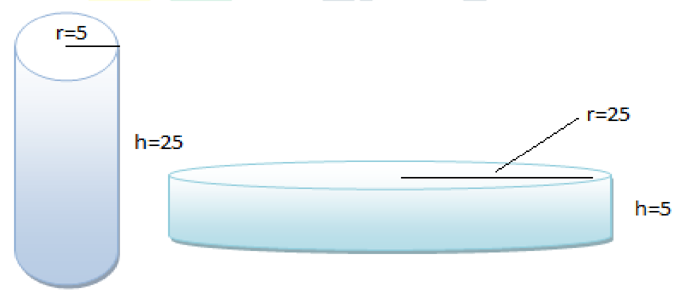
$$dv = \frac{\partial v}{\partial r} dr + \frac{\partial v}{\partial h} dh$$
$$= 2\pi rhdr + \pi r^2 dh = 2\times 5\times 25\times \pi dr + 5^2 \times \pi dh$$
$$= 250\pi dr + 25dh$$

Therefore, one single change in r brings around 250π in volume. one single change in h brings around 25π in volume. The volume of the can relative to small change of r is 10 times more sensitive than similar change in$h$. To make sure the accuracy of the size of the can, an engineer who is in charge of controlling the quality should pay more attention to the radius of the cans. In return, if we exchange the quantities of r and h and place $h = 5$ and $r = 25$, then the total differential of v is equal to the following.

$$dv = \frac{\partial v}{\partial r} dr + \frac{\partial v}{\partial h} dh$$
$$= 2\pi rhdr + \pi r^2 dh = 2\times 25\times 5\times \pi dr + 25^2 \times \pi dh$$
$$= 250\pi dr + 625dh$$

Now the volume relative to h is more sensitive than changes of r.

It is concluded from above solution that most sensitivity of the function relative to small changes belong to the variables whose partial derivatives are larger (3:179).



Shape (2) shows the sensitivity of volume of cylinder relative to change in radius (3:179)

Example 2

Suppose that the radius of the cylinder is 2m and its height is 3m. We want the error not to exceed $0.1m^3$. How precise the radius and the height should be measured. Suppose the errors of measurement of $dr\ and\ dh$ are equal.

Solution

$$v = \pi r^2 h \quad , \quad r = 2m \quad , h = 3m$$

$$dv = \frac{\partial v}{\partial r} dr + \frac{\partial v}{\partial h} dh \ \Rightarrow dv = 2\pi rhdr + \pi r^2 dh$$

$$\Rightarrow |dv| = |12\pi dr| + |4\pi dh| = 12\pi |dr| + 4\pi |dh|$$

Since $|dr| = |dh|$ and we get the following.

$$\Rightarrow |dv| = 16\pi |dr| \leq 0.1$$

Therefore: $$\Rightarrow |dr| dr \leq \frac{0.1}{16\pi} \approx 1.93 \times 10^{-3}$$

Application of Differential of factual quadratic functions in electric power

Example 1

The power consumed in electrical resistance by $p = \frac{E^2}{R}$ is determined by watt. If $E = 200$V and $R = 8$ Ohm, then how much would be the change of power in case E and R decrease 5V and 0.2 Ohm, respectively.

Solution

Considering the relation of P as a factual quadratic function, we calculate the differential.

$$dp = \frac{2E}{R} dE - \frac{E^2}{R^2} dR$$

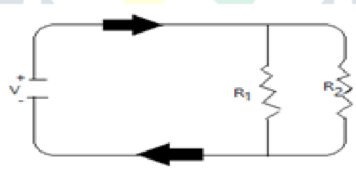
$$dp = \frac{2(200)}{8}(-5) - \left(\frac{200}{8}\right)^2 (-0.2) = -125\ Wat$$

Then P will decrease as much as 125V.

Example 2

If we close the resistance of R_1 and R_2 in parallel to bring change in the electrical resistance, the resistance of R is obtained from the following shape.

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$



Shape (3) shows the power cycle in which the resistances close in parallel (3:179).

a. Please show that the following equality is established

$$dR = \left(\frac{R}{R_1}\right)^2 dR_1 + \left(\frac{R}{R_2}\right)^2 dR_2$$

Please consider an Circuit similar to the above shape in which $R_1 = 100\ ohms$ and $R_2 = 400\ ohms$, but there is always insufficiency in production and the purchased resistance do not have the exact quantities. Is the quantity R relative to R_1 more sensitive or to R_2? Why?

