Study of Space Plasma with Multifractality and Intermittency Scaling during Solar Cycle 23

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Abstract: This study deals with the multifractality and intermittency scaling of plasma parameters (Proton Density and Proton Temperature) for January 1996 to December 2006 which is nearly the time span of Solar Cycle 23. The datasets of these parameters have been taken from OMNI Web server and the level of multifractality and intermittency is analyzed for solar wind plasma parameters. Our results show the inconsistent rate of the transfer of the energy flux which are clear indications of multifractal and intermittence behavior of solar wind plasma parameter turbulence in the inner heliosphere and describes the energy transfer is responsible for turbulence in various environments and their features having similar behavior like nonlinear multifractal systems. This result also shows that the degree of asymmetry for these scaled parameters have much better agreement with the real data for solar cycle 23 and data sampling shows weakly correlation with the phase of the solar activity.

***Index Terms* – Solar Wind Plasma Parameters, Solar Cycle, Multifractality Spectrum.**

i). Introduction

The analysis of solar wind plasma parameters are an example to study turbulence in various environments, including space and astrophysical plasmas (Bruno and Carbone, 2005). These are the charged particles released from the upper atmosphere of the sun known as corona. Normally these winds have two types slow solar wind most likely originates from equatorial regions of the solar corona while the fast wind is associated with coronal holes. The fast wind is relatively uniform and stable in nature while the slow wind is more turbulent and consequently quite variable in terms of velocities (Cho et. al., 2003). The Solar wind is a highly nonlinear system the energy at a given scale is not evenly distributed in space and shows as an example of turbulent and intermittent astrophysical plasma and we can observe how fluctuating parameters affected by intermittency alternate between burst of activity and quiescence (Burlaga, 1991; Marsch and Liu, 1994; Marsch and Tu, 1997; Sorriso et. al., 1999; Bruno et. al., 2003). Several classes of models have been developed to describe non uniform distribution of energy in the turbulent flow and due this nature of solar plasma multifractal model gives us to allow to look inside complex nature of intermittent turbulence (Mandelbrot, 1989).

These strong turbulences could be very useful in the mass transport into the cusp and may cause acceleration and heating of plasma (Savin et. al., 2002) have found that fluctuations at different frequencies suggesting multistage intermittent processes. Since the state of fully developed turbulence is reigning in the heliosphere, various methods of the turbulence theory are frequently used to study intermittency (Burlaga & Ness 1998; Bruno and Carbone 2005; Balsara and Kim, 2005). Along with this, solar wind is extensively studied with various traditional fractal techniques, such as analysis of fractal geometry (Falgarone and Puget, 1995). The multifractal nature of solar wind has been observed in the inner and outer heliosphere and depends on the various phases of the solar cycle and the nature of this departure is still unexplained (Marsch et al., 1996; Burlaga, 2001). A direct determination of the multifractal spectrum from the data is known to be a difficult problem but luckily, it appears that a certain kind of order does lie hidden away within the irregular solar wind fluctuations (Carbone, 1993; Marsch et. al., 1996; Macek, 2007). By using generalized dimensions and singularity spectra we may describe better for energy turbulence cascade and the degree of multifractality in the solar wind plasma (Meneveau and Sreenivasan, 1991) as we know that a fractals are generally self-similar and independent of scale with a particular fractal dimension which is a rough or disjointed geometrical point that can be subdivided in a reduced-size copy parts of the whole and at the same multifractal is an object that demonstrate various self-similarities which is point dependent resulting in the singularity spectrum therefore in a certain sense multifractal is a set of intertwined fractals.

