Analysis of Robust control methods for rigid robots

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Abstract: Robot manipulators play an efficient role in modern industry due to its enhanced precision, quality, productivity, and efficiency. Controlling these systems becomes difficult because of its non-linearity and time varying behaviors. Adaptive control methods and robust control methods are the efficient control mechanisms used to control uncertain systems. In adaptive control philosophy the controller is designed to learn the uncertain parameters associated with the system. Whereas, in robust control methods are easy to implement the controller has a fixed structure and yields an acceptable performance. In this paper, the various robust control techniques like the Linear-multivariable Approach, Passivity based approach, Variable structure controllers, the Robust Saturation Approach, the Robust Adaptive approach, which is used to control the motion of rigid robots are discussed and multiple papers have been summarized.

IndexTerms- Robust control, Rigid robots, Linear-multivariable Approach, Passivity-Based Approach, Variable-Structure Controllers, Robust Saturation Approach.

I INTRODUCTION

In the adaptive approach, the uncertain parameters of the particular system can be understood in designing a controller. In the robust approach, the controller structure is fixed- which gives efficient performance for a given uncertainty set. So, robust controllers are simpler to implement. To take advantage of both approaches, researchers have attempted to robustify certain adaptive controllers.

Linear-multivariable approach [1], Passivity based approach, Variable structure controllers, Robust Saturation Approach, Robust Adaptive approach are addressed. A linear-multivariable approach globally linearizes and decouples the robot's equations. The uncertain feedback terms shows because one does not have access to the exact inverse dynamics the linearization and decoupling will not be exact. This may be handled using techniques like multivariable linear robust control [2]. The passive nature of the robot is used to solve the same in Passivity based approaches [3]. Robust stability is conformed in Passivity-based methods of the closed-loop controller system. Even though uncertain parameter knowledge of the robot it tries to maintain the passivity of the closed-loop controller system. Variable Structure Controllers (VSS) are used in nonlinear processes. The robust Saturation Approach is used in controllers that are bound to uncertainty. Adaptive control design procedure for nonlinear systems with unknown nonlinearities and parametric uncertainty is known as the Robust Adaptive Approach.

II LINEAR-MULTIVARIABLE APPROACH

The linearization of nonlinear robot dynamics about the desired trajectory has come up in history many times and was popular [4], [5]. This concept was later developed to the global linearization of nonlinear robotics systems. The trajectory error vector [6] $e_1 = q - q_d, e_2 = e_1$.

From the following [7]:
\[
\dot{e} = A_e + B_v
\]

\[
A = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}
\]

\[
B = \begin{bmatrix} 0 \\ f \end{bmatrix}
\]

\[
e = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}
\]

\[
v = D(q)^{-1}[\tau - h(q,d)] - \delta_d……………………………………..(1.0)
\]

We can globally linearize the error system which is reduced for linear control v which gives the desired closed-loop performance
\[
\dot{d} = F_z + G_e
\]
\[
d = F_z + G_e
\]
\[
= C(s)e(t)……………………………………..(1.1)
\]

Where for a system of $f(s)$ with the, $e(t)$ as an input we can expect $v(t)$ as the output.
\[
F = G = H = 0
\]
\[
J = -K
\]
\[
J = -K……………………………………..(1.2)
\]

This is a static state-feedback controller leading to a nonlinear controller
\[
\tau = D(q)[\delta_d + v] + h(q,d)……………………………………..(1.3)
\]
Due to $d(q)$ the closed-loop system becomes
\[ \tau = D(q)[\dot{q} + v] + h(q, \dot{q}) \] (1.4)

Because of uncertainties in $D(q)$ and $h(q, \dot{q})$ the control law cannot be implemented, so we apply $\tau$ we are $\hat{D}$ and $\hat{h}$ are estimates of $D$ and $h$

\[ \tau = \tilde{D}[\dot{q} + v] + \tilde{h} \]

This leads to:
\[ \dot{e} = A_e + B(v + \eta) \]
\[ \eta = E(v + \dot{q}_0) + D^{-1}\Delta h \]
\[ E = D^{-1}D - I_n \]
\[ \Delta h = \hat{h} - h \] (1.5)

The disturbance caused by modeling uncertainties, parameter variation, and measurements with noise in them makes the vector $\eta$ is a nonlinear function of $e, v$ and is an internal disturbance [8]. For a given nonlinear perturbation $\eta$, linear multi-variable is a completely closed-loop system (FIG 1) that is stable to some extent [9]. So by making a suitable $c(s)$ value such that the $e(t)$ is stable.

\[ \dot{e} = A_e + B(v + \eta) \]
\[ v(t) = C(s)e(t) \] (1.6)

Reasonable assumptions are to be made for revolute-joint robots $d_1, d_2, \alpha, \beta_0, \beta_1, \beta_2$, are non-negative values

\[ (d_1)^{-1}In = \|D^{-1}\| < (d_1)^{-1}In \]
\[ \|E\| \leq \alpha \]
\[ \|A h\| \leq \beta_0 \|e\| + \beta_2 \|e\|^2 \] (1.7)

This should be modified for robots with prismatic joints.

