ON RADIO HERONIAN DD-DISTANCE MEAN LABELING OF SOME BASIC GRAPHS

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Abstract

A Radio Heronian Mean Dd-distance Labeling of a connected graph G is an injective map f from the vertex set $V(G)$ to the N such that for two distinct vertices u and v of G, $D^{\mathrm{Dd}}(u,v)+\left\lceil\frac{f(u)+\sqrt{f(u)f(v)}+f(v)}{3}\right\rceil \geq 1+diam^{\mathrm{Dd}}(G)$ where $D^{\mathrm{Dd}}(u,v)$ denotes the Dd-distance between u and v and $diam^{\mathrm{Dd}}(G)$ denotes the Dd-diameter of G. The radio heronian Dd-distance number of f, $rhmn^{\mathrm{Dd}}(f)$ is the maximum label assigned to any vertex of G. The radio heronian Dd-distance number of G, $rhmn^{\mathrm{Dd}}(G)$ is the minimum value of $rhmn^{\mathrm{Dd}}(f)$ taken over all radio heronian Dd-distance labeling f of G.

Keywords: Dd-distance, Radio heronian mean Dd-distance number.

1. Introduction

A graph $G=(V,E)$ we mean a finite undirected graph without loops or multiple edges. The order and size of G are denoted by p and q respectively. Graph labeling was introduced by Alexander Rosa in 1967. Radio mean labeling was introduced by S. Somasundaram and R. Ponraj in 2004. Harmonic mean labeling was introduced by S. Somasundaram and S S Sandhya in 2012.

The concept of D-distance was introduced by D. Reddy Babu et al. The concept of radio D-distance was introduced by T. Nicholas and K. John Bosco in 2017. The concept of radio mean D-distance was introduced by T. Nicholas and K. John Bosco in 2017. The concept of heronian mean labeling was introduced by S S Sandhya in 2017. The Dd-distance was introduced by A. Anto kinsely and P. Siva Ananthi [1]. For a connected graph G, $u-v$ path is defined as $D^{Dd}(u,v)=D(u,v)+\deg(u)+deg(v)$.

We are introduce the concept of radio heronian Dd-distance mean labeling of some basic graphs. A Radio Heronian Mean Dd-distance Labeling of a connected graph G is an injective map f from the vertex set $V(G)$ to the N such that for two distinct vertices u and v of G, $D^{\mathrm{Dd}}(u,v)+\left\lceil\frac{f(u)+\sqrt{f(u)f(v)}+f(v)}{3}\right\rceil \geq 1+diam^{\mathrm{Dd}}(G)$ where $D^{\mathrm{Dd}}(u,v)$ denotes the Dd-distance between u and v and $diam^{\mathrm{Dd}}(G)$ denotes the Dd-diameter of G. The radio heronian Dd-distance number of f, $rhmn^{\mathrm{Dd}}(f)$ is the maximum label assigned to any vertex of G. The radio heronian Dd-distance number of G, $rhmn^{\mathrm{Dd}}(G)$ is the minimum value of $rhmn^{\mathrm{Dd}}(f)$ taken over all radio heronian Dd-distance labeling f of G.

2. Main Result

Theorem 2.1

The radio heronian mean Dd-distance number of a complete graph K_n, $rhmn^{Dd}(K_n)=n$.

Proof:

Let $V(K_n)=\{v_{1,}v_{2,}v_{3,}\dots\dots\dots v_{n,}\}$ be the vertex set. Then $D^{Dd}(v_i,v_j), 1\leq i,j\leq n, i\neq j$.

It is $diam^{Dd}(K_n) = 3(n-1)$.

