**Review and Challenges of Various Bilevel Programming Problems**

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**Abstract:** Bilevel optimization is an uncommon kind of optimization where one problem is linked within another. The external optimization task is usually alluded to as the upper-level optimization task, and the inner optimization task is normally alluded to as the lower-level optimization task. These problems involve two kinds of factors, alluded to as the upper-level factors and the lower-level factors. A lot of new applied problems in the territory of vitality systems have as of late emerged that can be productively illuminated uniquely as blended integer bilevel programs. This includes problems in the domain of transportation, financial aspects, choice science, business, engineering, natural financial aspects and so forth. This paper presents different research takes a shot at bilevel problem, which likewise include a few applications and difficulties of bilevel programming problem.

**IndexTerms** - Transportation, Fixed Charge, Bilevel, Fractional Transportation.

I. INTRODUCTION

Bilevel optimization alludes to a various leveled problem in which optimization should be performed at two settled levels, in particular the upper level and the lower level. The point is to recognize the ideal of the upper level problem, subject to optimality of the corresponding lower level problem. A few problems from the domain of engineering, coordinations, financial aspects, and transportation have inherent settled structure which expects them to be displayed as bilevel optimization problems. Bilevel optimization ordinarily requires inordinate measure of capacity assessments since a lower level quest should be directed for evaluating every upper level arrangement. The assessments are particularly high when the problems are not appropriate for careful systems and transformative strategies are utilized instead. Reducing this computational exertion has been one of the key interests in this domain as of late.

Albeit a wide scope of uses fit the bilevel programming system, genuine usage are rare, due mainly to the absence of proficient algorithms for tackling medium-and huge scale bilevel programming problems (BLP). Solving a bilevel (all the more for the most part, various leveled) optimization problem, even in its least difficult structure, is a troublesome errand. A variety of elective strategies might be utilized dependent on the structure of the problem investigated; however there is no broad strategy that ensures assembly, execution, or optimality for each kind of problem.

Traffic arrange task under street organize helplessness and traffic blockage has become an intriguing issue for some analysts lately. Starting from the unpredictable system attributes of urban streets and multi-mode traffic arrange task, this work sets up a multi-mode bilevel transportation model dependent on organize weakness which can more readily reflect distinctive way decision conduct in a helpless network [2]. The best possible area of weigh-in-motion (WIM) stations in street systems is basic to successfully diminish the effect of overweight trucks. Truckers may rapidly get familiar with the areas of WIM stations, and take temporary re-routes to sidestep these checkpoints, with the guide of cutting edge route frameworks. This response should be considered in the area models for the organization of WIM frameworks. This work proposes a bilevel model to manage the WIM area problem in a street network [3]. Over the last several decades, the scale of highway transportation system has become larger than ever before. High-way Transportation System has become an indispensable part of human activities; it plays an important role in both economic and social development [4].

![Figure 1: BiLevel Linear Programming with Multiple Followers](image-url)

A huge measure of content information is recorded in the types of fix verbatim in railroad maintenance divisions. Productive content mining of such maintenance information assumes a significant job in detecting irregularities and improving deficiency determination effectiveness. Nonetheless, unstructured verbatim, high-dimensional information, and imbalanced flaw class circulation present difficulties for highlight determinations and shortcomings diagnosis [6]. The accuracy of issue finding for all issue classes, especially minority ones. Its exhibition has been approved by using a railroad maintenance informational collection gathered from 2008 to 2014 by a rail line partnership. It outflanks conventional methodologies [7].

