Power 3 Mean Labeling of Line Graphs

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Abstract:
In this paper we contribute some new results on Power 3 mean labeling of graphs. We investigate on some standard graphs that accept Power 3 mean labeling and proved that the Line graphs of these Power 3 mean graphs are also Power 3 mean graphs. We proved that the Line graphs of Path, Cycle, Comb, \(P_n \odot K_1, P_n \odot K_{1,2}\) are Power 3 mean graphs.

Key words: Graph, Power 3 mean graph, Line Graph, Path, Cycle, Comb, \(P_n \odot K_1, P_n \odot K_{1,2}\).

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1. Introduction:
All graphs in this paper are finite, simple, and undirected graph \(G = (V, E)\) with \(p\) vertices and \(q\) edges. For all detailed survey of Graph labeling, we refer to J.A.Gallian [1]. For all other standard terminology and notations we follow Harary [2]. The concept of mean labelling has been introduced by S.Somasundaram and R.Ponraj [3] in 2004. S.S.Somasundaram and S.S.Sandhya introduced Harmonic mean labeling [4] in 2012. S.S.Sandhya, S.Sreeji introduced Power 3 mean labeling.

In this paper we investigate the Line graphs of some standard Power 3 mean graphs Path, Cycle, Comb, \(P_n \odot K_{1,2}\). We will provide a brief summary of definitions and other information which are necessary for our present investigation.

A Path \(P_n\) is a walk in which all the vertices are distinct. A Cycle \(C_n\) is a Closed Path. The graph obtained by joining a single pendant edge to each vertex of a Path is called a Comb. A Complete Bipartite graph \(K_{m,n}\) is a bipartite graph with bipartition \((V_1, V_2)\) such that every vertex of \(V_1\) is joined to all the vertices of \(V_2\). Where \(|V_1| = m\) and \(|V_2| = n\). \(P_n \odot K_{1,2}\) is a graph obtained by attaching each vertex of \(P_n\) to the central vertex of \(K_{1,2}\).

Definition 1.1:
A graph \(G\) with \(p\) vertices and \(q\) edges is called a power 3 mean graph, if it is possible to label the vertices \(x \in V\) with distinct labels \(f(x)\) from \(1, 2, \ldots, q + 1\) in such a way that in each edge \(e = uv\) is labelled \(f(e = uv) = \left(\frac{x^2 + y^2}{2}\right)^\frac{1}{3}\) or \(\left(\frac{x^2 + y^2}{2}\right)^\frac{1}{2}\). Then, the edge labels are distinct. In this case \(f\) is called Power 3 Mean labelling of \(G\).

Remark 1.2:
If \(G\) is a Power 3 mean graph, then ‘1’ must be a label of one of the vertices of \(G\), since an edge should get label ‘1’.
Remark: 1.3

If u gets label ‘1’, then any edge incident with u must get label 1 (or) 2 (or) 3. Hence this vertex must have a degree \( \leq 3 \).

Definition 1.4:

Let \( G = V, E \) be a non-trivial graph. Now each edge in \( E \) can be considered as a set of two elements of \( V \). So \( E \) is a non-empty collection of distinct non-empty subsets of \( V \), such that their union is \( V \). So there is an intersection graph \( \Omega(E) \). Their graph \( \Omega(E) \) is called the line graph of \( G \) and is denoted by \( L(G) \).

We observe that the vertices of \( L(G) \) are the edges of \( G \). Further two vertices of \( L(G) \) are adjacent iff their corresponding edges are adjacent in \( G \). Thus the vertices \( a, b \) in \( L(G) \) are adjacent iff \( a = uv \) and \( b = vw \) are in \( G \).

Theorem 1.5: Any Path \( P_n \) is a Power 3 mean graph.

Theorem 1.6: Any Cycle \( C_n, n \geq 3 \) is a Power 3 mean graph.

Theorem 1.7: Any Comb \( P_n \bowtie K_1 \) is a Power 3 mean graph.

Theorem 1.8: \( P_n \bowtie K_{1,2} \) is a Power 3 mean graph.

Remark : 1.9 Line graph of path \( P_n \) is a Power 3 mean graph.

Remark : 1.10 Line graph of Cycle \( C_n \) is a Power 3 mean graph

Remark : 1.11 In Path and Cycle, Graph \( G \) and Line graph \( L(G) \) are isomorphic to each other.

2. Main Results:

Theorem 2.1:

The line graph of \( K_{1,3} \) is a Power 3 mean graph

Proof:

The graph \( K_{1,3} \) is displayed below.

![Figure 1](image)

Figure : 1

The Line graph of \( K_{1,3} \) is displayed below.
Theorem 2.2:

Line graph of Comb $P_n \bigcirc K_1$ is a Power 3 mean graph.

