Bayesian Preliminary Test Estimation (BPTE) of a change point in Weibull Sequence under LLF

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Abstract

When some items are put to test their lives on a life testing system or equipment, we see that after sometimes due to some abrupt change like shut down of the system, there is a break in sequence in recording their lives. This abrupt change can cause in dividing the sequence into two parts. For example If n items are put to test their lives then their lives will be $x_1, x_2, \ldots\ldots, x_n$. If there is one break in sequence, then sequence is divided into two parts. Suppose the change occurs at point mth, the sequence will be $x_1, x_2, \ldots\ldots, x_m$. and $x_m, x_2, \ldots\ldots, x_{n+1}$. Now the problem is how to detect and estimate the break point. In this paper, we have applied the Bayesian estimation method and test our detection by preliminary test estimation technique. The numerical comparison is also done by using R software

Keywords. Change-point analysis, abrupt change, CUSUM control Chart, Bayesian Estimation, Preliminary test estimator, LLF.

1. Introduction.

Change-point analysis has proven to be an efficient tool in understanding the essential information contained in meteorological data, such as rainfall, ozone level, and carbon dioxide concentration.

Physical systems manufacturing the items are often subject to random fluctuations which results in discontinuity at any point of time in any sequence or model. Such point on which discontinuity occurs is known as change point. The problem is of detecting change in sequences of life times and has inference on it. In such models, the main parameter of interest is the change point, which indicates when or where the change occurred. There are two fundamental problems of interest related to this parameter, viz., detection of a change and estimation. In general, an investigator first performs a test to detect a change and, if it is indicated, then the change point is estimated under a specified loss function.

In statistical quality control such studies are very much useful for the shifting in process mean for example cumulating sum(CUSUM) control chart are used in production process to detect in shift in target value, when small shift or change ($<1.5\sigma$) of interest occur, the cusum chart and the exponentially weighted chart are used. Montegomery (2001) and Wu et. al. (2004), discussed the procedure of CUSUM control in shifting in target value. Lim et. al. (2002), Wu and Tiau (2005) and Zhang and Wu (2005) considered the applications of CUSUM control charts.

The term structural change denotes a change in one or more of the parameters of a model. It is also employed to refer to a model, which has been mis-specified. Terms or phrases such as shift point, change point, transition function, switching regressions and two-phase regressions, although not identical in meaning to a structural change, are involved in some way with structural change. Change point models are used to describe discontinuous behavior in stochastic phenomena. The change point indexes where or when the shift occurs. It is a discrete random variable. The prior probability mass function of the shift point gives the nature of the change to be expected.

The Bayesian inferential applications can play an important role in study of such problem of change points. Many of statisticians like Chin and Broemeling (1980), Calabria and Pulcini (1994), Zacks (1983), Pandya and Jani (2006), Shah and Patel (2007,2009), Chib (1998), Altissemo and Corradi (2003) and Fiteni (2004) studied the change point Models in Bayesian framework. Broemeling (1985) and Broemeling and Tsurume (1987)are the useful references on structural change .

When a point estimate is required and alternative hypotheses lead to different estimates, an optimal Bayes estimate is obtained by minimizing posterior expected loss averaged over the hypotheses, with posterior probabilities used as weights. In order to reflect uncertainty regarding the validity of different hypotheses, Zellner and Vandaele (1975) suggested preliminary test estimation of the parameter under a specified loss function in Bayesian framework. Such a Bayesian preliminary test estimate (BPTE) incorporates prior information and is optimal relative to a given loss function. However, so far, no attempt has been made to study BPTE of the change point. Some of the literature includes Dey et al. (1998), Martin et al. (1988), Dey and Micheas (2000), Rios ,Insua and Ruggeri (2000), Micheas and Dey (2004), and the references therein.

In this paper we have discussed Bayesian Preliminary Test Estimation (BPTE) of a change point in Weibull sequence under linex loss function and examine its robustness through numerical simulation.

2. Statistical Model and Loss Function

Weibull distribution has extensively been used in life testing and reliability problems. The Weibull distribution is a continuous probability distribution. It is named after Waloddi Weibull who described it in detail in 1951, although it was first identified by Fréchet (1927) and first applied by Rosin & Rammler (1933) to describe the size distribution of particles in connection with his studies on strength of material. Weibull (1939,1951) showed that the distribution is also useful in describing the wear out of fatigue failures. Estimation and properties of the Weibull distribution is studied by many author’s like Kao (1959), Johnson;Kotz; Balakrishnan; (1994), Lieblein, and Zelen, (1956),Mann(1968).

