Higher Order Statistical Multifractality Spectrum of the Solar Wind Velocity and Interplanetary Magnetic Field

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Abstract*:* This study deals with the multifractality scaling of Solar Wind velocity and Interplanetary Magnetic Field (IMF) from January 1996 to December 2006 which is nearly the time span of Solar Cycle 23. The datasets of these parameters have been taken from OMNI Web server and the level of multifractality is analyzed for solar wind plasma parameters. Our results show the inhomogeneous rate of the transfer of the energy flux indicating multifractal behavior of solar wind plasma parameter turbulence in the inner heliosphere. This shows that the degree of asymmetry for the plasma parameters has much better agreement with the real data for solar cycle 23 and data sampling shows weakly correlation with the phase of the solar activity. This describes the energy transfer is responsible for turbulence in various environments and their features having similar behavior like nonlinear multifractal systems.

***Index Terms* – Solar Wind Plasma Parameters, Interplanetary Magnetic Field, Solar Cycle, Multifractality Spectrum.**

i) Introduction

The multifractality spectrum analysis of solar wind plasma parameters and Interplanetary Magnetic Field (IMF) are very important in context of solar terrestrial plasma relations with magnetosphere (Feder 1988; Takayasu 1989). When these functions show a heavy non-Gaussian tail it demonstrates a highly intermittent and multifractal process, tending to eruption-like energy release. This also indicates that the system, as a whole, is undergoing a non-linear dissipation with the energy interchange between different scales. So, a shape of the probability distribution function is a powerful diagnostics tool to study the nature of such signals (Horbury and Balogh, 2001; Pagel and Balogh, 2003; Yordanova et. al., 2009).

Now these days various observational techniques allows us to calculate the distribution function of the magnetic flux content in elements of the magnetic field of sun also and the theory of multifractality allows us an intuitive understanding of multiplicative processes and of the intermittent distributions of various characteristics of turbulence (Mandelbrot, 1989). As we know that the solar wind is also one of the heavy non-Gaussian tails and it demonstrates a highly intermittent and multifractal process (Bruno and Carbone, 2005). For these plasma systems energy at a given scale is not evenly distributed in space. Therefore, solar wind parameters in particular magnetic field and velocity, exhibit strong intermittent behavior both during solar minimum and solar maximum (Marsch and Tu, 1994; Marsch and Tu, 1997) at different heliocentric distance, in the inner as well as the outer heliosphere (Marsch and Liu, 1993; Sorriso et. al., 1999; Bruno et. al., 2001; Hnat et. al., 2003; Szczepaniak and Macek, 2008; Burlaga, 1991; Burlaga et. al., 1993; Burlaga, 2001).

The concept of multifractality was used in the context of scaling properties of intermittent turbulence in Solar wind plasma (Marsch and Tu, 1997; Bruno et. al., 2003). Many authors propose the observed scaling exponents, using simple and advanced models of the turbulence based on distribution of the energy flux between cascading eddies on various scales. Burlaga (1991) has been investigated the multifractal spectrum of magnetic field data using Voyager in the outer heliosphere and using Helios (plasma) data in the inner heliosphere (Marsch et. al., 1996).

The aim of our study is to examine the question of multifractal scaling properties of turbulence in the solar wind velocity and IMF. We analyzed the time series of the velocities and IMF of the solar wind from January 1996 to December 2006 it is nearly the time span of Solar Cycle 23. One reason to employ the MFDFA method is to avoid spurious detection of correlations that are artifacts of non-stationarities in the time series so we have investigated the multifractality of Solar wind plasma velocity and IMF during the Solar Cycle 23 in this study.

ii) Data

The data used for Multifractality Spectrum analysis for the Solar Wind Velocity and IMF are the daily counts of Solar wind plasma parameters. These datasets has been taken from January 1996 to December 2006 it is nearly the time span of Solar Cycle 23 and available online at the OMNIWeb data server (http://omniweb.gsfc.nasa.gov). The OMNIWeb data server is a venture of NASA`s (National Aeronautics and Space Administration) Space Physics Data Facility (SPDF). This Solar data has analyzed using the Multifractal Detrended Fluctuation Analysis (MFDFA) technique in order to characterize the intrinsic scaling property of Solar wind plasma parameters. This method allows a reliable multifractal characterization of non-stationary time series of Solar wind plasma parameters and IMF.