Multifractality is generally related to a probability measurement that has different fractal dimensions. So the study of multifractality is of great importance for space plasmas because it allows us to look at intermittent turbulence in the solar wind (Burlaga, 1991; Carbone, 1993; Marsch et al., 1996; Macek, 1998; Bruno et al., 2003). Many authors still attempt to recover the observed scaling exponents, using some simple and more advanced fractal and multifractal models of turbulence describing distribution of the energy flux between cascading eddies at various scales. In particular, the multifractal spectrum has been investigated using Voyager (Magnetic Field Fluctuations) data in the outer helio-sphere (Burlaga, 1991; Burlaga, 2001) and using Helios (plasma) data in inner helio-sphere (Marsch et. al., 1996). In this work, we are studying the multifractality and intermittency of the data provided by OMNI web server and understanding in detail the properties of intermittent turbulence using solar plasma parameters like proton density and proton temperature of the streams of the solar wind and found that the level of Multifractality and the level of intermittency are correlated for the various solar plasma parameters.

ii). Data

In this study the space plasma parameters have been analyzed from January 1996 to December 2006 which is nearly the time span of Solar Cycle 23. These datasets are the daily counts of Proton Density and Proton Temperature. For the multifractality and intermittency analysis of these parameters, the data has been taken from online, available at OMNIWeb data server (http://omniweb.gsfc.nasa.gov). The OMNIWeb data server is a venture of NASA`s (National Aeronautics and Space Administration) Space Physics Data Facility (SPDF). It leads in the design and implementation of multi-mission and multi-disciplinary data services.

iii). Methodology

The datasets of solar wind plasma parameters (Proton Density and Proton Temperature) have been analyzed from Multifractal Detrended Fluctuation Analysis (MFDFA) method to find the multifractal spectrum and intermittency of non-stationary time series. The MFDFA provides more accurate and precise result as compare to Wavelet Transform Modulus Maxima (WTMM) method especially when true fractal structure of data is unknown. It gives less biasing and being less likely to give a false positive result and due to this fact we used MFDFA in this chapter for the investigations of conditions and behavior of solar wind plasma parameters data during solar cycle 23. The MFDFA is designed for a data of finite length N, without requiring an $N \to \infty$ approximation for validity, so it is well suited for the analysis of solar wind plasma parameters. In this method solar wind data is treated as a one-dimensional line and assigns new values to each portion of data. It deals with the data having directional dependent scaling properties and the nonequivalence of the time and value axes. The assigned values are then assessed for multifractality and intermittency. The generalized form of MFDFA for solar wind plasma parameters data can be described as:

Let the X_j be a solar wind plasma parameter time series of length N, with compact support, i.e. $X_j = 0$ for an insignificant fraction of the values only. The profile at location $i, Y(i)$ is defined by taking the sum of deviation from the mean value and analytically given by (Kantelhardt et. al., 2002; Telesca et. al., 2004):

$$Y(i) = \sum_{k=1}^{i} \{Xj - \bar{X}\}, \quad i = 1, \ldots, N \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (1)$$

Subtraction of the mean $\bar{X}$ is not compulsory, because it would be eliminated in the preceding step. The profile $\mathrm{Y(i)}$ is divided into $\mathrm{Ns} \equiv \mathrm{int(N/s)}$ non-overlapping segments of equal size s. Since the length N of the series may not be multiple of the considered time scale s, an unequal and short part $(< s)$ of the profile may left at the end.

In order not to disregard this part of the series, the same procedure is repeated starting from the opposite end. Thus, $2\mathrm{N_S}$ segments are obtained altogether. Then local trend for each of the $2\mathrm{N_S}$ segments is calculated by a least-square fit of the series. Now the variance between the series $\mathrm{Y(i)}$ and the ordinate of the fitted polynomial $[\mathrm{y_v(i)}]$ is calculated as:

$$F^2(s, v) = \frac{1}{s} \sum_{i=1}^{s} \{Y\,[(v-1)s + i] - y_v i\}^2 \ldots \ldots \ldots \ldots \ldots (2)$$

Where indices i and v correspond to the original data points and the segment of size s respectively. The fluctuation function can be extended to include higher order moments (say q values) to analyze the scaling property of different ranges of fluctuations and also the detrending polynomial, yv can take any order n (linear, quadratic, cubic, etc.). The generalized fluctuation function $Fq(s)$ is thus defined by averaging over all segments to obtain the qth order fluctuation function as:

$$F_q(s) = \left\{ \frac{1}{2N_s} \sum_{v=1}^{2N_s} [F(s, v)]^{\frac{q}{2}} \right\}^{\frac{1}{q}} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3)$$