In general terms small-gain theorem [10], passivity theorem [10], and total stability theorem [11] are used to find the $c(s)$. Spong and Vidyasagar [8] assumed that the bound on the $\Delta h$ is linear and designed a class of linear compensator $c(s)$ using a factorization approach. Now by choosing an $R(s)$ which satisfies the design rules minimizing $\eta$. Because including better suitable quadratic bound will not make the $L_2$ unstable but will exclude any $L_2$ results unless reformulated and more assumptions are taken into the picture [12]. But by doing this noise in the measurements is not tolerated and the $L_2$ problem in a similar setting was able to show the boundness of the noisy signal.

Form Freund [13], and Tarn et al [14] static feedback compensators were used extensively

\[ v = C(s)e = -Ke \]
\[ \dot{e} = A_e + B(v + \eta) \] (1.8)

In this an extra control loop [14] to minimize the effects of $\eta$ or placing the poles in the left-half plane far enough to guarantee stability even when $\eta$ is present. The bound feedback control in [15] was used to get the $K_p$ value so that $K(sI - A + BK)^{-1}B$ is Strictly Positive Real (SPR). Francis and Wonham [16] model was used as an internal design for a linear controller with minimum by Kuo and Wang [17]. Because $\eta$ is a nonlinear function of $e$ and $v$, reducing $\eta$ does not always mean closed-loop stability.

In the field of robotics, feedback linearization has been used regularly and has been popular for some time now. The variety of linear techniques that can be used in the outer linear loop has been its biggest advantage. But because of large control efforts, many of the practical applications cannot be possible. This can be overcome by combining the local linearization approach with other techniques guaranteeing robust stability [4], [6].

![FIG-1](image-url)

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III Passivity-Based Approach

In models based on the Passive structure of a rigid robot $h(q, ̇q) = C(q, ̇q) ̇q + g(q), D(q) - 2C(q, ̇q)$ is skew-symmetric [3]:

$$D(q) ̇q + C(q, ̇q) ̇q + g(q) = τ$$

(2.1)

Theorem [18] is obtained because of this.

The Lagrange-Euler dynamical equations of a rigid robot define a passive mapping from $τ$ to $̇q$.

$$⟨̇q, τ⟩_T = ∫_0^T ̇𝑞^T 𝜋 𝑑𝑡 ≥ −β$$

(2.2)

Using this, the loop can be closed from $̇q$ to $τ$ with the passive system being asymptotic stable using the passivity theorem [10]. But the stability of $e_1$ is seen and not of $e_2$, but this can be solved a system maps $τ$ to a vector $r$ (i.e., filtered version of $e_1$) and the loop closing from $−r$ and $τ$ giving asymptotic stability to both $e_1$ and $e_2$ [18][19].

$$τ = D(q)a + C(q, ̇q) ̇q + g(q) - K_v( ̇q - ̇v)$$

(2.3)

$$v = q - r$$

(2.4)

$$r = −[sI + K(s)/s]e_1 = −F(s)^{-1}e_1$$

(2.5)

$$a = ̇v$$

(2.6)

where $F(s)$ is strictly proper and $K$ is a positive matrix, then asymptotic stability of $e_1$ and $e_2$ is guaranteed.

Considering the control law:

$$τ = −A(s)e_1 + u_2$$

(2.7)

Where $A(s)$ is an SPR transfer function, external input $u_2$ is bounded in $L_2$. Substituting (2.7) in (2.1) we get:

$$r = −A(s)e_1$$

(2.8)

The values of $e_1$ and $r$ are bound with $L_2$ and this can be deduced by choosing values of $A(s)$ and $u_2$. Since $A(s)^{-1}$ is SPR (inverse of SPR) we can say $e_1$ is asymptotically stable as $e_1 = −A(s)^{-1}r$.