Solution a

We perform differential from both sides of the relation.

$$-\frac{1}{R^2} dR = -\frac{1}{R_2{}^2} dR_1 - \frac{1}{R_2{}^2} dR_2$$

$$dR = \left(\frac{R}{R_2}\right)^2 dR_1 + \left(\frac{R}{R_2}\right)^2 dR_2$$

Solution b

$$dR = \left(\frac{R}{100}\right)^2 dR_1 + \left(\frac{R}{400}\right)^2 dR_2$$

$$dR = 10^{-4} R^2 dR_1 + 16 \times 10^{-4} dR_2$$

The quantity R is more sensitive relative to R_2 since it creates larger partial Derivative

Example 3

Pretend that the flow rate of I (ampere) in an electrical Circuit with the amount of voltage (V) and resistance R Ohm depends on $I = \frac{v}{R}$. If the voltage decreases from 24V to 23V and resistance decreases from 100 Ohm to 80 Ohm, is it added to the amount of *I* or is it reduced from it? Please state the R and V changes and the estimation of change in *I* in percentage?

Solution

The general differential for factual quadratic functions $z = f(x, y)$ is as follows.

$$dz = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy$$

$$I = \frac{v}{R} = \frac{24}{100} = 0.24 \; ohms$$

$$dI = \frac{1}{R} dv - \frac{v}{R^2} dR = \frac{1}{100}(-1) - \frac{24}{10000}(-20) = 0.038$$

$$\frac{\Delta v}{v} \times 100 = \frac{-1}{24} \times 100 = -4.17\%$$

$$\frac{\Delta R}{R} \times 100 = \frac{-20}{100} \times 100 = -20\%$$

$$\frac{\Delta I}{I} \times 100 = \frac{0.038}{0.24} \times 100 = 15.8\%$$

As it is observed, the flow rate increase as the voltage and the resistance decreases in electrical Circuit.

Discussion and Conclusion

Sciences have developed in relation to one another. All the development of sciences in different aspects are the result of their correlation. This correlation of sciences are with themselves and other sciences. It is concluded from the integration of results and achievements of sciences in different aspects, mathematics analyzes and evaluates relation, correlation and interaction of phenomenon of materialistic world, and provides the results in the form of function and formula for natural sciences. In this article, a small part of mathematics, which is differential of factual quadratic functions and their functions, was explored and these are the conclusions.

- The factual quadratic functions have been more effective than factual single functions in mathematics and math and their derivatives and differential have been more productive.
- The differential of factual quadratic functions have facilitated the approximate calculation of some of geometrical shapes.
- The sensitivity rate of a geometrical item with the change of one of its sides can be considered with the help of differential of factual quadratic functions.
- We can calculate electrical resistances in electrical cycle using differential of factual quadratic functions.

The use of differential in vital issues of humans in particular in engineering provides necessary facilities. The characteristic of these findings is the fact that the relation and interrelation of math is explained both independently and in relation to other sciences, and it is extendable in other aspects of natural sciences.

References

1. B.B. Damid Witch. (2008). Exercises and Mathematical Analysis Problems. Translated by Parviz Shahryari, Tehran: Amir Kabir Publishing Press.
2. Gorge B., Thomas & Rice, Feny. (2004). Calculation of Differential and Integral and analytical geometry (2nd Volume). Translated by Farzin Jamshidi and Murtaza Barati. Tehran: Safar Ashrafi Publishing Press.
3. Gorge B., Thomas & Rice, Feny. (2005). Manual for solving differential calculus. Tehran: Safar Ashrafi Publishing Press.
4. Gorge B., Thomas. (2010). Calculus of Differential and Integral. Translated by Ibrahim Amel Mehrabi. Tehran: Negar University Publishing Press.
5. Wir, Moris & Has Joel. (2009). Calculus of Differential and Integral. Translated by Sina Hedayateyan and Ibrahim Amel Mehrabi. Tehran: Negar University Publishing Press.
6. Farkho, Lida. (1991). General Math. First part. Tehran: Payam-e Noor University Publishing Press.
7. Farzin, Haji Jamshidi. (2007). General Math 2 & 3 Advanced. Tehran: Safar Ashrafi Publishing Press.
8. Najafi Khah, Mahdi. (2003). Calculus of Differential of Integral and Analytical Geometry. Tehran: Bahman Barna Publishing Press.
9. Nafar, Mahdi. (1996). Theory and Mathematics. Esfahan: Tarahan Publishing Press.
10. Hamta, Noor Mohmmad & Mirza, Mirza Mohammad. (2013). General Mathematics. Kabul: Polytechnic University.