Where the variable q can take any real value apart from zero. In case $q = 0$, the fluctuation function cannot be determined directly from $equation\,(3)$ because of diverging exponent. Thus, $F_0(s)$ is approximated by taking the logarithmic average as:

$$F_q(s) = \left\{ exp \frac{1}{4N_s} \sum_{v=1}^{2N_s} ln[F(s, v)] \right\} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (4)$$

To develop the relation between segment length q and fluctuation functions $F_q(s)$ above procedure was repeating several times for different values of segment length. Typically $F_q(s)$ will increase with increasing s. The scaling behavior of the fluctuation functions was determined by analyzing log-log plots $F_q(s)$ versus s for each value of q. Non-stationary is a frequent characteristic of composite variability and associated with various trends in the data patches having different local statistical properties (Kantelhardt et. al., 2001). The reason for detrending analysis is to remove the undue influence of larger scale on the statistics of solar wind plasma parameters data at the scale. The MFDFA method allows the detection of scaling property of a physical variable embedded in noisy data that can disguise true fluctuations of the series (Biswas et. al., 2012).

iv). Results and Discussion

In the present work, multifractality and intermittency of solar wind plasma parameters (Proton Density and Proton Temperature) have been analyzed from January 1996 to December 2006 for the time span of Solar Cycle 23. Analytical results of these plasma parameters are discussed in next subsections:

➢ Multifractality of Proton Density

Firstly we have analyzed the multifractal behavior of solar wind plasma parameter of proton density during the solar cycle 23. We calculated the overall Root Mean Square (F-Overall RMS) of proton density with the scale segment sample size of 512. Along with multifractal time series the monofractal time series and white noise has also been plotted with respect to sample size for the detailed analysis of variable time series of solar wind plasma parameter. The final plot of all three time series is demonstrated in Figure 1 and it is the singularity spectrum for the monofractal, white noise and multifractal time series for the selected segments of scale sample size of 512. The figure demonstrating that the monofractal and white noise spectrum following the line path and there were not any presence of deviations while the multifractality spectrum is omitting or missing the line path several times of proton density. This multifractality of proton density is showing the non homogeneous multifractal behavior during solar cycle 23.

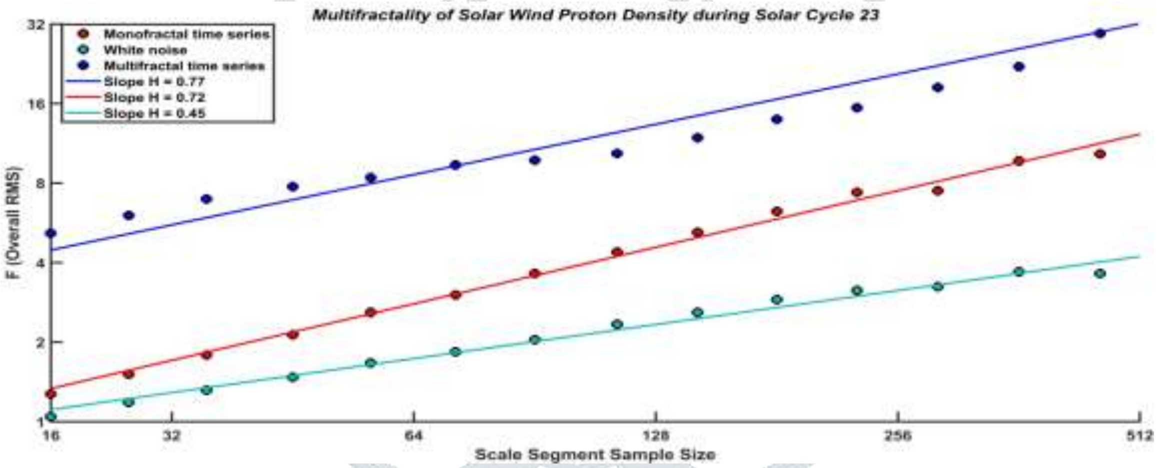

Fig. 1 – The Multifractality or Overall RMS (F) of Proton Density during the Solar Cycle 23.