iii) Methodology

In this study, we have analyzed the solar wind velocity and IMF data with Multifractal Detrended Fluctuation Analysis (MFDFA) method to find the multifractal spectrum of non-stationary time series. The MFDFA provides more accurate and precise result as compare to Wavelet Transform Modulus Maxima (WTMM) method especially when true fractal structure of data is unknown. It gives less biasing and being less likely to give a false positive result and due to the fact we used MFDFA in this work for the investigations of conditions and behavior of solar wind velocity and IMF during solar cycle 23.

The MFDFA is designed for a data of finite length$\mathrm{N}$, without requiring an $N \to \infty$ approximation for validity, so it is well suited for the analysis of solar wind velocity and IMF. In this method solar wind data is treated as a one-dimensional line and assigns new values to each portion of data. It deals with the data having directional dependent scaling properties and the nonequivalence of the time and value axes. The assigned values are then assessed for multifractality. The generalized form of MFDFA for solar wind data can be described as:

Let the X_j be a solar wind velocity and IMF time series of length N, with compact support, i.e. $X_j = 0$ for an insignificant fraction of the values only. The profile at location$i, Y(i)$ is defined by taking the sum of deviation from the mean value and analytically given by (Kantelhardt et. al., 2002; Telesca et. al., 2004):

$$Y(i) = \sum_{k=1}^{i} \{Xj - \bar{X}\}, \quad i = 1, \ldots, N \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (1)$$

Subtraction of the mean $\bar{X}$ is not compulsory, because it would be eliminated in the preceding step. The profile $\mathrm{Y(i)}$ is divided into $\mathrm{Ns} \equiv \mathrm{int(N/s)}$ non-overlapping segments of equal size s. Since the length N of the series may not be multiple of the considered time scale s, an unequal and short part $(< s)$ of the profile may leave at the end.

In order not to disregard this part of the series, the same procedure is repeated starting from the opposite end. Thus, $2\mathrm{N_S}$ segments are obtained altogether. Then local trend for each of the$2\mathrm{N_S}$ segments is calculated by a least-square fit of the series. Now the variance between the series $\mathrm{Y(i)}$ and the ordinate of the fitted polynomial $[\mathrm{y_v(i)}]$ is calculated as:

$$F^2(s, v) = \frac{1}{s} \sum_{i=1}^{s} \{Y[(v-1)s + i] - y_v i\}^2 \ldots \ldots \ldots \ldots \ldots (2)$$

Where indices i and v correspond to the original data points and the segment of size s respectively. The fluctuation function can be extended to include higher order moments (say q values) to analyze the scaling property of different ranges of fluctuations and also the detrending polynomial, yv can take any order n (linear, quadratic, cubic, etc.). The generalized fluctuation function $Fq(s)$ is thus defined by averaging over all segments to obtain the qth order fluctuation function as:

$$F_q(s) = \left\{ \frac{1}{2N_s} \sum_{v=1}^{2N_s} [F(s, v)]^{\frac{q}{2}} \right\}^{\frac{1}{q}} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3)$$

Where the variable q can take any real value apart from zero. In case $q = 0$, the fluctuation function cannot be determined directly from $equation\,(3)$ because of diverging exponent. Thus, $F_0(s)$ is approximated by taking the logarithmic average as:

$$F_q(s) = \left\{ exp \frac{1}{4N_s} \sum_{v=1}^{2N_s} ln[F(s, v)] \right\} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (4)$$

