CYLINDRICALLY SYMMETRIC UNIVERSE IN PRESENCE OF ELECTROMAGNETIC FIELD

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**Abstract**: Taking cylindrically symmetric metric we have found a non static cylindrically symmetric cosmological model which is spatially homogenous non-degenerate Petrov type - I. The Energy momentum tensor has been assumed to be that of a perfect fluid with an electromagnetic field. Various physical & geometrical properties of the model have been discussed.

**Introduction**: In recent years there has been a lot of interest in cosmological models in the presence of electromagnetic fields in general relativity. Cosmological models in the presence of a magnetic field have been studied by Zeldovich and Novikov[14] and Thorne[11]. Ginzburg[2] has studied the gravitational collapse of the magnetic star. Galaxies and interstellar spaces exhibit the presence of strong magnetic fields Zeldovich and Novikov[14]. Monoghan[7] and Seymour[9] have discussed the magnetic field in stellar bodies. Del[1] and Jacobs[4] have studied the behavior of the general Bianchi type - I cosmological model in the presence of the spatially homogeneous magnetic field. This work has been further extended by Tupper[12] to include Einstein-Maxwell fields in which the electric field is non-zero. He has also interpreted certain type -VI cosmologies with electromagnetic field Tupper[13] Roy and Prakash[8] taking the cylindrically symmetric metric of Marder[6] have constructed a spatially homogeneous cosmological model in the presence of an incident magnetic field which is also anisotropic and non degenerate Petrov type- I. Later on Singh and Yadav[10] assuming the energy momentum tensor to be that of perfect fluid with an electromagnetic field constructed a spatially homogeneous cosmological model. Some other researchers in this field are Yadav et. al.[15, 16], Saha and Rikhvitsky[17] and Saha and Visinescu[18].

In this paper, we have also constructed a non-static cylindrically symmetric cosmological model which is spatially homogeneous non-degenerate Petrov type-I assuming the energy momentum tensor to be that of a perfect fluid with an electromagnetic field. Various physical and geometrical properties of the model have been found.

The Field Equation and their Solutions:

We consider the most general cylindrically symmetric space time in the form given by

$$ds^2 = A^2(dt^2 - dx^2) - B^2 dy^2 - C^2 dz^2 \qquad\qquad (1.1)$$

where the metric potentials A, B, C are functions of time t alone. This ensures that the model is spatially homogeneous. The distribution consists of a perfect fluid and an electromagnetic field,

Thus

$$R_{ij} - 1/2g_{ij}R + \wedge g_{ij} = -k\left[(\rho+p)u_iu_j - pg_{ij} + E_{ij}\right] \tag{1.2}$$

$$g_{ij}u^iu^j = 1 \tag{1.3}$$

$$g_{ij} = g^{kI}F_{ik}F_{ij} - 1/4g_{ij}F_{mn}F^{mn} \tag{1.4}$$

$$F[ij;k] = 0 \tag{1.5}$$

$$F^{ij};j = j^i \tag{1.6}$$

where E_{ij} is the electromagnetic energy momentum tensor, F_{ij} is the electromagnetic field tensor, $\wedge$ is cosmological constant, J^i is current four vector and ρ and p are respectively the density and pressure of the distribution. The Co-ordinates are chosen to be commoving so that

$$u^1 = u^2 = u^3 = 0, u^4 = 1/A \tag{1.7}$$

We label the co-ordinates $(x, y, z, t) = (x^1, x^2, x^3, x^4)$

Without loss of any generality, we consider the case in which the electric and magnetic fields are in the x-direction. We write

$$F^2\,_{14}A^{-4} + F^2\,_{23}B^{-2}C^{-2} = L^2 \tag{1.8}$$

The diagonal components of the equation (1.2) may be written as

$$\frac{2}{A^2}\left[\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{C_{44}}{C} - \frac{A_4C_4}{AC} - \frac{A_4B_4}{AB} - \frac{A_4^2}{A^2}\right] = 2\wedge = -k[L^2 + (\rho+3p)] \tag{1.9}$$