➢ Intermittency of Proton Density

For the understanding of more comprehensive behavior of this multifractal spectrum we have plotted the intermittency or q-order exponent (Fq) for multifractal time series of proton density and shown in Figure 2. The slope gives the generalized exponent of multifractal time series of proton density with the scale segment sample size of 512 and the value of q is ranging from $-3\ to\ 3$. It is a clear visual that the slope is missing for every value of q i.e. for $q = -3, q = -1, q = 1\ and\ q = 3$. For each and individual value of q there were not any match between dot values and slopes and it is indicating that the multifractal behavior of proton density was highly disturbed and showing the non homogeneous multifractal behavior during solar cycle 23.

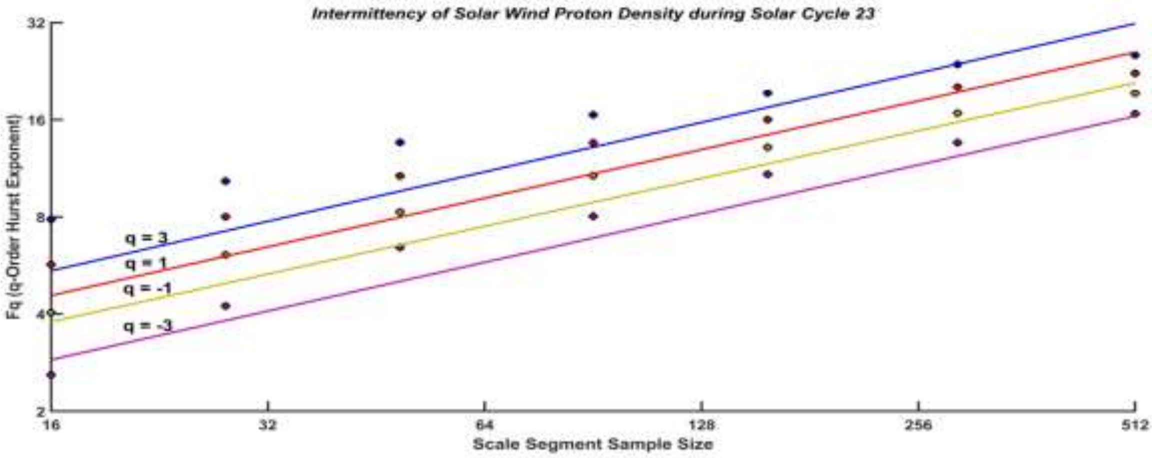

Fig. 2 – The Intermittency or q-order Exponent (Fq) of Proton Density during the Solar Cycle 23.

➢ Multifractality of Proton Temperature

Similarly in this section firstly we have analyzed the multifractal behavior of solar wind plasma parameter of proton temperature during the solar cycle 23. We calculated the overall Root Mean Square (F-Overall RMS) of proton temperature with the scale segment sample size of 512. Along with multifractal time series the monofractal time series and white noise has also been plotted with respect to sample size for the detailed analysis of variable time series of solar wind plasma parameter. The final plot of all three time series is demonstrated in Figure 3. It is the singularity spectrum for the monofractal, white noise and multifractal time series for the selected segments of scale sample size of 512. The figure demonstrating that the monofractal and white noise spectrum following the line path and there were not any presence of deviations while the multifractality spectrum is omitting or missing the line path several times of proton temperature. This multifractality behavior of proton temperature is showing the non homogeneous multifractal behavior during solar cycle 23.

