A NEW APPROACHE OF CRISP SETS AND FUZZY SETS BY USING BASIC OPERATIONS

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Abstract : Fuzzy sets have superior in a variety of and in much discipline. In fuzzy set theory, classical bivalent sets are regularly called crisp sets. Fuzzy set theory can be used in a spaciousz range of domain. The intend of this paper is to at hand; articulated the similarity of crisp sets and fuzzy sets. It can carry out operations like union, intersection, compliment and differences. The properties exhibited in the crisp set include commutativity, distritributivity, identity, transitivity and involution. Though, fuzzy sets also have the same properties.

I. INTRODUCTION

Fuzzy set theory was primarily proposed by Lotfi A.Zadeh in the year 1965. After that lot of theoretical growth has been done in a related field. A crisp relation represents the happening of association, interaction or interconnectedness between the elements of two or more sets. This concept can be general to allow for varies degree or strengths of association or interaction between elements. The paper is prepared as follows. We introduce basic concepts of Crisp sets related with fuzzy Sets. Degrees of association can be represented by membership grades in a normal set family member in the same way as degrees of set membership are stand for in the fuzzy set. This exertion is done with the aim of understanding the basic of fuzzy set and crisp sets. Fuzzy set and crisp set are the part of the separate set theories, where the fuzzy set equipment infinite-valued logic while crisp set employs bi-valued logic. Expert system principles were formulated premised on Boolean logic where crisp sets are used. But then scientists argued that human thinking does not always follow crisp "yes"/"no" logic, and it could be vague, qualitative, uncertain, imprecise or fuzzy in nature. This gave origination to the development of the fuzzy set theory to emulate human thinking. For an element in a universe, that consist of fuzzy sets can have a progressive transition among several degrees of membership. While in crisp sets the transition for an element in the universe between membership and non membership in a given set is rapid and well defined.

II METHOD OF ANALYSIS:

1.1 CLASSICAL SETS AND FUZZY SET

A classical (crisp) set is normally defined as a collection of elements or objects $x \in X$ that can be finite, countable, or over countable. Each single element can either belong to or not belong to a set A, $A \subseteq X$. In the former Case, the statement "x belongs to A" is true, whereas in the latter case this statement is false.Such a classical set can be described in different ways; one can either enumerate (list) the elements that belong to the set; describe the set analytically, for instance, by stating conditions for membership ($A = \{x/x \leq 5\}$); or define the member elements by using the characteristic function, in which 1 indicates membership and 0 nonmember ship. For a fuzzy set, the characteristic function allows various degrees of membership for the elements of a given set.A classical set is defined by crisp boundaries; i.e. there is no uncertainty in the prescription or location of the boundaries of the set as show in fig 1.1a where the boundary of crisp set A is an unambiguous line.

A fuzzy set, on the other hand, is prescribed by vague or ambiguous properties; hence its boundaries are ambiguously specified, as shown by the fuzzy boundary for set Ã in Fig 1.1b.

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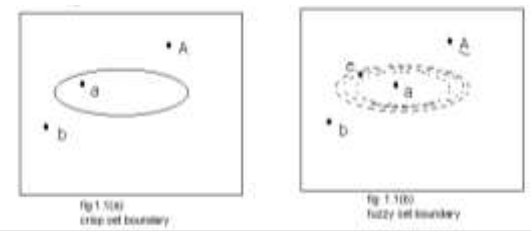
(Universe of Discourse)



Basis for comparison	Fuzzy Set	Crisp Set
Basic	Prescribed by vague or ambiguous properties.	Defined by precise and certain characteristics.
Property	Elements are allowed to be partially included in the set.	Element is either the member of a set or not
Applications	Used in fuzzy controllers	Digital design
Logic	Infinite-valued	bi-valued

1.1.1 BASIC CONCEPTS

In fuzzy set theory, normal or classical sets are called crisp set in order to distinguish them from fuzzy sets.

Definition 1.1

Universe of discourse X, as a set of objects all having the same uniqueness. The individual elements in the universe X will be denoted as X. Collections of elements within a universe are called sets and collections of elements within sets are called subsets. The total number of elements in a universe X is called its **cardinal number**, denoted by n_{x} , where $x \epsilon X$.

Definition 1.2

The null set, Φ as the set containing no elements, and the whole set, X, as the set of all elements in the universe. The null set is analogous to an impossible event, and the whole set is analogous to a certain event. All possible sets of X constitute a special set called the power set P(x).$P(X) = \{A/A \subseteq X\}$

Example 1.1

We have a universe comprised of three elements, X= {a,b,c}, so the cardinal number is $n_x=3$.