$$\frac{-2}{A^2}\left[\frac{A_{44}}{A} + \frac{A_4B_4}{AB} + \frac{A_4C_4}{AC} - \frac{A_4^2}{A^2}\right] + 2\wedge = -k[-L^2 + (\rho-p)] \tag{1.10}$$

$$\frac{-2}{A^2}\left[\frac{B_{44}}{B} + \frac{B_4C_4}{BC}\right] + 2\wedge = -k[L^2 + (\rho-p)] \tag{1.11}$$

$$\frac{-2}{A^2}\left[\frac{C_{44}}{C} + \frac{B_4C_4}{BC}\right] + 2\wedge = -k[L^2 + (\rho-p)] \tag{1.12}$$

where the suffix 4 indicates the ordinary differentiation with respect to time t after the symbols A, B, C. From these equations it is clear that L^2, ρ, p are each functions of time t alone. From equations (1.5) and (1.8) it follows that F_{23} is a constant and F_{14} is a function of time t only i.e.

$$F_{23} = k, F_{14} = \pm A^2(L^2 - k^2B^{-2}C^{-2})^{1/2} \tag{1.13}$$

where k is a constant.

The case when $F_{14} = 0$, which implies $J^i = 0$, we get, the model due to Roy Prakash[8] i.e. here assume that $F_{14} \neq 0$ and find the only non-vanishing component of J^i to be

$$J^i = \pm \frac{1}{A^2 BC} \cdot \frac{\partial}{\partial t} \left[BC(L^2 - k^2 B^{-2} C^{-2})^{1/2} \right] \tag{1.14}$$

Equation (1.14) shows that is space like, unless

$L^2 = mB^{-2} C^{-2}$ where m is a constant.

In which case $J^i = 0$. The 4-current is in general the sum of the convection current and conduction current Greenburg [3] and Licknerowicz[5].

We have

$$J^i = \epsilon_0 u^i + \lambda u_j F^{ij} \tag{1.15}$$

where ϵ_0 is the rest charge density and λ is the conductivity. In the case considered here we have $\epsilon_0 = 0$ i.e., magneto hydrodynamics. From equations (1.13), (1.14) and (1.15), we find that the conductivity is given by

$$\lambda = -\frac{1}{A} D_4 D^{-1} \tag{1.16}$$

where $D = BC(L^2 - k^2 B^{-2} C^{-2})^{1/2}$

The requirement of positive conductivity in equation (1.16) puts further restrictions on A, B, C. Hence in the magneto hydrodynamics case metric functions are restricted not only by the field equations and energy conditions they are also restricted by the requirement that the conductivity be positive for a realistic model.

The equations (1.9)-(1.12) are four equations in six unknown A, B, C, p and L In order to determine them, two more conditions have to be imposed on them. For this we assume that the space time is of degenerate Petrov type-1, the degeneracy being in y and z directions. This requires that $C^{12}_{12} = C^{13}_{13}$. This condition is identically satisfied if B =C.

However, we shall take the metric potentials to be unequal. We further assume that F_{14} is such that

$$L^2 = f^2 B^{-3} C^{-3} \tag{1.17}$$

where f is a constant.