To develop the relation between segment length q and fluctuation functions $F_q(s)$ above procedure was repeating several times for different values of segment length. Typically $F_q(s)$ will increase with increasing s. The scaling behavior of the fluctuation functions was determined by analyzing log-log plots $F_q(s)$ versus s for each value of q. If the series X_j has long-range power-law correlated, $F_q(s)$ increases, for large values of s, as a power-law

$$F_q(s) \propto s^{h(q)} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (5)$$

The exponent $h(q)$ may depend on q and known as generalized Hurst exponent. For $q = 2, h(q)$ is identical to the well-known Hurst exponent H and provide information about the average fluctuation of the series. For the positive values of $q, h(q)$ can define the scaling performance of the segments with large fluctuations. And these large fluctuations are exemplified by a smaller scaling exponent $h(q)$ for multifractal time series. On the other hand, for negative values of $q, h(q)$ describes the scaling behavior of the segments with small fluctuations, characterized by a larger scaling exponent.

The simplest way to analyze the solar wind velocity and IMF data is to link the generalized fluctuation function and the standard box counting formalism of multifractal analysis. For original dataX_j, the mass distribution probability in the vth segment of size s unit, $P_s(v)$ is written as:

$$F_s(v) = \sum_{k=(v-1)s+1}^{vs} X_j = Y(vs) - Y(v-1) \ldots\ldots\ldots\ldots\ldots (6)$$

The mass scaling function $\tau(q)$ is then defined by partition function $\mu(q,s)$ as:

$$\mu(q,s) = \sum_{v=1}^{N/s} |P_s(s)|^q \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

The mass scaling function is related to the generalized Hurst scaling function, $h(q)$ as Yu et. al. (2011):

$$\tau(q) = qh(q) - 1 - qH' \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Where $H' = h(1) - 1$ is called the non conservation parameter and proceed to the $f(\alpha)$ spectrum by the Legendre transforms:

$$\alpha(q) = \frac{d\tau(q)}{dq} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

$$f[\alpha(q)] = \alpha q(q) - \tau(q) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6.10)$$

A plot of $f(\alpha)$ and α is the multifractal spectrum for the solar wind velocity and IMF data. Here α is the singularity strength or Holder exponent, while $f(\alpha)$ denotes the dimension of the subset of the series that is characterized by α. We can directly relate α and $f(\alpha)$ to $h(q)$ as:

$$\alpha = h(q) + qh'(q) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

$$and$$

$$f(\alpha) = q[\alpha - hq] + 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

Non-stationary is a frequent characteristic of composite variability and associated with various trends in the data patches having different local statistical properties (Kantelhardt et. al., 2001). The DFA method reduces the effect of non-stationarities on scaling property of data.

The reason for detrending analysis is to remove the undue influence of larger scale on the statistics of solar wind plasma velocity and IMF data of solar cycle 23 at the scale. The MFDFA method allows the detection of scaling property of a physical variable embedded in noisy data that can disguise true fluctuations of the time series of solar cycle 23 (Marsch et. al., 1996; Biswas et. al., 2012).

iv) Results and Discussion

In this study we have analyzed the solar wind velocity and IMF data by using MFDFA methodology. The datasets analyzed from January 1996 to December 2006 which is nearly the time span of Solar Cycle 23. Analytical results are discussed in next subsections:

- **Multifractality of Solar Wind Velocity**

Firstly we have observed the multifractal behavior of solar wind velocity during solar cycle 23. We calculated the overall Root Mean Square (F-Overall RMS) of solar wind velocity with the scale segment sample size of 1024. The monofractal time series, white noise and multifractal time series has been plotted with respect to sample size of 1024 and it is demonstrated in Figure 1. It is the singularity multifractal spectrum for the monofractal, white noise and multifractal time series for the selected segments of scale sample size. The figure demonstrating that the monofractal and white noise spectrum following the line path and there were not any presence of deviations while the multifractality spectrum is omitting several time of solar wind velocity. It is showing the non homogeneous multifractal behavior during solar cycle 23.