The time-dependent origin-destination (TDOD) request estimation problem targets estimating dynamic interest that speaks to the watched traffic stream designs in a transportation arrange. Blunders in TDOD request are regularly engendered into the system yields causing inconsistent planning and operational policies [8]. Various genuine problems in engineering, coordinations, financial matters, transportation and so forth should be displayed as bilevel optimization problems because of involvement of a chain of importance of leaders, and therefore the problem is of noteworthy research interest. Ensuring lower level optimality for every
arrangement makes the problem computationally intensive as far as number of capacity assessments required [9]. Quick increase in the size and unpredictability of the problems emerging from these domains has incited dynamic interest in the structure of productive algorithms for bilevel optimization. While Memetic Algorithms (MAs) have been very effective in solving single level optimization problems, there have been not many investigations exploring their application in bilevel problems. MAs basically endeavor to combine preferences of worldwide and nearby inquiry techniques to find ideal arrangements with low computational cost (work evaluations) [12]. The right now most hazardous materials transportation scheduling models mainly expect to minimize all out expenses in request to remove into both transportation from pocket costs and hazard related costs which are seen as static. In this work, we consider the following hazardous materials transportation routing procedures problem: a given arrangement of hazardous materials shipments must be delivered over a street transportation organize in request to move a given measure of unsafe materials from a particular origin point to a particular destination within the time limitation [19].

II. LITERATURE SURVEY

Kaushal, B., et al.[2020] Bilevel Programming Problem (BLPP) is a various leveled optimization problem. Here, the constraint set of the upper level problem, called the pioneer, is ascertainment by the lower level problem, called the follower. In this work, a BLPP is viewed as where the pioneer's problem is a fractional transportation problem and the follower's problem is a fixed charge transportation problem. Subsequently, a bilevel fixed charge fractional transportation problem (BLFCFTP) is figured. The problem is additionally explained through graphical portrayal. The problem originates from a natural way of life industry which is utilizing its pre-owned vegetable cooking oil for the creation of biodiesel. The mathematical model of the problem is displayed and is likewise explained through graphical portrayal. The problem pertains to an evolved way of life industry wherein the pre-owned vegetable cooking oil is used for the creation of biodiesel. Another model statements Indore city where waste is changed over into bio-CNG which is utilized to run open vehicle. [1]

H. Wen-jun et al.[2019] The upper level is an element of maximizing the limit of a street arrange under the investment constraints of the traffic specialists. Its motivation is to minimize the effect of the system activity proficiency and execution after the links or ways are harmed. The lower level is a multi-mode traffic organize equilibrium model, which is basically a voyager conduct task problem. Finally, a numerical model is given to show the viability of the multimodal traffic model dependent on defenselessness. The examination results show that the two-level model considering helplessness index diminishes the effect of unusual occasions on the exhibition of street organize, and is helpful for the foundation of an increasingly solid and stable street network.[2]

C. Lu, et al.[2018] The model includes an upper level and various lower level models individually, speak to the basic leadership conduct of the law implementation authorities and truckers for various origin-destination sets. The upper level model determines the ideal number and areas of WIM stations in order to minimize the harm because of evquivocal overweight trucks, considering the truckers' course decisions. The lower level models mimic the truckers' course decisions because of the WIM areas determined by the upper level model. A heuristic is created to effectively unravel the bilevel model, as it is hard to obtain the specific arrangement. The proposed model and heuristic are assessed using a test instance produced dependent on the Nevada Street arrange. The outcomes show that the heuristic outflanks an old style approach dependent on k most limited ways. [3]

A. Mishra et al.[2018] Presents Intelligent Transport Systems (ITS) which provide transport solutions by utilizing state-of-the-art information and telecommunication technologies. It is an integrated system of people, roads and vehicles, designed to significantly contribute to improve road safety, efficiency and comfort, as well as environmental conservation through realization of smoother traffic by relieving traffic congestion. This paper aims to elucidate various aspects of ITS-it's need, the various user applications, technologies utilized and concludes by emphasizing the case study of IBM ITS. [4]