The radio heronian mean Dd-distance condition is $D^{\mathrm{Dd}}(u,v) + \left\lceil \frac{f(u)+\sqrt{f(u)f(v)}+f(v)}{3} \right\rceil \geq 1 + diam^{\mathrm{Dd}}(G)$

Now $D^{\mathrm{Dd}}(v_1,v_2) + \left\lceil \frac{f(v_1)+\sqrt{f(v_1)f(v_2)}+f(v_2)}{3} \right\rceil \geq 1 + diam^{\mathrm{Dd}}(K_n)$

$f(v_1) + \sqrt{f(v_1)f(v_2)} + f(v_2) \geq 1,$ which implies $f(v_1) = 1$ and $f(v_2) = 2$

So $f(v_i) = i, 1 \leq i \leq n.$

Hence $rhmn^{Dd}(K_n) = n.$

Theorem: 2.2

The radio heronian mean D-distance number of a path, $rhmn^{Dd}(P_n) \leq \begin{cases} n, & 2 \leq n \leq 7 \\ 2n-7, & n \geq 8 \end{cases}$

Proof:

Let $V(P_n) = \{v_{1,}v_{2,}v_{3,} \dots \dots \dots v_n\}$ be the vertex set and $E(P_n) = \{v_iv_{i+1}, 1 \leq i \leq n-1\}$ be the edge set.

Then $D^{Dd}(v_1,v_n) = D^{Dd}(v_n,v_2) = n+1, D^{Dd}(v_i,v_{i+1}) = 7, 1 \leq i \leq n$

So $diam^{\mathrm{Dd}}(P_n) = n+1$

Without loss of generality, let $f(v_1) < f(v_2) < \cdots \dots \dots \dots \dots . < f(v_{n-1})$.

We shall check the radio heronian mean Dd-distance condition

$D^{\mathrm{Dd}}(u,v) + \left\lceil \frac{f(u)+\sqrt{f(u)f(v)}+f(v)}{3} \right\rceil \geq 1 + diam^{\mathrm{Dd}}(G)$

Now, $D^{\mathrm{Dd}}(v_1,v_n) + \left\lceil \frac{f(v_1)+\sqrt{f(v_1)f(v_n)}+f(v_n)}{3} \right\rceil \geq 1 + diam^{\mathrm{Dd}}(P_n)$

$f(v_1) + \sqrt{f(v_1)f(v_2)} + f(v_2) \geq 1,$ which implies $f(v_n) = n-7$ and $f(v_1) = n-6$

$D^{\mathrm{Dd}}(v_1,v_2) + \left\lceil \frac{f(v_1)+\sqrt{f(v_1)f(v_2)}+f(v_2)}{3} \right\rceil \geq 1 + diam^{\mathrm{Dd}}(P_n)$

$f(v_1) + \sqrt{f(v_1)f(v_2)} + f(v_2) \geq 2n-8,$ which implies $f(v_1) = n-6$ and $f(v_2) = n-5$

$D^{\mathrm{Dd}}(v_2,v_3) + \left\lceil \frac{f(v_2)+\sqrt{f(v_2)f(v_3)}+f(v_3)}{3} \right\rceil \geq 1 + diam^{\mathrm{Dd}}(P_n)$

$f(v_1) + \sqrt{f(v_1)f(v_2)} + f(v_2) \geq 2n-10,$ which implies $f(v_2) = n-5$ and $f(v_3) = n-4$

Therefore $f(v_i) = n+i-7, 1 \leq i \leq n$

Hence $rhmn^{Dd}(P_n) \leq \begin{cases} n, & 2 \leq n \leq 7 \\ 2n-7, & n \geq 8 \end{cases}$

Theorem 2.3

The radio heronian mean Dd-distance number of a star, $rhmn^{Dd}(K_{1,n}) \leq \begin{cases} n+1, & 2 \leq n \leq 4 \\ 2n-3, & n \geq 5 \end{cases}$

Proof:

Let $V(K_{1,n}) = \{v_0, v_{1,}v_{2,}v_{3,} \dots \dots \dots v_n\}$ be the vertex set and $E(K_{1,n}) = \{v_0v_i, 1 \le i \le n-1\}$ be the edge set.