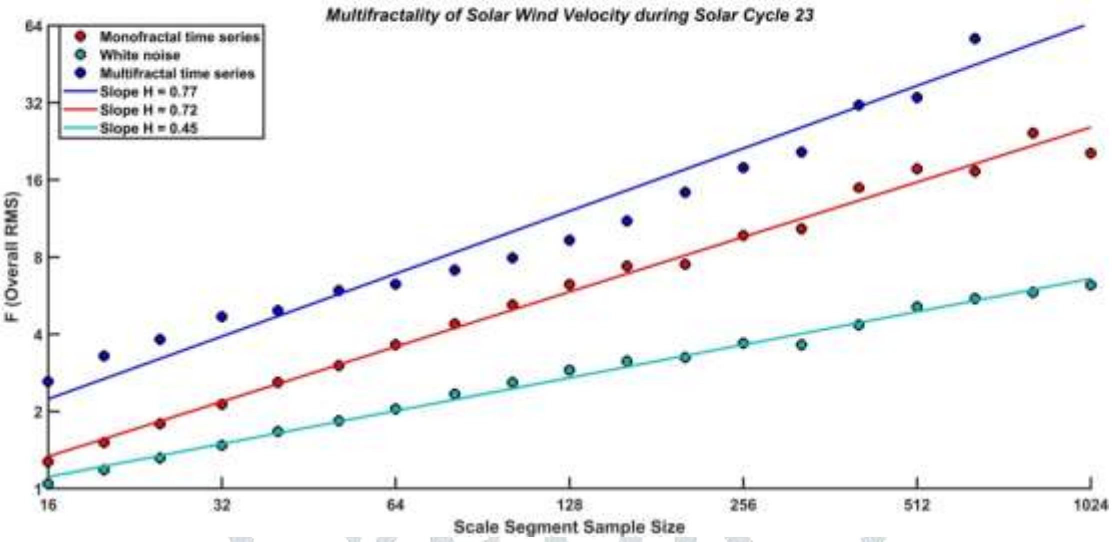

Fig. 1 – The Multifractal Overall RMS $(F)$of Solar Wind Velocity during the Solar Cycle 23.

➢ q-order Hurst Exponent of Solar Wind Velocity

The next two figures are also showing the strong agreement of similar behavior. The Figure 2 displaying the q-order Hurst exponent (Hq) of solar wind velocity during solar cycle 23.

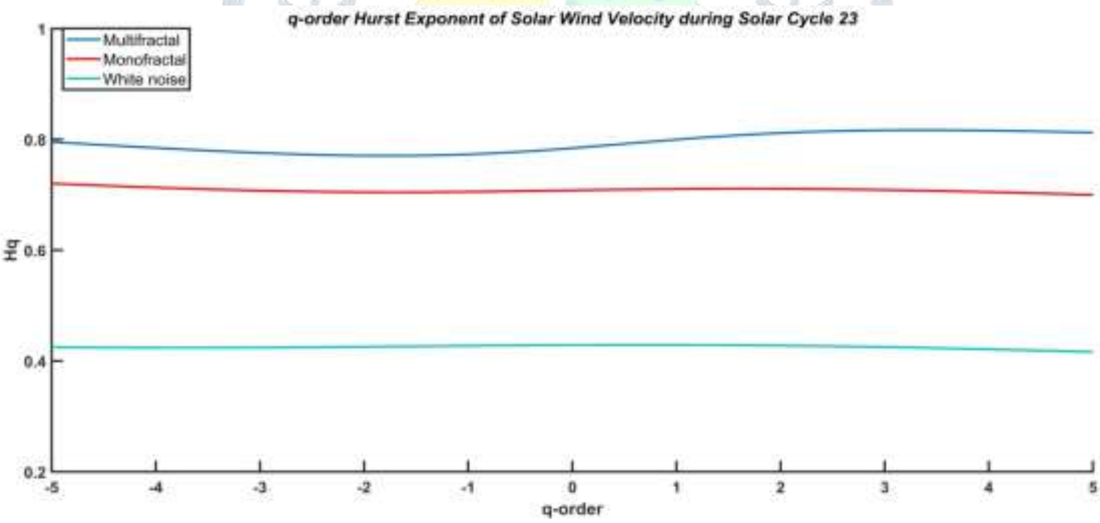

Fig. 2 – The q-order Hurst Exponent (Hq) of Solar Wind Velocity during the Solar Cycle 23.