M. M. Islam et al.,[2017] The utilization of surrogate modeling to accomplish this objective has so far been hardly examined. In this work, we present a surrogate helped optimization approach toward addressing this exploration hole. The methodology utilizes surrogates of numerous sorts in request to give adaptability of approximating various kinds of capacities all the more precisely. The calculation is additionally fortified using specific re-assessment of promising arrangements and intermittent settled neighborhood search. The presentation of the proposed calculation is exhibited on twenty five standard benchmark problems. The outcomes are contrasted and various other set up transformative and crossover algorithms to exhibit the adequacy of the proposed approach in obtaining serious outcomes using generally less capacity assessments. [5]

F. Wang et al.[2017] In this investigation, a bi-level optimization problem is proposed where the upper level is an Ordinary Least-Squared (OLS) blunder minimization problem that minimizes the deviation between the assessed and watched traffic volumes from SCATS, while the lower level produces task extents network using a mesoscopic reproduction based unique client equilibrium model. The interior point conjugate slope technique, as a precise inclination strategy, is applied to take care of the TDOD request estimation problem. The obtained outcomes feature the ability of the proposed approach in improving the presentation of a powerful enormous scale organize model of Melbourne CBD. [6]

S. Shafiei et al.[2016] The time-dependent origin-destination (TDOD) request estimation problem targets estimating dynamic interest that speaks to the watched traffic stream designs in a transportation arrange. Blanders in TDOD request are frequently spread into the system yields causing untrustworthy planning and operational arrangements. In this examination, a bi-level optimization problem is proposed where the upper level is an Ordinary Least-Squared (OLS) mistake minimization problem that minimizes the deviation between the assessed and watched traffic volumes from SCATS, while the lower level creates task extents grid using a mesoscopic reproduction based powerful client equilibrium model. The interior point conjugate slope technique, as an accurate inclination strategy, is applied to take care of the TDOD request estimation problem. The obtained outcomes feature the capacity of the proposed approach in improving the presentation of a powerful enormous scale arrange model of Melbourne CBD. [7]

M. M. Islam et al.,[2016] In various genuine problems, there could likewise exist different followers, which might be dependent on one another. In this work, we stretch out BLMA to take care of problems with numerous followers. Various benchmark multifollower problems from the writing are tackled using the proposed calculation, alluded to as BLMAMF, and contrasted with as of late revealed outcomes with show the viabiliy of the methodology. [8]

A. Y. S. Lam et al.[2016] In this work, propose another open transportation framework dependent on AVs. It deals with an armada of AVs to oblige transportation demands, offering point-to-point administrations with ride sharing. We center around the two significant problems of the framework: scheduling and affirmation control. The previous is to design the most conservative
timetables and courses for the AVs to fulfill the permissible solicitations, while the last is to determine the arrangement of allowable solicitations among all solicitations to create greatest benefit. [9]

Z. Guo et al.,[2015] This work tends to an inventory network scheduling problem with thought of creation and transportation tasks. This problem is figured as a bilevel blended integer nonlinear program. An advancement procedure based bilevel optimization model is created to deal with this problem. The presentation of the proposed model is assessed by numerical examinations dependent on certifiable information and industrial-size problems. Test results show that the proposed model can take care of the investigated problem successfully. [10]

A. Sinha et al.,[2015] within the sight of different goals, the resulting arrangement set is a Pareto-ideal boodocks that comprises of ideal choices of the pioneer and corresponding ideal reactions from the follower. The power's goals are to boost its incomes through tolls and minimize the contamination levels. The system clients' destinations are to minimize travel cost and travel time. [11]

M. M. Islam, H. K et al.,[2015] In this work, present another settled methodology for solving bilevel optimization problems. The exhibited approach utilizes memetic calculation at the upper level, while a worldwide or a neighborhood search technique is utilized in the lower level during different periods of the hunt. The exhibition of the proposed approach is contrasted and two built up approaches, NBLEA and BLEAQ, using SMD benchmark problem set. The numerical examinations exhibit the advantages of the proposed approach both as far as precision and computational cost, establishing its potential for solving bilevel optimization problems. [12]