In the case of time-varying trajectories $[q_0^T q_d^T]T$, the error $e_2$ is bounded but not asymptotically stable [20]. Regardless of the robot’s parameters, a controller is guaranteed to be robust until $A(s)$ is SPR and $u_2$ is $L_2$.

$$τ = −K_v(q)e_1 - K_v(q)e_2 + g$$

(2.9)

Even if $D(q)$ is not known stability is guaranteed by the passivity of the robot and feedback law and the accommodation of contact forces and larger uncertainties makes it’s a good approach whereas the requirement of
The $D(q)$ value to be known to find the $K_1$ and $K_2$ values (2.9) on which the closed-loop performance depends ends up as a drawback.

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### IV Variable-Structure Controllers

Many nonlinear processes are controlled by VSS theory, taking this approach error to ‘switching surface’ needs to be done only once, after that the system goes into ‘sliding mode’ where modeling disturbances and uncertainties will not affect the mode [21],[22]. The point regulation problem ($q_d = 0$) was solved in the first-ever publication of the VSS Theory by Young [22] using $\bar{\tau}_i = \tau + i, \text{if} si(e_{i0}, qd > 0)\\\bar{\tau}_i, \text{if} si(e_{i0}, qd) < 0$ .........................................................(3.0)

Where $i$ goes from 1 ... $n$ for an $n$ - link robot, and $s_i$ are the switching planes, $s_i(e_{i0}, q_i) = e_i e_{i0} + q_i, c_i > 0$

Bounds on uncertainties and sliding surfaces $s_1, s_2, ..., s_n$ are shown using the hierarchy, once $\tau^+$ and $\tau^-$ are found to drive the error signal to the intersection of sliding surfaces, the error will slide to zero. By this sliding mode, this model eliminates nonlinear coupling of joints which was an issue in [23],[24],[25]. These also have a discontinuous $s_i = 0$ creating chattering which my trigger unmodeled high-frequency dynamics.

Slotine modified the VSS controller called “suction control” to solve this problem [26], [27]. The problem was solved by allowing sliding surfaces to be time-varying. In the first step, the control law forces the path towards the sliding surface. In the second step, the controller itself is smoothed inside a possibly time-varying layer. This helps in the optimal balance between bandwidth and precision.

The VSS controller in which inversion of the inertia matrix was avoided in [28], [29]. Though it’s only theoretically impressive it does not exploit the physics of the robots and the asymptotic stability of the error is sacrificed to avoid chattering.

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Using the auxiliary saturating controller to compensate for unknown elements present in the dynamics of the robot is discussed in this.

\[ D(q)\ddot{q} + C(q, \dot{q})\dot{q} + Z(q, \dot{q}) = \zeta \]  

Where \( D(q) \) and \( C(q, \dot{q}) \) are defined in (2.1) and \( Z(q, \dot{q}) \) is representing friction, gravity, and bounded torque disturbances. Because the controller is only dependent on uncertainty bounds it is considered as a robust controller.

\[ d_i d_n = D(q)d_2 l_n \]

\[ \parallel C(q, \dot{q})\dot{q} + Z(q, \dot{q}) \parallel \leq \zeta_0 + \zeta_1 \parallel e \parallel + \zeta_2 \parallel e \parallel^2 \]  

Where \( d_i \)'s and \( \zeta_i \)'s are positive scalar constants and the trajectory error is defined.

To prove the ultimate boundedness of \( e \) Spong [30] used Lyapunov stability theory, Spong’s controller is representative of this class and given as:

\[ \zeta = (2d_1d_2)(d_1 + d_2)^{-1} \left[ d_2 - K_2 e_2 - K_1 e_1 - v_r \right] + \dot{C}(q, \dot{q}) + \dot{Z}(q, \dot{q}) \]

Where \( V_r = \begin{cases} \left( B^T Pe \right) \left( \parallel B^T Pe \parallel \right)^{-1} \rho, & \text{if } \parallel B^T Pe \parallel > \varepsilon \rho, \\ \left( B^T Pe \right) \rho^{-1} \rho, & \text{if } \parallel B^T Pe \parallel \leq \varepsilon \end{cases} \]

And \( \rho = (1 - \alpha)^{-1} \left[ a \parallel d_d \parallel + \parallel K_1 \parallel e_1 + \parallel K_2 \parallel e_2 + \left( d_1 \right)^{-1} \Phi \right] \)

\[ \Phi = \beta_0 \parallel e \parallel + \beta_2 \parallel e \parallel^2 \]

\[ \alpha = (d_2 - d_1)(d_2 + d_1)^{-1} \]  

Where matrix \( P \) is symmetric,

\[ A^T P + P A = -Q \]

The positive-definite solution of the Lyapunov equation where \( Q \) is symmetric and positive-definite matrix.