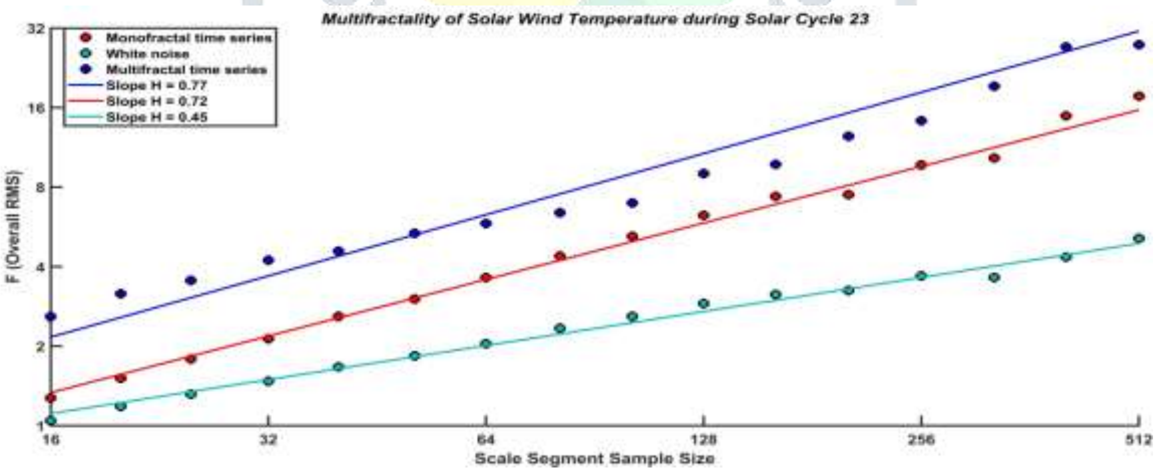

Fig. 3 – The Multifractality or Overall RMS (F) of Proton Temperature during the Solar Cycle 23.

➢ Intermittency of Proton Temperature

For the understanding of more comprehensive behavior of this multifractal spectrum we have plotted the intermittency or q-order exponent (Fq) for multifractal time series of proton temperature and shown in Figure 4. The slope gives the generalized exponent of multifractal time series of proton temperature with the scale segment sample size of 1024 and the value of q is ranging from $-3\ to\ 3$. It is a clear visual that the slope is missing for every value of q i.e. for $q = -3, q = -1, q = 1\ and\ q = 3$. For each and individual value of q there were not any match between dot values and slopes and it is indicating that the multifractal behavior of proton temperature was highly disturbed and showing the non homogeneous multifractal behavior during solar cycle 23.

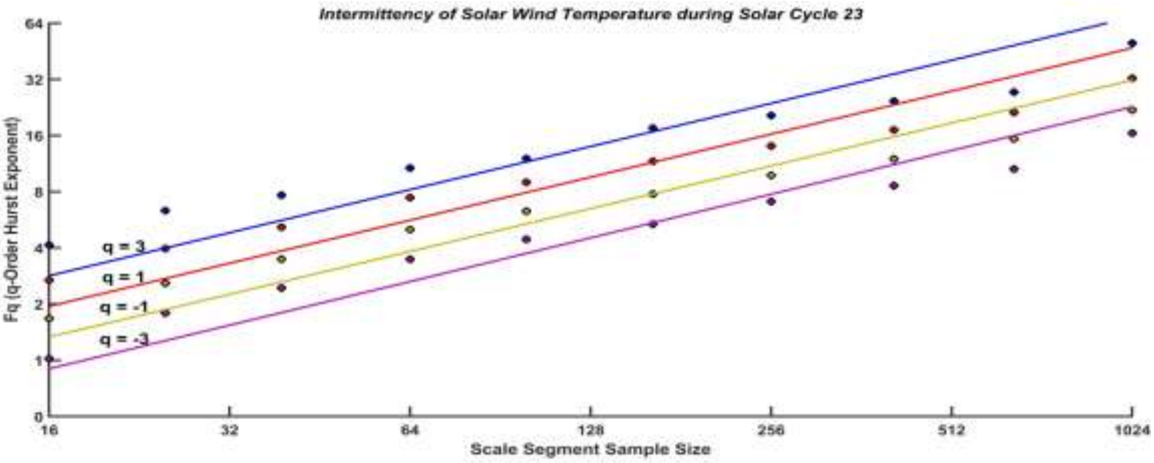

Fig. 4 – The Intermittency or q-order Exponent (Fq) of Proton Temperature during the Solar Cycle 23.