The Figure 2 is displaying the multifractal, monofractal and white noise hurst exponent behavior with respect to q-order. The value of q-order has been taken from -5 to 5 for the analysis of hurst exponent behavior of solar wind velocity during solar cycle 23. It was noticed from Figure 2 that the monofractal and white noise time series characterized by a single exponent over all time scales, Hq is independent of q. For multifractal time series, Hq shows variations with respect to q. The different scaling of small and large fluctuation will yield a significant dependence of Hq on q. Therefore, for positive value of q, Hq describes the scaling behavior of the segments with large fluctuations while for negative value of q, Hq shows small fluctuations.

➢ q-order Mass Exponent of Solar Wind Velocity

Similar behavior has been recorded during the calculation of q-order mass exponent (tq) of solar wind velocity and it is shown in Figure 3.

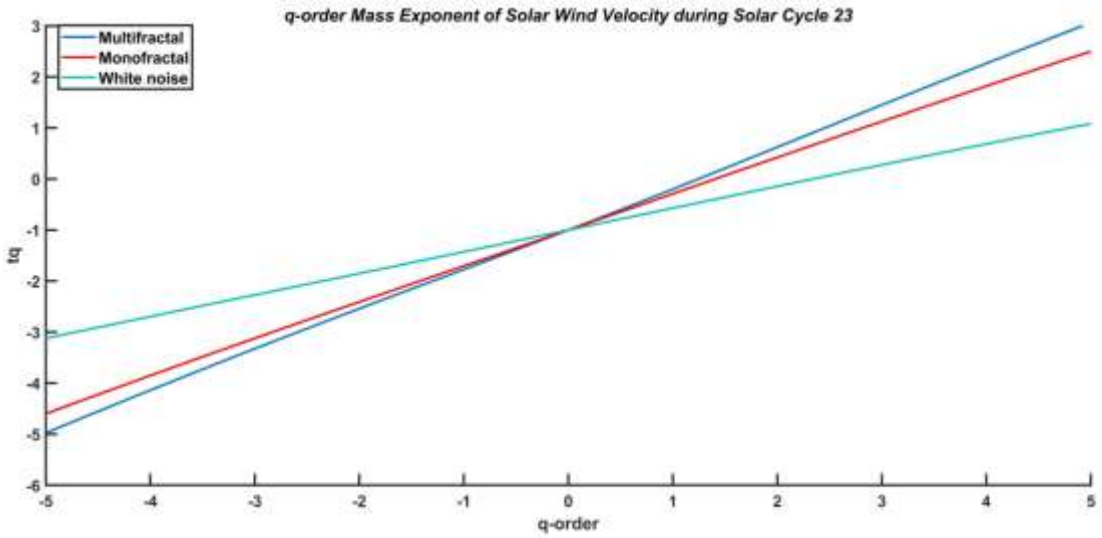

Fig. 3 – The q-order Mass Exponent (tq) of Solar Wind Velocity during the Solar Cycle 23.

The multifractal, monofractal and white noise mass exponent (tq) behavior with respect to q-orderis shown in Figure 3. The value of q-order has been taken from -5 to 5 for the analysis of mass exponent behavior of solar wind velocity during solar cycle 23. It was noticed from figure that the monofractal and white noise time series characterized by a single exponent over all time scales, tq is independent of q. For multifractal time series, tq shows variations with respect to q. The tq value of multifractality starts from -5 for the q-value of -5 while it touches the value of highest 3 for the q-value of 5, which is quit higher as compared to the values of monofractal and white noise. Therefore the solar wind velocity is showing the non homogeneous multifractal behavior during solar cycle 23.

