Mathematical Modelling on Air Quality Management

Akansha Agarwal[1], Dr. Rajan Singh[2], Dr. B.k Singh[3]

[1,2,3]Department of Mathematics IFTM University, Moradabad.

Abstract

The aim and idea of this paper is to manage air quality and its implications on economic activity. The basic approach of this paper is based on the concept of least cost solution subject to meet prescribed set of standard performance. We also analyze whether the decision is fair to every individual. We are also presenting the general model of pollution control.

Keywords: - Air pollution, Partial Derivatives, AQI, LPP.

Introduction

Air pollution management modelling is a numerical tool used to describe the causal relationship between emissions, meteorology, atmospheric concentrations, deposition, and other factors. Air pollution measurements give important, quantitative information about ambient concentrations and deposition, but they can only describe air quality at specific locations and times, without giving clear guidance on the identification of the causes of the air quality problem.

Mathematical modelling defined here as the use of equations to describe or simulate process in a system which is inherent in applying knowledge and is indispensable for science and societies .air quality management is the process to determine what atmospheric concentration of a particular pollutant is acceptable and to determine what level of source emission of that pollutant in that area can be allowed if the specified concentration is not exceeded and to develop regulations to ensure that the level of emission is reduced and maintained at or below level. Air pollution modelling, instead, can give a more complete deterministic description of the air quality problem, including factor analysis. The management of ambient air pollution is difficult but important. It involves identification of the sources of material emitted into air, the quantitative estimation of the emission rate of the pollutants, the understanding of the transport of the substance from source to downwind. Effective management and planning of resources and environmental system is a reason of concerns in the past decades since contamination and resource-scarcity problems have led to a variety of impacts and liabilities. Mathematical models are recognized and effective tools that could examine economic, environmental, and ecological impact of alternative pollution-control and resources-conservation actions, and thus aid planners or decision-makers in formulating cost-effective management policies. The management of air pollution and its implications on economic activity and quality of life is important. Many researchers such as heaney[1],teller[2] and seinfield and keyn[3] attempted to identify the problems related to environment quality management. Due to the non - existence of a real world they gain little success in implementing their proposal. To overcome these difficulties dantzing and wolfe[4] suggested to create small sectors and the environmental problem of these sectors to become a component of the total regional problem .but for the developing countries this type of decentralized process of environmental control will not be beneficial suggested by baumal and fabian[5].they suggested that it may create a problem for particular interest of individual to determine the quality of pollutant discharged .it is therefore, necessary to establish a guideline of regional activity. Derfman and jacob[6] have argued that the optimization models can be used to find a number of alternatives for control policies of environment. Here we will discuss some optimization techniques by which we handle air quality management problems.

Air pollution management

Air pollution management is divided into three categories.

1. The first category consist of those in which we know the standard of air at a particular place and time. The objective of this type is to minimize the cost of control to achieve standard air quality.
2. The second problem is that in which resources available for control of pollution has a limit and the objective is to minimize pollutant concentration.
3. This type of problem will be a utility function type in which utility function is defined as the cost of control and cost of pollutant damage may be added and minimized.

This type of approach was first studied by kohn[7,8] and gorret al[9].here we will take a similar approach of how burning of coal with high sulphur content pollutes the environment.

Let each pollution producing activity p, has a set of feasible control C_p

We consider x_{pq} as the output level of activity using the control combination q. We also define

S_p=total level of pollution output p to be produced

c_{pq}= the cost of producing each unit of output p using control method q,

$O_{pq,l}$ =output of pollutant l produced by each unit of production of output p with control method q,

E_l=maximum allowable emission of pollutant l.

Then the pollution control model can be expressed as

Min$\sum c_{pq}x_{pq}$

Subject to $\sum x_{pq} = S_p and \sum O_{pq,l}x_{pq} \leq E_l$

This is a linear programming problem and can be solved by simplex method.

Again, if the resources are fixed and the problem is to minimize pollution level, then we can write as

Min$\{\sum O_{pq,l}x_{pq}\}$

Subject to $\sum c_{pq}x_{pq} \leq R$

$$\sum x_{pq} = S_p$$

Where R =Resources available for pollution control.