The probability density function of Weibull distribution is given as

$$f(x) = \frac{\theta}{\sigma} x^{(\theta-1)} \exp\left(-\frac{x^{\theta}}{\sigma}\right) \quad ; \qquad x, \theta, \sigma > 0 \quad , \tag{2.1}$$

Where $'\sigma'$ is the scale and ‘θ’ is shape parameters.

The most widely used loss function in estimation problems is quadratic loss function given as $(\hat{\sigma}, \sigma) = k(\hat{\sigma} - \sigma)^2$, where $\hat{\theta}$ is the estimate of θ, the loss function is called quadratic weighed loss function. If k=1, we have

$$L(\hat{\sigma}, \sigma) = (\hat{\sigma} - \sigma)^2 \quad , \tag{2.2}$$

known as squared error loss function (SELF).

3. The Detection of Change Point.

Suppose $x_1, x_2, \dots, x_m, x_{(m+1)}, \dots, x_n$ is a sequence of independent random variables such that

$$x_i = \begin{cases} f_1(x_i\,; \sigma_1, \theta_1); i = 1,2, \dots \dots \dots \dots \dots, m \\ f_1(x_i\,; \sigma_2, \theta_2), i = (m+1), \dots \dots \dots, n \end{cases} \tag{3.1}$$

Here $x_1, x_2, \dots, x_n$ ($n \geq 3$) be a sequence of observed life times. First let observations $x_1, x_2, \dots\dots, x_n$ have come from Weibull distribution with probability density function (pdf) as

$$f(x) = \frac{\theta}{\sigma} x^{(\theta-1)} \exp\left(-\frac{x^{\theta}}{\sigma}\right) \quad ; \qquad x, \theta, \sigma > 0 \quad , \tag{3.2}$$

Let ‘m’ is change point in the observation, which breaks the distribution in two sequences as ($x_1, x_2, \dots, x_m$) & ($x_{(m+1)}, \dots, x_n$).

The probability density functions of the above sequences are

$$\mathrm{f}_1(\mathrm{x}) = \frac{\theta_1}{\sigma_1} x_i^{\theta_1 - 1} \exp\left(-\frac{x_i^{\theta_1}}{\sigma_1}\right) \quad ; \qquad x, \sigma_1, \theta_1 > 0 \quad , \tag{3.3}$$

$$f_2(x) = \frac{\theta_2}{\sigma_2} x_i^{\theta_2 - 1} \exp\left(-\frac{x_i^{\theta_2}}{\sigma_2}\right) \quad ; \qquad x, \sigma_2, \theta_2 > 0 \quad , \tag{3.4}$$

This can be written with Weibull sequence before and after change point 'm'

$$x_i = \begin{cases} \frac{\theta_1}{\sigma_1} x_i^{\theta_1 - 1} \exp\left(-\frac{x_i^{\theta_1}}{\sigma_1}\right) & i = 1, \dots \dots \dots \dots, m \\ \frac{\theta_2}{\sigma_2} x_i^{\theta_2 - 1} \exp\left(-\frac{x_i^{\theta_2}}{\sigma_2}\right) & i = (m+1), \dots \dots \dots, n \end{cases} \tag{3.5}$$

4. Likelihood, Prior and Posterior.

The joint likelihood function of the Weibull sequences of before and after change point 'm' is given by

$$l(\sigma_1, \sigma_2, p|x) = \prod_{i=1}^{m} f_1(x_i|\sigma_1) \prod_{(m+1)}^{n} f_2(x_i|\sigma_2), \tag{4.1}$$

$$l(\sigma_1, \sigma_2, p|x) = \prod_{i=1}^{m} \frac{\theta_1}{\sigma_1} x_i^{\theta_1 - 1} \exp\left(-\frac{x_i^{\theta_1}}{\sigma_1}\right) \prod_{(m+1)}^{n} \frac{\theta_2}{\sigma_2} x_i^{\theta_2 - 1} \exp\left(-\frac{x_i^{\theta_2}}{\sigma_2}\right) \tag{4.2}$$

The joint prior for 'm' is given by

$$g(m|x) = \iint_{\sigma_1, \sigma_2} g(\sigma_1, \sigma_2, m|x) d\sigma_1 d\sigma_2 \,; \tag{4.3}$$

$$s.t. \quad \sigma_1 \in \Theta_1 \;; \quad \sigma_2 \in \Theta_2 \quad and \quad m = 1,2, \dots .(n-1).$$

With a change point at 'm', where m is unknown, using the equations (4.2) and (4.3), the joint posterior distribution is given by

$$h(\sigma_1, \sigma_2, p|x) = l(\sigma_1, \sigma_2, m). g(\sigma_1, \sigma_2, m); \sigma_1 \in \theta_1, \sigma_2 \in \theta_2, \tag{4.4}$$

such that m=1,2,…(n-1)

Detection of Change Point.