Then $D^{Dd}(v_i, v_j) = 4, 1 \le i, j \le n, i \ne j$

$D^{Dd}(v_0, v_i) = n+2, 1 \le i \le n.$

So $diam^{\mathrm{Dd}}(P_n) = n+2$

Without loss of generality, let $f(v_1) < f(v_2) < \cdots \dots \dots \dots \dots . < f(v_{n-1}).$

We shall check the radio heronian mean Dd-distance condition

$$D^{\mathrm{Dd}}(u,v) + \left\lceil \frac{f(u)+\sqrt{f(u)f(v)}+f(v)}{3} \right\rceil \ge 1 + diam^{\mathrm{Dd}}(G)$$

Now, $D^{\mathrm{Dd}}(v_0, v_1) + \left\lceil \frac{f(v_0)+\sqrt{f(v_0)f(v_1)}+f(v_1)}{3} \right\rceil \ge 1 + diam^{\mathrm{Dd}}(K_{1,n})$

$f(v_0) + \sqrt{f(v_0)f(v_1)} + f(v_1) \ge 1,$ which implies $f(v_0) = n-3$ and $f(v_1) = n-2$

$D^{\mathrm{Dd}}(v_1, v_2) + \left\lceil \frac{f(v_1)+\sqrt{f(v_1)f(v_2)}+f(v_2)}{3} \right\rceil \ge 1 + diam^{\mathrm{Dd}}(K_{1,n})$

$f(v_1) + \sqrt{f(v_1)f(v_2)} + f(v_2) \ge 2n-3,$ which implies $f(v_1) = n-2$ and $f(v_2) = n-1$

$D^{\mathrm{Dd}}(v_2, v_3) + \left\lceil \frac{f(v_2)+\sqrt{f(v_2)f(v_3)}+f(v_3)}{3} \right\rceil \ge 1 + diam^{\mathrm{Dd}}(K_{1,n})$

$f(v_1) + \sqrt{f(v_1)f(v_2)} + f(v_2) \ge 2n-3,$ which implies $f(v_2) = n-1$ and $f(v_3) \ge n-2$

Therefore $f(v_3) = n,\ \ f(v_i) = n+i-3,\ \ 0 \le i \le n$

Hence $rhmn^{Dd}(P_n) \le \begin{cases} n+1, & 2 \le n \le 4 \\ 2n-3, & n \ge 5 \end{cases}$

Theorem: 2.4

The radio heronian mean Dd-distance number of a subdivision of a star,

$$rhmn^{Dd}(S(K_{1,n})) \le \begin{cases} 3, n = 1 \\ 2n+1, n \ge 2 \end{cases}$$

Proof:

Let $V(S(K_{1,n})) = \{v_0, v_{1,}v_{2,}v_{3,} \dots \dots \dots v_n, u_1, u_2, \dots u_n\}$ be the vertex set , where v_0 is the apex vertex.

Let $E(S(K_{1,n})) = \{v_0v_i, v_iu_i\ 1 \le i \le n-1\}$ be the edge set.

Then $D^{Dd}(v_i, u_i) = 4, 1 \le i \le n, D^{Dd}(v_0, u_i) = n+3, 1 \le i \le n$

So $diam^{\mathrm{Dd}}(P_n) = n+1$

Without loss of generality, let $f(v_0) < f(v_1) < f(v_2) < \cdots \dots \dots \dots \dots . < f(v_n).$

We shall check the radio heronian mean Dd-distance condition

$$D^{\mathrm{Dd}}(u,v) + \left\lceil \frac{f(u)+\sqrt{f(u)f(v)}+f(v)}{3} \right\rceil \ge 1 + diam^{\mathrm{Dd}}(G)$$

Now, $D^{\mathrm{Dd}}(v_0,u_1)+\left\lceil\frac{f(v_0)+\sqrt{f(v_1)f(u_1)}+f(u_1)}{3}\right\rceil \geq 1+diam^{\mathrm{Dd}}(S(K_{1,n})$

