Roots of a number

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Abstract

To find a square root of a number which is perfect square in similar way to find a cube root of a number which is perfect cube we have to make some mathematical operations but we may solve such problems by observation. In this paper the analysis of square of number and cube of a number is made. This analysis results to find a square root of a number which is perfect square or cube root of a number which is perfect cube. Such a discussion may be extended for fourth root or fifth root of a number.

Key words:- Square root, Cube root, Fourth root.

I. Introduction

There is a method of finding prime factors when we have to find out the square root of a number which is perfect square number, cube root of a number which is perfect cube number and fourth root of a number which is perfect fourth power of some number. Sometimes this method is difficult to find various root of a number. In this paper some facts are discussed so that we can find various root of a number by observation. These facts and observation are easy and logical to find various roots of a number.

Square Root of a perfect square number

In traditional method to find square root of a number we find the all prime factors of a given number. Then we write it as product of square of each prime factor separately. Then selecting one-one factor from each term we obtain square root of a number which is perfect square.

Some Observations in finding Square Root of a perfect square number

1) Square of two digit number contains three or four digits. Similarly square of three digit number contain five or six digits.

2) The squares of first nine natural numbers are 1, 4, 9, 16, 25, 36, 49, 64, & 81. Observe that 1, 4, 5, 6, 9 are digits at unit place. Hence no perfect square number has 2, 3, 7 & 8 at its unit place.

3) Square of 1 and 9 has 1 at its unit place. Square of 2 and 8 has 4 at its unit place. Square of 3 and 7 has 9 at its unit place. Square of 4 and 6 has 6 at its unit place. Square of 5 has 5 at its unit place.

4) Conversely for any **perfect square number,** digit at its unit place and 'digit at units' place' of square root are related as follows.

Unit place digit of Perfect square number	Unit place digit of Square root
1	1 or 9
4	2 or 8
5	5
6	4 or 6
9	3 or 7

5) Number with odd numbers of zeros as consecutive right most digits is not a perfect square number.

Square root by observation

This method is most suitable for perfect square number which has almost four digit.

1) Group the numbers in two parts. Right part (RP) will consists of unit and tens place and left part will have remaining one or two digits.

2) Select a number p (say) whose square is nearest less than last part.

3) Observe the unit place digit of RP and choose possible unit place digit of square root. Let d_1 and d_2 be the digits as in the table.

4) Then square root of given number is pd_1 or pd_2. Observe $pd_1 \leq p5^2 \leq pd_2$.

5) This comparison gives exact square root.

Ex:- Find the square of 784.

Solution:-

1) This is three digits number whose square root has to be found out. Now the square of 20 is 400 and square of 30 is 900 hence square root of 784 is greater than 20 but less than 30.

2) Divide 784 in to two parts left part is 7 right part is 84. Unit place of right part is 4 hence unit place of a number which is square root of 784 is 2 or 8.

3) Now square root of 784 is greater than 20 but less than 30 hence tenth place of square root is 2 and unit place is 2 or 8. Hence possible square roots are 22 or 28. But square of 25 is 625 (which is easy to find out) and 784 is greater than 625 hence square root of 784 is 28.

Ex:- Find the square of 4356.

Solution:-

1) This is four digits number whose square root has to be found out. Now the square of 60 is 3600 and square of 70 is 4900 hence square root of 4356 is greater than 60 but less than 70.

2) Divide 4356 in to two parts left part is 43 right part is 56. Unit place of right part is 6 hence unit place of a number which is square root of 4356 is 4 or 6.

3) Now square root of 4356 is greater than 60 but less than 70 hence tenth place of square root is 6 and unit place is 4 or 6. Hence possible square roots are 64 or 66. But square of 65 is 4225 and 4356 is greater than 4225 hence square root of 4356 is 66.

Perfect cube

Observe the following table -

Number	1	2	3	4	5	6	7	8	9	10
Cube	1	8	27	64	125	216	343	512	729	1000
Digital root of cube	1	8	0	1	8	0	1	8	0	1

Observing the above table one can easily observe the following properties of a cube –

(1) If Unit digit of a number is 1, 4, 5, 6 or 9 the unit digit of its cube is same.

(2) If unit digit of a number is 2, 3, 7 or 8 , the unit digit of its cube is 8, 7, 3 or 2 resply.

(3) By method of multiplication by Observation method we can say that if the unit place of a number is 0 the its cube contains three zeros at the end.

(4) The digital root of a cube is 0, 1 or 8.

It is observed that these properties are found true for all the cubes.

If there are necessary number of zeroes (three times a positive integer) on at the end of given number , we can neglect them and we find the cube root of the remaining part and then adjust one third of the neglected zeros on the right .

Cube root of a perfect cube

First we shall discuss to find the cube root of a number with at most 6 digits and having no zero at the end..

Step 1 : Deciding the unit digit of the cube root.

If the unit digit of given number is 1, 4, 5, 6 or 9 the unit digit of its cube root must be the same 1, 4, 5, 6 or 9 respectively. But if its unit digit is 2, 3, 7 or 8 (which is not observed as the unit digit of a perfect cube), then unit digit of its cube root will be 8, 7, 3 or 2 respectively.

We can tabulate this fact as following –

Unit digit of theNumber	1	2	3	4	5	6	7	8	9
Unit digit of its Cuberoot	1	**8**	**7**	4	5	6	**3**	**2**	9

Step 2 : Deciding the tenth place digit of the cube root.

Now neglect the last three digits of the given number and find the greatest Number whose cube is not greater than the remaining part of the given number. This will be the Tens digit of possible cube root. Adjust the unit digit we found before, which will raise the possible cube root of given number.

Step 3 : By any suitable method find the cube of the obtained cube root and confirm the final cube root.

Example:- Find the cube root of 54872.

The digital root of given number is 8. So it may be a perfect cube. Its unit digit is 2. Hence the unit digit of its cube root will be 10 – 2 = 8.

Now neglecting its last three digits remaining part is 54 and as $3^3 < 6 < 4^3$, the Tens digit will be 3. Hence possible cube root of given number6859 can be 38

Guiding numbers for the cube of 38 are 27 | 72 | 192 | 512 , Hence –

$38^3 = 27 \mid 216 \mid 576 \mid 512 = 27 \mid 216 \mid 576 + 51 \mid (2)$

$= 27 \mid 216 + 62 \mid (72) = 27 + 27 \mid (872) = 54872$

So required cube root = 38.

Example:- Find the cube root of 175616 .

The digital root of given number is 8. So it may be a perfect cube. Its unit digit is 6. Hence the unit digit of its cube root will be = 6.

Now neglecting its last three digits remaining part remains 175 and as $5^3 < 175 < 6^3$, the Tens digit will be 5.

Hence possible cube root of given number can be 56.

Guiding numbers for the cube of 56 are 125 | 150 | 180 | 216 , Hence –

$56^3 = 125 \mid 450 \mid 540 \mid 216 = 125 \mid 450 \mid 540 + 21 \mid (6)$

$= 125 \mid 450 + 56 \mid (16) = 125 + 50 \mid (616) = 175616$.

So cube root of 175616 is 56.

Conclusion

1) The method of finding the square root of a perfect square number and cube root of a perfect cube number is perfect and easy.

2) Unit place of a number always gives us unit place of square root or cube root of a number.

3) This result can be extended for fourth root and fifth root of a number.

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