Proof:

Let $G$ be a graph obtained from a Path $P_n = u_1u_2 \ldots u_n$ by joining the vertex $u_i$ to $v_i; 1 \leq i \leq n$. 

Graph $G$ of Comb $P_5 \bigcirc K_1$ is displayed below.

Let $e_i$ be the vertices of $L(G)$. The Line graph $L(G)$ of Comb $P_5 \bigcirc K_1$ is shown in figure 9.

In general, the Line graph $L(G)$ of Comb $P_n \bigcirc K_1$ is shown in figure 10.

Let $u_i, v_i$ be the vertices of Line graph $L(G)$. 
Define a function $f(\mathcal{G}) \rightarrow \{1,2, \ldots, q + 1\}$ by,

$f(u_1) = 1$; $f(u_2) = 2$; $f(u_i) = 3i - 2$; $3 \leq i \leq n - 1$

$f(u_n) = f(u_{n-1}) + 2$; $f(v_i) = 3i$; $1 \leq i \leq n$

Edges are labeled with, $f(u_1u_2) = 1$; $f(u_iu_{i+1}) = 3i - 3$; $2 \leq i \leq n - 2$

$f(u_{n-1}u_n) = f(u_{n-1})$; $f(v_iu_{i+1}) = 3i - 1$; $1 \leq i \leq n - 3$; $f(v_iu_{i+2}) = 3i + 1$

Hence $L(G)$ of Comb $P_n \odot K_3$ is a Power 3 mean graph.

**Example : 2.3** Power 3 mean labeling of Line graph of Comb $P_5 \odot K_1$ is shown below.

![Figure 6]

**Theorem 2.4:**

Line graph of $P_n \odot K_{1,2}$ is a Power 3 mean graph.

**Proof:**

Graph $G$ of $P_4 \odot K_{1,2}$ is displayed below.

![Figure 7]

The Line graph $L(G)$ of $P_4 \odot K_{1,2}$ is shown in figure : 13

![Figure 8]
In general, the Line graph of $P_4 \circ K_{1,2}$ is shown in figure : 14

Let $L(G)$ be the line graph of $P_n \circ K_{1,2}$ and $u_i, v_i, w_i$ be the vertices of $L(G)$.

Define a function $f: V(G) \rightarrow \{1, 2, ..., q + 1\}$ by

$f(u_1) = 1$; $f(u_i) = 6i - 9; 2 \leq i \leq n - 1$;

$f(u_n) = f(u_{n-1}) + 4$

$f(v_1) = 2$;

$f(v_i) = 6i - 7; 2 \leq i \leq n - 1$;

$f(w_i) = 6i + 1; 1 \leq i \leq n - 3$

Edges are labeled with $f(u_1 u_2) = 2$;

$f(u_i u_{i+1}) = 6i - 5; 2 \leq i \leq n - 2$

$f(u_{n-1} u_n) = f(u_{n-1}) + 2$

$f(u_1 v_1) = 1$;

$f(u_i v_i) = 6i - 8$;

$f(v_1 u_2) = 3$;

$f(v_2 u_3) = 8$

$f(v_i u_{i+1}) = 6i - 4; 3 \leq i \leq n - 2$;

$f(v_n u_{n+1}) = f(v_n) + 1$;

$f(w_i u_{i+1}) = 6i - 1; 1 \leq i \leq n - 3$

$f(w_i u_{i+1}) = 6i; 1 \leq i \leq n - 3$;

$f(w_i u_{i+2}) = 6i + 3; 1 \leq i \leq n - 3$

Thus, $f$ admits Power 3 mean labeling of $G$. Hence $L(G)$ of $P_n \circ K_{1,2}$ is a Power 3 mean graph.

Example : 2.5 Power 3 mean labeling of Line graph $P_4 \circ K_{1,2}$ is shown below.
Theorem 2.9:

Line graph of \( K_3 \odot K_1 \) is a Power 3 mean graph.

Proof:

The graph of \( K_3 \odot K_1 \) is shown below.

In the above figure, the vertices and edges are get distinct labels.

Hence Line graph of \( K_3 \odot K_1 \) is a Power 3 mean graph.

3. Conclusion:

The study of Power 3 mean labelling of Line graphs is important due to its diversified applications. Line graphs of all Power 3 Mean Graphs are not Power 3 Mean Graphs. It is very interesting to investigate graphs which admits Power 3 Mean Labeling. In this Paper, We proved that Line Graph of Path, Cycle, Comb, Star, \( P_n \odot K_{1,2} \) are Power 3 Mean Graphs. The derived results are demonstrated by means of sufficient illustrations which provide better understanding. It is possible to investigate similar results for several other graphs.

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