➢ Multifractality of IMF

In this section we have analyzed the multifractal behavior of IMF during the solar cycle 23. We calculated the overall Root Mean Square (F-Overall RMS) of IMF with the scale segment sample size of 512. Along with multifractal time series the monofractal time series and white noise has also been plotted with respect to sample size for the detailed analysis of variable time series of IMF. The final plot of all three time series is demonstrated in Figure 4.

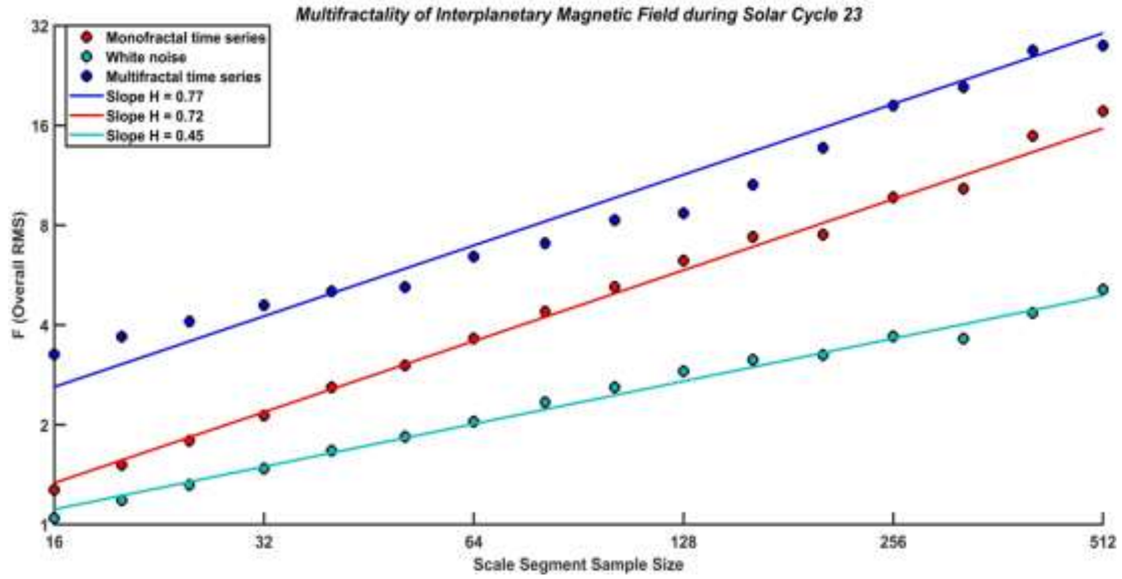

Fig. 4 – The Multifractality or Overall RMS (F) of IMF during the Solar Cycle 23.

The Figure 4 demonstrates the multifractal overall RMS (F) profile of IMF during the solar cycle 23. It is the singularity spectrum for the monofractal, white noise and multifractal time series for the selected segments of scale sample size of 512. The figure demonstrating that the monofractal and white noise spectrum following the line path and there were not any presence of deviations while the multifractality spectrum is omitting or missing the line path several times of IMF during the time lapse of solar cycle 23. This multifractality of IMF is showing the non homogeneous multifractal behavior several times during the minima of solar cycle 23 and maxima of solar cycle 23.

- **q-order Hurst Exponent of IMF**

The next two figures are also showing the strong agreement of similar behavior. The Figure 5 displaying the q-order Hurst exponent (Hq) of IMF during solar cycle 23. It is displaying the multifractal, monofractal and white noise hurst exponent behavior with respect to q-order. The value of q-order has been taken from -5 to 5 for the analysis of hurst exponent behavior of IMF during solar cycle 23. It was noticed from figure that the monofractal and white noise time series characterized by a single exponent over all time scales, Hq is independent of q. For multifractal time series, Hq shows variations with respect to q. The different scaling of small and large fluctuation will yield a significant dependence of Hq on q. Therefore, for positive value of q, Hq describes the scaling behavior of the segments with large fluctuations while for negative value of q, Hq shows small fluctuations and similar to monofractal time series.