v). Conclusions

The solar wind is now observed mostly near Earth, it is an essential part of the Sun's evolution. The solar wind mainly consists of two components slow and fast solar wind which generated from areas of closed magnetic fields and opens magnetic fields in the Sun respectively and called coronal holes. The location of coronal holes varies with the solar cycle. During a solar minimum, coronal holes are concentrated at the poles, whereas during the solar maximum, huge coronal holes appear at low latitudes, near the solar equator and due to which Earth becomes a direct target for fast solar wind streams emanated from low-latitude coronal holes (Pagel and Balogh 2003). As to the usefulness of multifractality and intermittency notion for various physical aspects of understanding of solar phenomena, it is worth to mention the long standing problem of appearance of low plasma conductivity in the corona especially during a flare. As the solar activity increases the slow solar wind becomes somewhat more multifractal and the fast wind is slightly less multifractal. On other hand it seems that the degree of asymmetry of dimension spectrum for the slow wind is rather anti correlated with phase of solar activity (Szczepaniak and Macek 2008).

We have investigated the fluctuations of various solar wind plasma parameters like Proton Density and Proton Temperature as measured in the turbulent boundary layers. The multifractality and intermittency method has been used to estimate the scaling behavior of the partition function. We have found that their features are similar to those founds in the nonlinear multifractal systems (Schroeder, 2000). If the corresponding intermittency parameter is high, Taylor and Cargill (2002) suggested that plasma flows when the magneto-sheath interacts with the magnetopause indentation at cusp under northward IMF conditions. They have shown that when the plasma velocity is in excess of the fast mode magneto-sonic wave speed, a highly turbulent and boundary layer forms which enters the cusp indentation. At times, the IMF is stable with large positive By and negative Bz, which suggests that Polar spacecraft senses plasma on open field lines flowing toward the magnetotail. For such configuration of IMF the reconnection in the vicinity of the sub-solar point affects the cusp structure. In our results the scaling behavior of the multifractality and intermittency of solar plasma parameters reveal that the magnetic field has a multifractal structure. This leads to the conclusion that the turbulence is dominated by flow eddies (Takayasu, 1989).

Our results shows the inhomogeneous rate of the transfer of the energy flux indicating multifractal and intermittent behavior of solar wind plasma parameter turbulence in the inner heliosphere. For this we have demonstrated in plot the three scaling parameters a much better agreement with the real data is obtained. By investigating the OMNI web data we have shown that as the solar activity increases the solar wind becomes somewhat more multifractal and more asymmetric. Which shows that the degree of asymmetry of the multifractality and intermittency for solar cycle 23, samples is rather weakly correlated with the phase of the solar activity. It is describing intermittent energy transfer for analysis of turbulence in various environments (Yordanova et. al., 2009). We analyzed that the multifractal and intermittency spectrum of the solar wind plasma parameters attractor is consistent with that for the multifractal measure of the generalized two scale plot. Thus these results show multifractal structure of the solar wind in the inner heliosphere. Hence there exists an inertial manifold for the solar wind, in which the system has multifractal structure and where noise is certainly not dominant. The multifractal structure convicted by the wind, might probably be related to the complex topology shown by the magnetic field at the source regions of the solar wind. So the ideas of multifractality and intermittency are share deep common roots, which is caused by natural evolution of non-linear dynamical dissipative systems. Our Sun is one of such systems, and solar physics can gain much by utilizing these concepts, along with a rich set of developed tools, in further understanding of our closest star.

vi). Acknowledgements

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vii). References

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