H. Zhou et al.,[2014] Linear bilevel programming has been read for a long time and applied in various domains, for example, transportation, financial aspects, engineering, condition, and media communications. Be that as it may, there is absence of consideration of the effects on powerful basic leadership with unexpected or unordinary occasions brought about by erratic regular habitat or human exercises (for example Tidal wave, seismic tremor, and vindictive or psychological oppressor assaults). Truly these occasions could happens all the more frequently and have progressively critical effects on basic leadership in an increasingly mind boggling and dynamic world. [13]

C. Feng et al.,[2014] This work exhibits an augmentation of the two-organize transportation planning problem for a huge scale hydropower development venture where bi-level programming and an equalization of the acceptable degree between chiefs are considered. For solving the complex and non-linear bilevel programming model, an interactive fluffy programming strategy and a molecule swarm optimization with uninvolved assembly calculation are planned as a combined arrangement approach. Finally, the outcomes and examination of a functional model are introduced to exhibit the reasonableness and effectiveness of the proposed model and optimization strategy. [14]

X. Qiu et al.,[2013] Supply Center point in Industrial Park (SHIP) can give transportation administration sharing by dispatching a vehicle to circle numerous makers to cover their conveyance prerequisites. This work talks about how Supply Center in Industrial Park (SHIP) and producers interact to enhance their choices on transportation pricing and milk-run process duration, and the conveyance calendars of crude materials. This problem is demonstrated as a bilevel program with the SHIP as the pioneer and makers as followers. A numerical report is led to examine the influence of major parameters. [15]

L. Jia et al.,[2013] Transportation-dispersion planning problems have been one of the significant themes in the board science. This work contemplates a bilevel multiobjective transportation-dispersion planning model in which the distributing organization has two goals and the manufacturing organization has one goal. For the proposed model, we present a crossover calculation to take care of this problem, the calculation utilizes the controlled elitism and the dynamic crowding separation to improve the NSGA-II, and hence a changed NSGA II is given. A handy problem is displayed to approve the proposed calculation. [16]

T. Sugiyama et al.,[2012] The railroad group rostering problem is a scheduling problem to find a lot of lists that is plausible task of team obligations satisfying a few work conditions. We propose a novel disintegration way to deal with take care of railroad team rostering problems with the goal of reasonable work condition in request to decrease the computational exertion. The upper level ace problem determines a task of group obligations to the arrangement of programs and the lower level subproblem produces a practical succession of lists including a few work conditions. [17]

R. Minciardi et al.,[2012] The operational administration of traffic streams, constrained by various chiefs (that don't trade information) through a system, offers ascend to a typical modeling structure that may find application within various research regions: street traffic control, unsafe materials transportation, media transmission systems, vitality frameworks. In this work, a general choice design is considered and an application is given to the instance of the administration of armadas of vehicles that transport unsafe materials (hazardous materials). The considered engineering considers the nearness of various chiefs. The problem is likewise described by the nearness of a few (conceivably conflicting) targets. For the situation of hazardous materials transportation, such destinations might be the decrease of monetary expenses and the containment of the hazard (for vehicles and infrastructures). [18]

Li Yanhong et al.,[2011] This work investigates the thoughts of traveler transport structure optimization within exhaustive vehicle hall using bi-level planning procedures. And afterward a bi-level multi-target programming model is built up in this work, where the upper level considers the manufacturers and government's anxiety, for example, framework effectiveness, transport limit, capital investment. [19]

G. Hongli, et al.,[2011] This work expounds the birth and advancement of bilevel programming integrating lists of sources home and abroad. At that point examination a few solving algorithms of general bilevel programming, multi-objective bilevel programming and introduce their present applications in transportation, asset allotment, coordinations and different angles quickly. The last piece of the work is the further possibilities of its improvement. [20]

M. Mesbah, et al.,[2011] The point of this work is to reallocate the street space between private vehicle and travel modes so the framework is enhanced. A bilevel programming approach is adjusted for this reason. The upper level involves a target work from the framework chiefs' point of view, though at the lower level, a clients' viewpoint is demonstrated. To consider the significant impacts of a need arrangement, three models are utilized: 1) a modular split; 2) a client equilibrium traffic task; and 3) a travel task. A genetic calculation (GA) approach is utilized, which empowers the technique to be applied to enormous systems. [21]