Let the hypothesis for detecting change point 'm' is

$H_o: m = n \quad Vs \quad H_1: m \neq n$

Let us assume that the prior probability mass function of the change point 'm' is

$$g(m) = \begin{cases} p, & if\ m = n \\ \frac{(1-p)}{n-1} & if\ m \neq n \end{cases} \quad ; \quad o < p < 1, p \text{ is known probability}; \tag{4.5}$$

Let us assume that the scalar parameters $\sigma_1$and σ_2 and the change point 'm' are independent of each other.

Let us take prior of scalar parameter σ_1 as natural conjugate gamma prior given by,

$$g(\sigma_1) = \begin{cases} \frac{b_1^{a_1}}{\Gamma a_1} \sigma_1^{-(a_1+1)} e^{-b_1/\sigma_1} \,; & \sigma_1 > 0, (a_1, b_1) > 0 \\ 0 & , \quad \text{Otherwise} \end{cases}, \tag{4.6}$$

The prior of scalar parameter σ_2 as natural conjugate gamma prior given by

$$g(\sigma_2) = \begin{cases} \frac{b_2^{a_2}}{\Gamma a_2} \sigma_2^{-(a_2+1)} e^{-b_2/\sigma_2}, & \text{where } \sigma_2 > 0 \text{ and } (a_2, b_2) > 0 \\ 0 & , \quad \text{Otherwise} \end{cases}, \tag{4.7}$$

Again with independent σ_1, σ_2 and 'm', we have under null hypothesisH_0, the joint prior as

$$g(\sigma_1, \sigma_2, m) = g(\sigma_1). g(m) \quad , \tag{4.8}$$

However under alternative hypothesis H_1, the joint prior is given by

$$g(\sigma_1, \sigma_2, m) = g(\sigma_1)\, g(\sigma_2) g(m), \tag{4.9}$$

Now the joint likelihood is given by

$$l(\sigma_1, \sigma_2, m|\underline{x}) = \begin{cases} \prod_{i=1}^{n} f_1(x_i\,; \sigma_1)\ ; & if\ m = n \\ \prod_{i=1}^{m} f_1(x_i\,; \sigma_1) \prod_{i=m+1}^{n} f_2(x_i\,; \sigma_2); & if\ m \neq n \end{cases}; \tag{4.10}$$

This is derived as

$$l(\sigma_1, \sigma_2, m|\underline{x}) = \begin{cases} \prod_{i=1}^{n} \frac{\theta_1}{\sigma_1} x_i^{\ (\theta_1-1)} \exp\left(-\frac{\sum x_i^{\ \theta_1}}{\sigma_1}\right); \\ \prod_{i=1}^{n} \frac{\theta_1}{\sigma_1} x_i^{\ (\theta_1-1)} \exp\left(-\frac{\sum x_i^{\ \theta_1}}{\sigma_2}\right) \prod_{i=m+1}^{n} \frac{\theta_2}{\sigma_2} x_i^{\ (\theta_2-1)} \exp\left(-\frac{\sum x_i^{\ \theta_2}}{\sigma_2}\right) \end{cases} \tag{4.11}$$

Combining the equations(4.5) ,(4.8),(4.9) and (4.11),we get the joint posterior of $\sigma_1,\ \sigma_2\quad and\quad m$ as

$$h(\sigma_1, \sigma_2, m|\underline{x}) = \begin{cases} p\ g(\sigma_1) \prod_{i=1}^{n} f_1(x_i\,; \sigma_1)\ d\sigma_1\,; & if\ m = n \\ \frac{(1-p)}{(n-1)} \prod_{i=1}^{m} f_1(x_i\,; \sigma_1) \prod_{i=m+1}^{n} f_2(x_i\,; \sigma_2)\ g(\sigma_1, \sigma_{2_1}) d\sigma_1 d\sigma_2; & if\ m \neq n \end{cases} \tag{4.12}$$