$f(v_0)+\sqrt{f(v_0)f(u_1)}+f(u_1) \geq 1,$ which implies $f(v_0)=1$ and $f(u_1)=3$

$D^{\mathrm{Dd}}(u_1,u_2)+\left\lceil\frac{f(u_1)+\sqrt{f(u_1)f(u_2)}+f(u)}{3}\right\rceil \geq 1+diam^{\mathrm{Dd}}(S(K_{1,n}))$

$f(u_1)+\sqrt{f(u)f(u_2)}+f(u_2) \geq 2n-5,$ which implies $f(u_1)=3$ and $f(u_2)=n+3$

Therefore $f(u_i)=n+i+1, 2 \leq i \leq n.$

$D^{\mathrm{Dd}}(u_n,v_1)+\left\lceil\frac{f(u_n)+\sqrt{f(u_n)f(v_1)}+f(v_1)}{3}\right\rceil \geq 1+diam^{\mathrm{Dd}}(S(K_{1,n}))$

$f(u_n)+\sqrt{f(u_n)f(v_1)}+f(v_1) \geq 2n-5,$ which implies $f(u_n)=2n+1$ and $f(v_1)=n+2$

Therefore $f(v_i)=n+i+1, 1 \leq i \leq n$

Hence $rhmn^{D}(S(K_{1,n})) \leq \begin{cases} 3, n=1 \\ 2n+1, n \geq 2 \end{cases}$

Theorem: 2.5

The radio heronian mean Dd-distance number of a fan graph, $rhmn^{Dd}(F_n) \leq \{2n, \quad n \geq 2$

Proof:

Let $V(F_n)=\{v_0,v_1,v_2,v_3,\dots\dots\dots v_n\}$ be the vertex set and $E(F_n)=\{v_0v_i, v_iv_{i+1}, 1 \leq i \leq n\}$ be the edge set.

Then $D^{Dd}(v_0,v_1)=2n+2, D^{Dd}(v_1,v_2)=n+5$

So $diam^{\mathrm{Dd}}(P_n)=2n+2$

Without loss of generality, let $f(v_0)<f(v_1)<f(v_2)<\cdots\dots\dots\dots\dots.<f(v_n)$.

We shall check the radio heronian mean Dd-distance condition

$D^{\mathrm{Dd}}(u,v)+\left\lceil\frac{f(u)+\sqrt{f(u)f(v)}+f(v)}{3}\right\rceil \geq 1+diam^{\mathrm{Dd}}(G)$

Now, $D^{\mathrm{Dd}}(v_0,v_1)+\left\lceil\frac{f(v_0)+\sqrt{f(v_0)f(v_1)}+f(v_1)}{3}\right\rceil \geq 1+diam^{\mathrm{Dd}}(F_n)$

$f(v_0)+\sqrt{f(v_0)f(v_1)}+f(v_1) \geq 1,$ which implies $f(v_0)=n$ and $f(v_1)=n+1$

$D^{\mathrm{Dd}}(v_1,v_2)+\left\lceil\frac{f(v_1)+\sqrt{f(v_1)f(v_2)}+f(v_2)}{3}\right\rceil \geq 1+diam^{\mathrm{Dd}}(F_n)$

$f(v_1)+\sqrt{f(v_1)f(v_2)}+f(v_2) \geq 2n+1,$ which implies $f(v_1)=n+1$ and $f(v_2)=n+2$

$d^{\mathrm{Dd}}(v_2,v_3)+\left\lceil\frac{f(v_2)+\sqrt{f(v_2)f(v_3)}+f(v_3)}{3}\right\rceil \geq 1+diam^{\mathrm{Dd}}(F_n)$

$f(v_2)+\sqrt{f(v_2)f(v_3)}+f(v_3) \geq 2n+2,$ which implies $f(v_2)=n+2$ and $f(v_3)=n+4$

Therefore $f(v_i)=n+i, 1 \leq i \leq n$

Hence $rhmn^{Dd}(F_n) \leq \{2n, \quad n \geq 2$

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