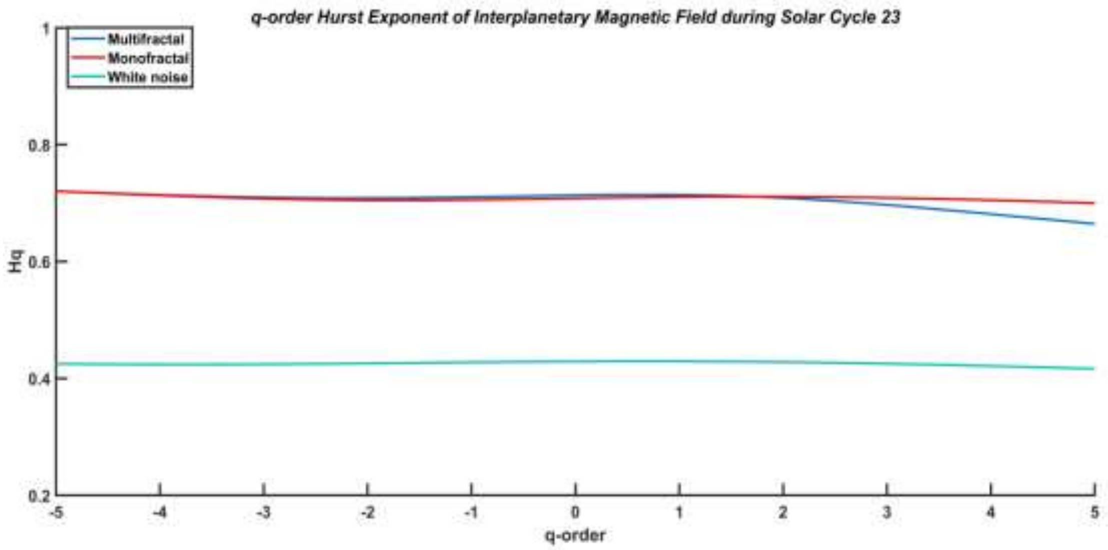

Fig. 5 – The q-order Hurst Exponent (Hq) of IMF during the Solar Cycle 23.

➢ q-order Mass Exponent of IMF

Similar behavior of IMF has been recorded during the calculation of q-order mass exponent (tq) and it is shown in Figure 6. The multifractal, monofractal and white noise mass exponent (tq) behavior with respect to q-order is shown in figure. The value of q-order has been taken from -5 to 5 for the analysis of mass exponent behavior of IMF during solar cycle 23.