K. Deb et al.,[2010] The examination indicates an away from of developmental algorithms in solving such troublesome problems of down to earth significance contrasted with their typical arrangement by a computationally costly settled method. The examination opens up numerous issues identified with multi-objective bilevel programming and ideally this investigation will
propel Emotional and different specialists to give more consideration to this significant and troublesome problem solving action. [22]

### Table 1: Summary of literature survey

<table>
<thead>
<tr>
<th>Sr No.</th>
<th>Author Name &amp; Year</th>
<th>Proposed Work</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Kaushal, B IEEE 2020</td>
<td>Bilevel fixed charge fractional transportation problem</td>
<td>Reduction of waste and decreased natural contamination.</td>
</tr>
<tr>
<td>2</td>
<td>H. Wen-jun IEEE 2019</td>
<td>Multi-mode bilevel transportation model based on network vulnerability</td>
<td>Reduces the effect of anomalous occasions</td>
</tr>
<tr>
<td>3</td>
<td>C. Lu, IEEE 2018</td>
<td>Proposes a bilevel model to deal with the WIM location problem</td>
<td>Results show that the heuristic outflanks</td>
</tr>
<tr>
<td>4</td>
<td>F. Wang IEEE 2017</td>
<td>Bilevel feature extraction-based text mining</td>
<td>Enhances the exactness of shortcoming analysis</td>
</tr>
<tr>
<td>5</td>
<td>M. M. Islam IEEE 2016</td>
<td>Proposed a bilevel memetic algorithm</td>
<td>Demonstrate the adequacy of the methodology</td>
</tr>
<tr>
<td>6</td>
<td>Z. Guo IEEE 2015</td>
<td>Bilevel mixed integer nonlinear program</td>
<td>Proposed model can take care of the investigated problem adequately</td>
</tr>
<tr>
<td>7</td>
<td>H. Zhou IEEE 2014</td>
<td>Bilevel multi-follower programming with a virtual follower</td>
<td>Better bring about instance of expanded model and approach</td>
</tr>
<tr>
<td>8</td>
<td>K. Deb IEEE 2010</td>
<td>Multi-objective bilevel programming</td>
<td>Improved populace sizing and termination criteria</td>
</tr>
</tbody>
</table>

### III. CHALLENGES AND APPLICATIONS

#### A. Challenges

1. **Allocation Models**

   Bilevel programming has additionally filled in as a reasonable choice for modeling designation problems where two-hierarchized levels with various targets are involved. At each level, the leader expects to streamline his own interest. The predefined existing chain of command permits that the upper level has total information about the lower level's choice on the portion, yet not on the other way around way. Specifically, bilevel programming offers an advantageous structure for dealing with the portion problems.

2. **Information Protection**

   The strategies and approaches solving bilevel programming problems additionally are genuine in the zones of information assurance and cybersecurity. Be that as it may, the arrangements in these comparative zones have some exceptional highlights, particularly, concerning certain cryptographic applications. One of these highlights is the following. There exists a standard (single-level) mathematical formalization of the cryptographic problem, yet it has been appeared to manage a few blemishes. Subsequently, the proposed strategies and approaches dependent on the bilevel programming procedures eliminates those deficiencies and improve the processing of problems of the information assurance and cybersecurity on the new quality level.

   A significant and basic problem that shows up in these kinds of circumstances is the distribution of assets or the portion of gatherings in the entire procedure considered. Consequently, we are going to isolate this writing audit in two ways: first, the past works done where the ideal assignment of assets are portrayed, and afterward, the works identified with ideally dispense clients, circulation focuses, plants, or different gatherings involved in a particular inventory network are refereed.