The power set is P(X) = { Φ , {a},{b},{c},{a,b},{a,c},{b,c},{a,b,c}}

Cardinality of the power set, denoted $n_{p(x)}$, is establish as $n_{p(x)} = 2^{n\,x} = 2^3 = 8$

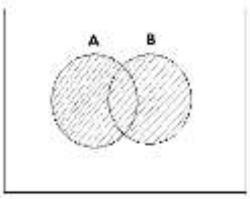
1.1.2 Operations on classical sets

Let A and B be two sets on the universe X. Then the follows operations are defined as follows:

UNION:

The union between the two sets, denoted by A∪B, represents all those elements in the universe that reside in (or belong to) either the set A, the set B, or both sets A and B. The union is the exclusive or operation. $A \cup B = \{ x / x \in A \text{ or } x \in B \}$

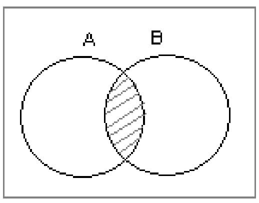
Figure:



A∪B

INTERSECTION: The intersection of the two sets , denoted A∩ B , represents all those elements in the universe X that simultaneously reside in (or belong to) both sets A and B.$A \cap B = \{ x / x \in A \text{ or } x \in B \}$

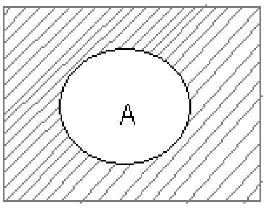
Figure



A∩B

COMPLEMENT: The complement of a set A, denoted Ā , is defined as the collection of all elements in the universe that do not reside in the set A. $\bar{A} = \{x / x \notin A, x \epsilon X\}$

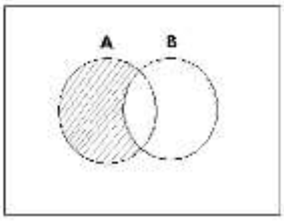
Figure



Ā

DIFFERENCE: The difference of a set A with respect to B, denoted A/B, is defined as the collection of all elements in the universe that reside in A and that do not reside in B simultaneously. $A/B = \{x / x \epsilon A \text{ and } x \notin B\}$

Figure



A/B

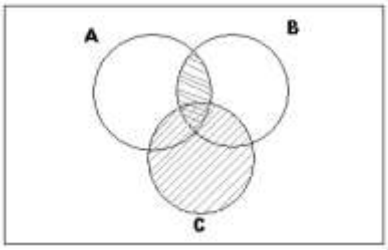
1.1.2 Fundamental properties of classical set operations:

Certain properties of sets are important because at their influence on the manipulation of sets.

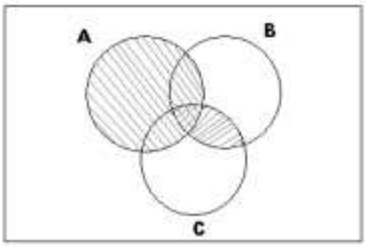
Involution	$\bar{\bar{A}}$=A
Commutativity	A∪B=B∪A, A∩B=B∩A
Associativity	A∪(B∪C)=(A∪B)∪C, A∩(B∩C)=(A∩B)∩C
Distributivity	A∪(B∩C)=(A∪B)∩(A∪C), A∩(B∪C)=(A∩B)∪(A∩C)
Idempotency	A∪A = A,A∩A=A
Absorption	A∪(A∩ B)=A,A∩(A∪B)=A
Absorption by X and Φ	A∪X=X,A $\cap \Phi = \Phi$
Identity	A∪ Φ=A,A∩X=A
Transitivity	IF A⊆B⊆C then A⊆C

Figures:

Venn diagrams for (a) (A∩B)∩C AND (b) A∩$(B \cap C)$

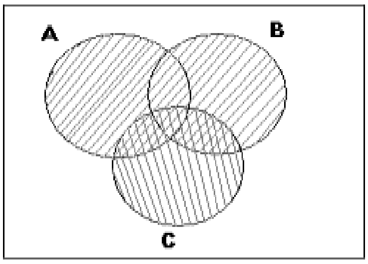


(a)

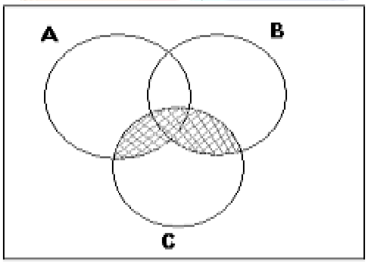


(b)

Venn diagrams for (a) (A∪B)∩C AND (b) (A∩ C)∪$(B \cap C)$

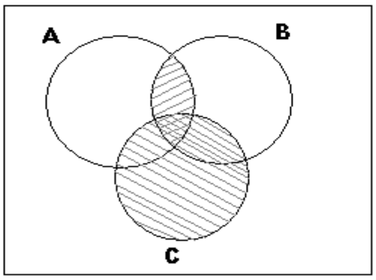


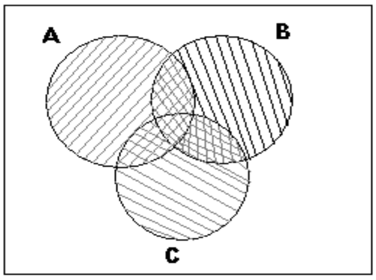
(a)