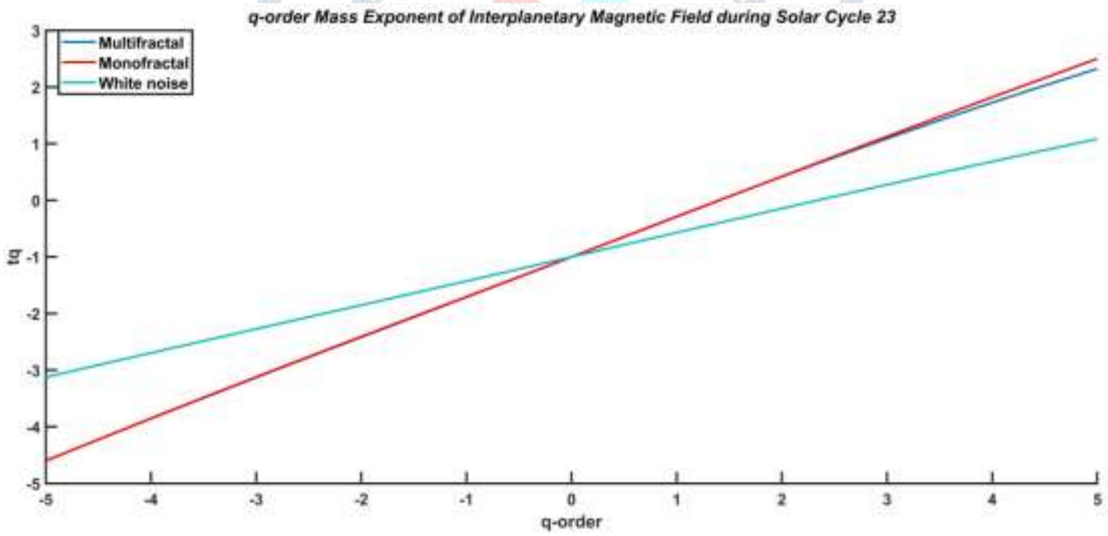

Fig. 6 – The q-order Mass Exponent (tq) of IMF during the Solar Cycle 23.

It was noticed from Figure 6 that the monofractal and white noise time series characterized by a single exponent over all time scales and tq is independent for the value of q. For multifractal time series, tq shows variations with respect to q. The tq value of multifractality starts from -4.5 for the q-value of -5 while it touches the highest value at the q-value of 5, which is quite similar to the various values of monofractal time series. Therefore the solar wind velocity is showing the non homogeneous multifractal behavior during solar cycle 23.

v) Conclusions

In this study, we attempted to demonstrate the multifractal properties of solar wind velocity and IMF plasma parameters using multifractal spectrum analysis. This multifractal property of solar wind velocity and IMF parameters is a key significance in studying solar wind plasma turbulence for a symmetric scaling. These fluctuations in Solar wind velocity and IMF are not quite of a random nature but result shows its non-linear dynamic behavior. It is worth noting that idea of multifractality is applied also throughout a broad range of scientific studies from interstellar medium (Falgarone and Puget, 1995; Cho et. al. 2003; Kritsuk and Norman 2004; Balsara & Kim 2005) to biological and social systems (Schroeder 2000).

We have studied the inhomogeneous rate of the transfer of the energy flux indicating multifractal behavior of solar wind turbulence in solar plasma. In particular, we have identified somewhat smaller degree of multifractality during solar cycle 23 for the disturbed solar wind and IMF; where turbulence evolves indicate more perturbation. By investigating OMNI data for solar wind fluctuation we have shown that the degree of multifractality and asymmetry of the perturbed solar wind velocity and IMF exhibit geomagnetic storm dependence with some symmetry with respect to the ecliptic plane. Both quantities seem to be correlated during solar cycle. The multifractal singularity spectra become roughly symmetric.

The fluctuation in multifractality of solar wind velocity and IMF is observed various times during solar cycle 23. The multifractal overall RMS (F) of solar wind velocity and IMF is a singularity multifractal spectrum for the monofractal, white noise and multifractal time series for the selected segments of scale sample size. It was recorded that the monofractal and white noise spectrum following the line path and there were not any presence of deviations while the multifractality spectrum is omitting several time of solar wind velocity and IMF. The q-order hurst exponent (Hq) and q-order mass exponent (tq) are also displaying these similar variations in multifractality of solar wind velocity and IMF for the q-value ranging from -5 to 5. The q-order hurst exponent (Hq) shows more variability in multifractality spectrum as compared to monofractal and white noise. Similarly the multifractality of q-order mass exponent (tq) is showing more highest and lowest variability as compared to monofractal and white noise. Our results provide direct supporting evidence the Solar wind plasma parameter is likely to have multifractal structure.

This variability in multifractality spectrum are possibly related to the transition from the region where the interaction of the fast and slow streams takes place to a more homogeneous region of the pure fast solar wind. Basically, the multifractal spectra for solar wind velocity and IMF are consistent with the generalized for both positive and negative, but rather with different scaling parameters for sizes of eddies in the ecliptic while in most cases it is sufficient to reproduce the spectrum of the fast solar wind in the solar plasma. Hence this model appears to be good tool for describing of solar wind turbulence and allowing for a unifying description of its multifractal characteristics. Therefore, we may propose this cascade model describing intermittent energy transfer for analysis of turbulence in various environments (Krasnoselskikh et. al., 2013).

vi) Acknowledgements

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vii) References

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