One should choose an efficient point which minimizes the damage function of pollution. If such damage function f is given then problem of minimizing cost due to damage and control can be formulated as

$$\mathrm{Min}[(\sum a_{pq,l}x_{pq}) + \sum c_{pq}x_{pq}]$$

Subject to $\sum x_{pq} = S_p$.

Model of pollution control

Let P' be the pollution level of a certain environmental conditions and is considered to be a flow related to the gross output, and the capital k invested to control pollution. Then

$P' = F(x,\text{k})$ (1)

The derivative of the functions are as follows

$\frac{\partial P\prime}{\partial x} > 0 \quad \frac{\partial^2 P\prime}{\partial x^2} \geq 0$ (2)

$\frac{\partial P\prime}{\partial K} < 0 \quad \frac{\partial^2 P\prime}{\partial K^2} < 0$ (3)

$\frac{\partial^2 P\prime}{\partial x \partial K} = 0$ (4)

Assumption (2) implies that each unit of pollution control capital decreases pollution but that the decrease of each succeeding pollution unit is less that the preceding unit of pollution level.

Assumption (3) implies that the pollution increases with output at a non-decreasing rate. Since it is not a prior i obvious that size of stock of pollution control capital would affect the marginal increase of pollution per unit of output the cross partial derivative have been assumed to be zero.

The flow of pollution is assumed to affect utility. We have assumed that the social economic welfare at any point of time is measured strictly as a concave function u of current consumption c and current pollution level p. We define the utility function as

$U = U(c, P')$ (5)

Utility function has following properties

$U_c > 0 \quad U_{cc} < 0 \quad \lim_{c \to 0} U_c(c, P') = \infty$

$U_{P\prime} < 0 \quad U_{pp} < 0 \quad \lim_{P\prime \to 0} U_{P\prime}(c, P') = 0$

$U_{cp} = 0$

The economy has a fixed productive capacity and produces a constant level of output y which can be devoted to consumption or to anti-pollution activity E,i.e.

$$Y = c + E$$

The pollution accumulation can be expressed as

$$P' = z(c) - f(P') + \alpha$$

Z(c) the pollution control function, which has following properties,$z' > 0, z'' > 0$

$$z(c_0) = 0, z(c) < 0, if\ c < c_0$$

$$, z(c) > 0, if\ c > c_0$$

The problem of maximizing utility flow can be discussed as follows:

$$max \int_0^{\infty} e^{-\partial t} U(c, P') dt$$

Subject to $P' = z(c) - f(P') + \alpha$

$$P'(0) = P_0$$

This can be solved by using pentryagin's maximum principle (hadley and kemp). The necessary condition of optimal solution is that there exist a continuous function $\emptyset(t)$ such that if

$$H = U(c, P') + \emptyset[z(c) - f(P') + \alpha]$$

$$\frac{\partial H}{\partial c} = U_c + \emptyset z' = 0$$

$$\dot{\emptyset} = (\partial + f')\emptyset - U_p\ ; P' = z(c) - f(\dot{P'}) + \alpha\ ; P(0)^{\cdot} = P_0$$

The constant variable can be interpreted as social price or imputed cost of pollution. The expression of can be represented as the difference between the additional utility from one more unit of output used for consumption and the decrease in utility dur to pollution resulting from additional output.

The basic approach of this paper is based on concept of least cost solution, subject to meet a prescribed set of performance standard. This least cost control process might comprise a wide variety of onsite control as well as offsite control.

We consider a region whose environment control has been undertaken and we partitioned it in four areas. Each study has two options

- Onsite control
- Offsite control

The following notations is used

X_{ij}= decision variable number of units of control j selected for area i.

$\overline{X_{ij}}$ = upper bound on i and j

c_{ij}= unit cost for control j in area i

D_{ij}=quantity of commodity originates in area i

$\overline{Q_i}$ = maximum allowable release of pollutant from area i

Z_i=total control cost to area i

α_i=reduction in control cost to area i if $\overline{Q_i}$ is increased by one unit.

t_j =unit cost of transporting pollutant from area j to central control location

c=unit cost of central control

$\overline{W}$= maximum available control at central facility.