And the marginal posterior of 'm' is given by

$$h(m|\underline{x}) = \begin{cases} P\ \int g(\sigma_1) \prod_{i=1}^{n} f_1(x_i\,; \sigma_1) d\sigma_1\,; & if\ m = n \\ \frac{(1-P)}{(n-1)}\ \iint \prod_{i=1}^{m} f_1(x_i\,; \sigma_1) \prod_{i=m+1}^{n} f_2(x_i\,; \sigma_2) d\sigma_1 d\sigma_2; & if\ m \neq n \end{cases}; \tag{4.13}$$

with constant of proportionality

$$[D(x)]^{-1} = P \int g(\sigma_1) \prod_{i=1}^{n} f_1(x_i\,; \sigma_1) d\sigma_1 + \frac{(1-P)}{(n-1)} \sum_{m=1}^{n-1} \iint \prod_{i=1}^{m} f_1(x_i\,; \sigma_1) \prod_{i=(m+`1)}^{n} f_2(x_i\,; \sigma_2) g(\sigma_1 \sigma_2) d\sigma_1 d\sigma_2$$

(4.14)

Which is derived as

$$\boldsymbol{h(m|\underline{x})} = \begin{cases} \mathbf{p} \int \left[\frac{\mathbf{b_1^{a_1}}}{\boldsymbol{\Gamma}\mathbf{a_1}} \boldsymbol{\sigma_1}^{-(\mathbf{a_1}+1)} \mathbf{e}^{-\mathbf{b_1}/\sigma_1} \frac{\boldsymbol{\theta_1}}{\boldsymbol{\sigma_1}} \prod_{i=1}^{n} \boldsymbol{x_i}^{\ (\theta_1-1)} \exp\left(-\frac{\sum \boldsymbol{x_i}^{\ \theta_1}}{\boldsymbol{\sigma_1}}\right)\right] \boldsymbol{d\sigma_1}; \\ \frac{(\mathbf{1-p})}{(\boldsymbol{n-1})} \iint \left[\left\{\prod_{i=1}^{m} \{\frac{\boldsymbol{\theta_1}}{\boldsymbol{\sigma_1}} \boldsymbol{x_i}^{\ (\theta_1-1)} \exp\left(-\frac{\sum \boldsymbol{x_i}^{\ \theta_1}}{\boldsymbol{\sigma_2}}\right) \frac{\boldsymbol{\theta_2}}{\boldsymbol{\sigma_2}} \prod_{i=(m+1)}^{n} \boldsymbol{x_i}^{\ (\theta_2-1)} \exp\left(-\frac{\sum \boldsymbol{x_i}^{\ \theta_2}}{\boldsymbol{\sigma_2}}\right) * \frac{\mathbf{b_1^{a_1}}}{\boldsymbol{\Gamma}\mathbf{a_1}} \boldsymbol{\sigma_1}^{-(\mathbf{a_1}+1)} \mathbf{e}^{-\mathbf{b_1}/\sigma_1} \frac{\mathbf{b_2^{a_2}}}{\boldsymbol{\Gamma}\mathbf{a_2}} \boldsymbol{\sigma_2}^{-(\mathbf{a_2}+1)} \mathbf{e}^{-\mathbf{b_2}/\sigma_2}\right\}\right] \boldsymbol{d\sigma_1 d\sigma_1} \end{cases}$$

(4.13)

On simplifying we get

$$h(m|\underline{x}) = \begin{cases} \frac{p\theta_1 a_1 b_1^{a_1} \prod_{i=1}^{n} x_i^{\ (\theta_1-1)}}{\left(b_1 + \sum x_i^{\ \theta_1}\right)^{(a_1+1)}} \\ \frac{(1-p)}{(n-1)} * \frac{\theta_1 a_1 b_1^{a_1} \prod_{i=1}^{m} x_i^{\ (\theta_1-1)}}{\left(b_1 + \sum x_i^{\ \theta_1}\right)^{(a_1+1)}} * \frac{a_2 \theta_2 b_2^{a_2} \prod_{i=(m+1)}^{n} x_i^{\ (\theta_2-1)}}{\left(b_2 + \sum x_i^{\ \theta_2}\right)^{(a_2+1)}} \end{cases}; \tag{4.15}$$