It becomes clear that \( v_r \) depends on \( K_1 \) and \( K_2 \) through this however my interfere with the ability to adjust the Serov gains, so:

\[ \zeta = -K_2 e_2 - K_1 e_1 - v_r \left( \rho, e_1, e_2, \varepsilon \right) \]

\[ \rho = \delta_0 + \delta_1 \parallel e \parallel + \delta_2 \parallel e \parallel^2 \]

Where \( \delta \)'s are the positive scalars Note \( \rho \) is no longer contains the servo gains so \( K_1 \) and \( K_2 \) can be varied without tampering auxiliary control \( v_r \). If the initial error is \( e(0) = 0 \) and \( k_3 \) is chosen as \( k_2 = 2K_2 = K_1d_n \) the tracking error may be bounded as

\[ \parallel e \parallel \leq \left[ \varepsilon \left( 2k_2 + \frac{3d_2}{2} \right) \varepsilon (k_3d_2)^{-1} \right]^{(12)} \]

\[ k_d > \parallel T_d \parallel \]

A similar simulation of a controller using Manutee R3 was made by Corless [31] and also by Chen which required an acceleration measurement in [32]. Saturating-type feedback derived from Lyapunov-stability theory was used by Gilbert and Ha in [33] to guarantee the ultimate boundedness of tracking error.

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VI Robust Adaptive Approach

Combining adaptive and robust control concepts a scheme was derived by Slotine [34] in which vector of estimated parameters and \( Y(,) \) is a \( nXr \) regression matrix is given by \( \Phi \).
\[
\frac{d\phi}{dt} = -y^T(.)[e_1 + e_2] \tag{5.0}
\]

The tracking error should be asymptotically stable. If there is no error in (2.1). However may become unbounded in presence of bounded disturbance or unmodeled dynamics [35],[36]. Parameter estimates remain bounded this was shown by Slotine in [37] if one uses

\[
\tau = \tau_a + k_d sgn(e_1 + e_2) \tag{5.1}
\]

Where \(\tau_a\) is explained in (5.2) and is a positive scalar constant following \(k_d > \|T_d\|\) condition. To compensate for both unmodeled dynamics and bounded disturbances, Reed introduced \(\sigma - \) in [38] modifying the original work done by Loannou in [39]. The law now looks like

\[
\frac{d\phi}{dt} = -y^T(.)[e_1 + e_2] - \sigma \phi \tag{5.2}
\]

Where

\[
\sigma = \begin{cases} 
0; & \text{if } \|\hat{\phi}\| < \phi_0 \\
\|\hat{\phi}\| (\phi_0)^{-1}; & \text{if } \phi_0 < \|\hat{\phi}\| < 2\phi_0 \\
1; & \text{if } \|\hat{\phi}\| > 2\phi_0
\end{cases}
\]

And

\[
\phi_0 > \|\phi\| \tag{5.3}
\]

Using this it was shown that tracking error and all closed-loop signals are bounded.

The other approach by Singh [40] combining Spong’s controller in (4.2) to estimate \(\beta_0, \beta_1, \beta_2\) uncertainty in (4.3), making knowledge of the exact size of uncertainties not necessary. Certain instability in adaptive control robots was addressed by Spong and Ghorbel in [41].

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**VII Conclusion**

The major experimental robust control approaches for a rigid robot were reviewed. Though the controllers were Robust in nature and could handle a range of uncertain parameters the choice of which control to use is a complex question. The Linear multivariable approach can be used where specifications of the device are linear in nature (i.e. Damping ratio, percent overshoot, etc.). But the main drawback of this is that it may lead to high gain control laws to get robust control. When it comes to the Passive controller though they are easy to build in a practical but their performance is less than desired. With that, the robust version of the passive controller does not exploit the physics as the adaptive version which may cause some issues in a few scenarios. VSS theory is used in many nonlinear processes where, error to ‘switching surface’ needs to be done only once, after that the system goes into ‘sliding mode’ where modeling disturbances and uncertainties will not affect anymore. In the Robust Saturation approach, the auxiliary saturating controller is used to compensate for the unknown elements present in the dynamics of the robot.

Robust Adaptive Approach is used for a class of nonlinear systems with both parametric uncertainty and unknown nonlinearities. Though robot dynamics are nonlinear in nature some successful controllers have used the robot’s physics. This information becomes very useful when force control comes into the picture.

**REFERENCES**