From equations (1.11) and (1.12) we have

$$\frac{B_{44}}{B} - \frac{C_{44}}{C} = 0 \tag{1.18}$$

Equation (1.18) with condition $C^{12}_{12} = C^{13}_{13}$ gives

$$\frac{A_4}{A} \left(\frac{C_4}{C} - \frac{B_4}{B} \right) = 0 \tag{1.19}$$

Since $B \neq C$, equation (1.19) gives

$$\wedge = M \text{ (a constant)} \tag{1.20}$$

From equations (1.10), (1.11) and (1.20) we have

$$\frac{B_{44}}{B} \pm \frac{B_4 C_4}{BC} = kL^2 M^2 \tag{1.21}$$

Equation (1.18) on integration gives

$$B_4 C - BC_4 = n \tag{1.22}$$

where n being an arbitrary constant of integration putting $B/C = \alpha$ and $BC = \beta$, equation (1.22) goes to the from

$$\left(\frac{\alpha_4}{\alpha} \right) \beta = n \tag{1.23}$$

and equation (1.21) turns into

$$\frac{1}{\beta} \left[\left(\frac{\alpha_4}{\alpha} + \frac{\beta_4}{\beta} \right) \beta \right]_4 = 2kL^2 M^2 \tag{1.24}$$

From equations (1.23) and (1.24), we have

$$\frac{\beta_{44}}{\beta} = 2kL^2 M^2 \tag{1.25}$$

which, after the use of condition $L^2 = f^2 B^{-3} C^{-3}$ reduces to

$$\beta_{44} = \frac{2kf^2 M^2}{\beta^2} \tag{1.26}$$

Equation (1.26) on integration yields

$$[\beta_4]^2 = \frac{b}{\beta}(\beta - a) \tag{1.27}$$

where

$$a = \frac{4kf^2 M^2}{b} \tag{1.28}$$

and b is an arbitrary constant which we shall take to be unity. Clearly from equations (1.23) and (1.27); we have

$$\frac{d\alpha}{\alpha} = \frac{n}{\beta^{1/2}} \cdot \frac{d\beta}{(\beta - a)^{1/2}} \tag{1.29}$$

Integrating of equation (1.29) gives

$$\alpha = b \left[\beta^{1/2} + (\beta - a)^{1/2} \right]^{2n}, \tag{1.30}$$

b being a constant of integration.

Therefore,

$$B^2 = b\beta[\beta^{1/2} + (\beta - a)^{1/2}]^{2n} \tag{1.31}$$

$$C^2 = \beta / b[\beta^{1/2} + (\beta - a)^{1/2}]^{2n} \tag{1.32}$$

where b is constant, $\beta = BC$ and $a = 4kf^2$

Consequently the line element (1.1) takes the form

$$ds^2 = A^2 \left[\frac{d\beta}{\left(\dfrac{d\beta}{dt}\right)^2} - dx^2 \right] - B^2 dy^2 - C^2 dz^2 \tag{1.33}$$

which, by using equations (1.20), (1.27), (1.31) and (1.32) as takes the form

$$ds^2 = N^2 \left[\left(\frac{\beta}{\beta - a} \right) dt^2 - dx^2 \right] - b\beta \left[\beta^{1/2} + (\beta - a)^{1/2} \right]^{2n}_{dy^2}$$

$$- \frac{\beta}{b} \left[\beta^{1/2} + (\beta - a)^{1/2} \right]^{-2n}_{dz^2} \tag{1.34}$$

The transformation $x \to x, \sqrt{b}y \to \gamma, \dfrac{1}{\sqrt{b}}z \to Z, \beta \to (a + T)$ reduces the metric equation (1.34) to the

form

$$N^2 \left[\left(\frac{a + T}{T} \right) dT^2 - dx^2 \right] - (a + T) \left[T^{1/2} + (a + T)^{1/2} \right]^{2n} dy^2$$

$$- (a + T) \left[T^{1/2} + (a + T)^{1/2} \right]^{-2n} dz^2 \tag{1.35}$$

Some Physical Features: Pressure and density for the model equation (1.34) are given by

$$k.\rho = 1/4(T - a)^{-2} - g^2\phi(T)^2 + kf^2/2(T - C)^{-1} - \wedge \tag{1.36}$$

$$k.\rho = 1/4(T - a)^{-2} - g^2\phi'(T)^2 + 3kf^2/2(T - C)^{-1} + \wedge \tag{1.37}$$

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