The posterior in favour of the null hypothesis H_0 is

$$O(H_0|x) = p[(m = n|x)]/p\{m \neq n|x\}, \tag{4.16}$$

$$= \frac{p \int g(\sigma_1) \prod_{i=1}^{n} f_1(x|\sigma_1) d\sigma_1}{(1-p)/(n-1) \sum_{m=1}^{(n-1)} \iint \prod_{i=1}^{m} f_1(x|\sigma_1) \prod_{i=(m+1)}^{n} f_2(x|\sigma_2) g(\sigma_1, \sigma_2)\, d\sigma_1 d\sigma_2}$$

$$= \frac{p\int \frac{b_1^{a_1}}{\Gamma a_1}\sigma_1^{-(a_1+1)}e^{-b_1/\sigma_1}\frac{\theta_1}{\sigma_1}\prod_{i=1}^{m} x_i^{(\theta_1-1)} \exp\left(-\frac{\sum x_i^{\theta_1}}{\sigma_1}\right)d\sigma_1}{\frac{(1-p)}{(n-1)}\sum_{m=1}^{(n-1)} \iint \left\{\frac{\theta_1}{\sigma_1}\prod_{i=1}^{m} x_i^{(\theta_1-1)} \exp\left(-\frac{\sum x_i^{\theta_1}}{\sigma_2}\right) * \frac{\theta_2}{\sigma_2}\prod_{i=m+1}^{n} x_i^{(\theta_2-1)} \exp\left(-\frac{\sum x_i^{\theta_2}}{\sigma_2}\right)\frac{b_1^{a_1}}{\Gamma a_1}\sigma_1^{-(a_1+1)}e^{-b_1/\sigma_1}\frac{b_2^{a_2}}{\Gamma a_2}\sigma_2^{-(a_2+1)}e^{-b_2/\sigma_2}\right\}d\sigma_1 d\sigma_1} \tag{4.17}$$

$$= \frac{\frac{p\theta_1 a_1 b_1^{a_1}\prod_{i=1}^{n} x_i^{(\theta_1-1)}}{\left(b_1+\sum x_i^{\theta_1}\right)^{(a_1+1)}}}{\frac{(1-p)}{(n-1)}\sum_{m=1}^{(n-1)}\left\{\frac{\theta_1 a_1 b_1^{a_1}\prod_{i=1}^{m} x_i^{(\theta_1-1)}}{\left(b_1+\sum x_i^{\theta_1}\right)^{(a_1+1)}} * \frac{a_2\theta_2 b_2^{a_2}\prod_{i=(m+1)}^{n} x_i^{(\theta_2-1)}}{\left(b_2+\sum x_i^{\theta_2}\right)^{(a_2+1)}}\right\}}; \tag{4.18}$$

$$O(H_0|x) = \frac{\frac{p\prod_{i=1}^{n} x_i^{(\theta_1-1)}}{\left(b_1+\sum x_i^{\theta_1}\right)^{(a_1+1)}}}{\frac{(1-p)}{(n-1)}\sum_{m=1}^{(n-1)}\left\{\frac{\prod_{i=1}^{m} x_i^{(\theta_1-1)}}{\left(b_1+\sum x_i^{\theta_1}\right)^{(a_1+1)}} * \frac{a_2\theta_2 b_2^{a_2}\prod_{i=(m+1)}^{n} x_i^{(\theta_2-1)}}{\left(b_2+\sum x_i^{\theta_2}\right)^{(a_2+1)}}\right\}}; \tag{4.19}$$

The hypothesis H_0 is not accepted, if the Posterior odds are less than 1.

5. Bayesian Preliminary Test Estimation (BPTE) of the Change Point

Suppose $x_1, x_2, \dots\dots, x_m, x_{(m+1),} \dots\dots, x_n$ is a sequence of independent random variables such that

$$x_i = \begin{cases} f_1(x_i\,; \sigma_1, \theta_1); i = 1,2, \dots\dots\dots\dots\dots m \\ f_2(x_i\,; \sigma_2, \theta_2), i = (m+1), \dots\dots\dots n \end{cases} ; \tag{5.1}$$

The change point 'm' is an unknown discrete random parameter. Further suppose that the scalar parameters σ_1, σ_2 and 'm' are independent of each other.

Let p_0 denote the posterior probability of the hypothesis $H_0: m = n$ of no change so that $(1 - p_0)$ is the posterior probability of the alternative hypothesis $H_1: m \neq n$ of a change.