3. **Software Implementation**

   With the ongoing increase in information innovation in current society, the problem of information security happened to uncommon desperation. The most troublesome undertaking is to give secure handling and capacity of basic and secret information for government and privately owned businesses, banks, and different frameworks. An answer for this problem is to execute frameworks that accommodate information secrecy, integrity, validness, and availability by methods for cryptographic programming and cryptographic equipment dependent on certain methodologies making utilization of bilevel programming.

   Simultaneously, cryptoanalytical strategies taking preferred position of the advancement in capacities of present day PCs request high prerequisites on the security parameters of current cryptosystems with the utilization of the notable systems and gadgets of bilevel programming. Also, the increased measure of information prepared in current information frameworks needs a very elevated level execution of the cutting edge cryptosystems. Subsequently, the timing prerequisites to cryptographic applications have increased drastically; that is, imminent cryptoalgorithms must give productive processing of mass information while applying bilevel programming and, simultaneously, a significant level of security.

#### B. Applications

Bilevel optimization problems are generally found in various genuine problems. This includes problems in the domain of transportation, financial aspects, choice science, business, engineering, natural financial matters and so on. A portion of the pragmatic bilevel problems examined in the writing are quickly talked discussed.

1. **Toll setting problem**

   In the field of transportation, bilevel optimization normally shows up in the cost setting problem. Consider a system of parkways that is worked by the administration. The administration needs to augment its incomes by choosing the ideal cost setting for the thruways. Be that as it may, the administration can boost its incomes just by taking the parkway clients’ problem into...
account. For some random duty structure the parkway clients take care of their own optimization problem, where they minimize their traveling costs by deciding between utilizing the expressways or an elective course.

2. Structural optimization

Auxiliary optimization problems comprise of two levels of optimization task and are generally alluded as mathematical programming problems with equilibrium constraints (MPEC). The upper level goal in such problems may involve cost minimization or weight minimization subject to limits on removals, stresses and contact powers.

3. Defense applications

Bilevel optimization has various applications in resistance, as vital hostile and cautious power structure plan, vital aircraft power structure, and assignment of strategic airplane to missions. The hostile substance for this situation might be viewed as a pioneer and the protective element for this situation might be viewed as a follower.

4. Solution methodologies

Bilevel optimization problems are difficult to fathom. One arrangement technique is to reformulate bilevel optimization problems to optimization problems for which vigorous arrangement algorithms are accessible. Broadened Mathematical Programming (EMP) is an augmentation to mathematical programming dialects that gives a few watchwords to bilevel optimization problems. These comments encourage the programmed reformulation to Mathematical Projects with Equilibrium Constraints (MPECs) for which develop solver innovation exists. EMP is accessible within GAMS.

5. Evolutionary bilevel optimization

For complex bilevel problems, old style techniques flop because of troubles like non-linearity, discreteness, non-differentiability, non-convexity and so forth. In such circumstances, developmental strategies, however computationally demanding, could be an elective instrument to balance a portion of these troubles and lead to a surmised ideal arrangement.

6. Multi-objective bilevel optimization

A bilevel optimization problem can be summed up to a multi-objective bilevel optimization problem with different destinations at one or the two levels.

7. Transportation Problem

This problem originates from an evolved way of life industry which is utilizing its pre-owned material for the creation of another material. Improvement of the mathematical model of this problem is likewise challenging.

IV. CONCLUSION

In this paper, we present an overview of Bilevel Programming and Application territory, firmly identified with applied problems, for example, flammable gas irregularity money out problem, transportation problem, cost optimization problem, and others. Late outcomes and patterns in the blended integer bilevel programming models with linear target capacity and constraints are additionally portrayed. Many open inquiries despite everything exist in Bilevel Programming hypothesis, particularly according to applications. New points/questions emerge as, use of non smooth variational investigation. Numerous new applications are discovered; much is yet open regarding arrangement algorithms; significant are additionally blended discrete bilevel optimization problems.

REFERENCES