In this problem we have taken a simple situation to make us understand the concept without getting into computational difficulties. This result can be extended more easily to realistic cases where multiple central control facilities exist. The objective of this model is to minimize the total cost of onsite and offsite control.

Mathematically, this problem can be formulated as

Z=min $\sum c_{ij}x_{ij} + (c + t_j)Q$

Subject to $\sum X_{ij} + \alpha_i = D_i$

$Q_i \leq \overline{Q_\iota}$; $X_i \leq \overline{X_{\iota J}}$ for all j

$Q_i \geq 0$, $X_{ij} \geq 0$

This type of model can be solved using linear programming method, if q is known.

Air quality index (AQI) values	Levels of health concern	Colours
When the AQI is in this range:	*Air quality conditions are:*	*As symbolized by this colour*
0 to 50	Good	Green
51 to 100	Moderate	Yellow
101 to 150	Unhealthy for sensitive groups	Orange
151 to 200	Unhealthy	Red
201 to 300	Very unhealthy	Purple
301 to 500	Hazardous	Maroon

- “Good" AQI is 0 to 50. Air quality is considered satisfactory, and air pollution poses little or no risk.
- "Moderate" AQI is 51 to 100. Air quality is acceptable; however, for some pollutants there may be a moderate health concern for a very small number of people. For example, people who are unusually sensitive to ozone may experience respiratory symptoms.
- "Unhealthy for sensitive groups" AQI is 101 to 150. Although general public is not likely to be affected at this AQI range, people with lung disease, older adults and children are at a greater risk from exposure to ozone, whereas persons with heart and lung disease, older adults and children are at greater risk from the presence of particles in the air.
- "Unhealthy" AQI is 151 to 200. Everyone may begin to experience some adverse health effects, and members of the sensitive groups may experience more serious effects.
- "Very Unhealthy" AQI is 201 to 300. This would trigger a health alert signifying that everyone may experience more serious health effects.

- "Hazardous" AQI greater than 300. This would trigger a health warnings of emergency conditions. The entire population is more likely to be affected.

REFRENCES

1. Heaney,J.P., Carter, B.J. and Pyatt, E. E. Equivalent price for upstream BOD removal. Water Resource Research,I,3(1971).
2. Teller, A. The use of linear programming to estimate the cost of some alternative air pollution abatement policies. Proceedings IBM Scientific computer symposium, New York(1968).
3. Seinfield, J.H. and Kyen, C.P. Determination of optimal air pollution control strategies. Socio-Economic Planning science,5,(1971).
4. Dantzing, G.B. and Wolfe, P. Decomposition principle for linear programming. Operational Research 8,1(1940).
5. Baumal, W.J. and Fabian, T. Decomposition pricing for decentralization and external economics. Management Science,II,1(1964).
6. Derfman, R. and Jacoby, H.D. A model of public decisions illustrated by water pollution policy problem in the analysis and evaluation of public expenditure: the PPB systems. Submitted to the subcommittee on economy in Govt., U.S. Congress, Joint Economic Committee,1,Washington D.C., U.S. Govt. Printing office(1969).
7. Heaney, J.P. and Sheikh, H. Game theoretic approach to equitable regional environmental quality management :In Mathematical Analysis of decision problem in Ecology. Springer-Verlag, Berlin(1973).
8. Luce, R. and Raiffa, H. Game and decision. Wiley ,New York(1957).
9. Loehman, E.and Whinstane, A. A new theory of pricing and decision making for public investment. The Bell Journal of Economics and Management Science,2,2(1971).
10. Koh, R.E. Linear Programming model for air pollution control . J. Air Pol. Control Assoc.20(1970).
11. Koh, R.E. Optimal air quality standards. Econometrica, Nov.(1971).
12. Abramar, R. Optimization model for regional air pollution control. Proceedings of NATO Conf., Turkey (1973).
13. Loucks, D.P. Environmental noise management. Proceedings of NATO Conf., Turkey (1973).