The posterior expected loss under the linex loss function $L(m, \widehat{m})$ with change point 'm' is given by

$$E(L(m, \widehat{m}|x) = P_0 E(L(m, \widehat{m}|H_0 x) + (1 - P_0)E\big(L(m, \widehat{m}|H_1 x)\big) \tag{5.2}$$

$$= P_0 L(n, \widehat{m}) + (1 - P_0)E\big(L(m, \widehat{m}|H_1 x)\big) \tag{5.3}$$

Thus the BPTE $\widehat{m}$ of change point 'm' under linex loss function is

$$L_u(m, \widehat{m}) = v(\exp(u(\widehat{m} - m) - u(\widehat{m} - m) - 1)\,; \qquad v > 0, u \neq 0$$

is given by

$$\widehat{m}_u = -\frac{1}{u}\log\{P_0 e^{-un} + (1 - p_0)E(e^{-un}|H_1, x)\},$$

$$= -\frac{1}{u}\log\left\{e^{-un} + \frac{1}{(1+K_{01})}\left(E(e^{-um}|H_1, x) - e^{-un}\right)\right\}; \tag{5.4}$$

Which is equals to

$$\widehat{m}_u = -\frac{1}{u}log\left[K_{01}e^{-un} + \frac{1}{(1+K_{01})}\frac{(1-p)}{(n-1)} * \sum_{m=1}^{(n-1)}\{e^{-um}exp\big((\theta_1 - 1)\sum_{i=1}^{m} logx_i\big) *\right.$$

$$* \, exp\left((\theta_2 - 1)\sum_{i=(m+1)}^{n} logx_i\right) * \frac{\theta_1 b_1^{a_1}}{\left(b_1+\sum_{i=1}^{m} x_i^{\theta_1}\right)^{a_1}} \frac{\theta_2 b_2^{a_2}}{\left(b_2+\sum_{i=(m+1)}^{n} x_i^{\theta_1}\right)^{a_2}}\Bigg] \quad (5.5)$$

Provided expectation exists. Here $K_{01} = \frac{p_0}{(1-p_0)}$ is the posterior odds ratio (POR) in favour of H_0. It is to note that K_{01} close to 1 suggests that H_0 is more or less as likelihood as H_1 a posteriori while if this ratio is large, we regard H_0 as relatively more likely than H_1.

For $K_{01} = 0$, that is the posterior odds ratio indicates a change in the sequence. BPTE $\widehat{m_u}$ will reduce to the Bayes estimate under linex loss. However, for large values of $K_{01}, \widehat{m_u}$ would be close to n.

As observed by Zeller and Vandale (1975), it may interest to recall that (i) $\widehat{m}_u$ is a continuous function of the observations (ii) prior information about m under H_1 can be induced through use of an appropriate prior probability mass function and (iii) there is no arbitraries in the choice of the classical significance level.

Numerical Illustration.

Consider a sequence of 20 independent observations of Weibull distribution with σ =2 and θ =1.5, which are generated such that the first ten are from Weibull distribution where mean of first ten observation is σ_1= 1.731334. The last ten observations are again drawn from Weibull distribution where mean of last ten observation is σ_2= 1.067129

1.45	3.03	2.42	2.14	0.46	2.65	3.03	0.89	1.27	1.16
0.62	1.66	0.83	1.54	1.53	0.62	0.79	1.11	1.67	0.45

Mean(x) =1.465699, Var(x) =0.6659064,

Table (1)

Bayesian Preliminary Test Estimate of m under Linex Loss function

p → u ↓	0.00	0.01	0.05	0.25	0.5	0.75	0.95	0.99
-2	20	20	20	20	20	20	20	20
20	18	18	18	18	18	17	17	16
25	14	14	14	14	14	14	14	13
30	11	11	11	11	11	11	11	11
35	10	10	10	10	10	10	10	9
40	9	9	9	9	9	9	9	9

The following observations are made from Table (1)

1. For u > 0, BPTE of change point ‘m’ started decreasing and provide an under-estimate of ‘m’ and vice-versa, which shows that, overestimation, is more serious than underestimation. It seems to be true because, in particular, for small values of ‘p’ reflecting less faith in the hypothesis of no change.
2. For fixed value of ‘p’, The BPTE of change point ‘m’ decreases as u increases from -2 and greater values of u=20, 25, 30, 35 and 40. However, for fixed u, as p increases, the estimate BPTE of change point ‘m’ decreases. The effective range of u is from 25 to 35. We observe here that BPTE of change point ‘m’ is near the ‘true’ change point m=12.
3. For u = −2, from p = 0.00 to p = 0.99, the BPTE of change point ‘m’ become constant, it means that we are almost sure of H_0.

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