(b)

Venn diagrams for (a) (A∩B)∪C AND (b) (A∪C) ∩ (BUC)





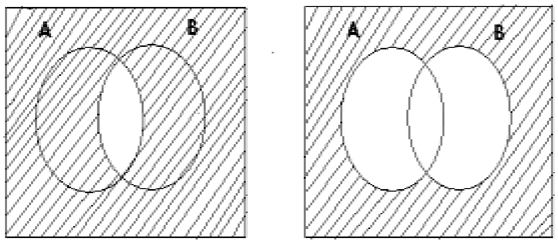
(a) (b)

Two special properties of set operations are known as the excluded middle lows and De Morgan's laws.

Excluded Middle Laws:

Law of the excluded middle: A∪Ā=X , Law of contradiction: A∩Ā= Φ

De Morgan Laws: $\overline{A_1 \cup A_2} = \bar{A}_1 \cap \bar{A}_2$, $\overline{A_1 \cap A_2} = \bar{A}_1 \cup \bar{A}_2$



$\overline{\mathbf{A \cap B}}$ $\overline{\mathbf{A \cup B}}$

1.1.3 Mapping of classical sets to functions

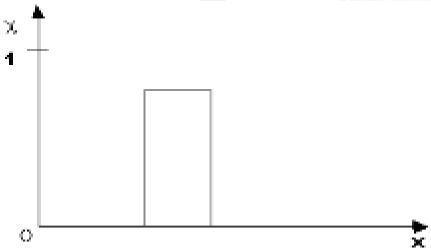
Mapping is an important concept in relating set – theoretic forms to function – theoretic representations of information. Suppose X and Y are two difference universes of discourse (information) . If an elements x is contained in X and corresponds to an element y contained in Y. It is generally termed a mapping from X to Y, or f: X → Y

Definition: 1.3 characteristic functions

The characteristic function χ_A is defined by

$$\chi_A(x) = \begin{cases} 1, & x \in A \\ 0, & x \notin A \end{cases}$$

Where χ_A expresses membership in set A for the element x in the universe. Membership thought is a mapping from an element x in universe X is two elements in universe Y; i.e., to the element 0 or 1. As shown in figure.



Membership function is a mapping for crisp set A

Definition 1.4

For any set A defined on the universe X. there exists a function – theoretic set, called a value set, denoted V(A), under the mapping of the characteristic function, χ. By convention, the null set is assigned the membership value 0 and the whole set X is assigned the membership value 1.

Example 1.2

Continuing with the example (Example 1.1) of a universe with three elements, X ={a,b,c}, we desire to map the elements of the power set of X, i.e., P(X), to a universe Y, consisting of only two elements

i.e., Y = {0,1} As, the elements of the power sets are enumerated.

P(x)={Φ, {a},{b},{c},{a,b},{b,c},{c,a},{a,b,c}}

Thus, the element in the value set V (A) as determined from the mapping are.

V{p(x)} = {{0,0,0},{1,0,0},{0,1,0},{0,0,1},{1,1,0},{0,1,1},{1,0,1},{1,1,1}}

For example, the third subset in the power set P(X) is the element b , for this subset there is no a, so a value of 0 goes in the 1st position of the date triplet; there is a b, so a value of 1 goes in the second position of the date triplet; there is no C, so a value of 0 goes in the third position of the data triplet. Similarly all other entries are made.

1.1.5 Functions theoretic operations

Define two sets A and B, which is on the universe X. The symbol

- ∨ - maximum operator
- ∧ - minimum operator

Union of these two sets in terms of function. Theoretic terms is given by,

UNION: $A \cup B \rightarrow \chi_{A \cup B}(x) = \chi_A(x) \vee \chi_B(x) = \max(\chi_A(x), \chi_B(x))$

The intersection of these two sets in function theoretic terms is given by,

INTERSECTION $A \cap B \rightarrow \chi_{A \cap B}(x) = \chi_A(x) \wedge \chi_B(x) = \min(\chi_A(x), \chi_B(x))$

The complement of a single set of universe X, say A is given by,

COMPLEMENT $\bar{A} = \chi_{\bar{A}}(x) = 1 - \chi_A(x)$

For two sets on the same universe, say A and B if one set (A) is contained in another set (B), then

CONTAINMENT $A \subseteq B \rightarrow \chi_A(x) \leq \chi_B(x)$

1.2FUZZY SETS

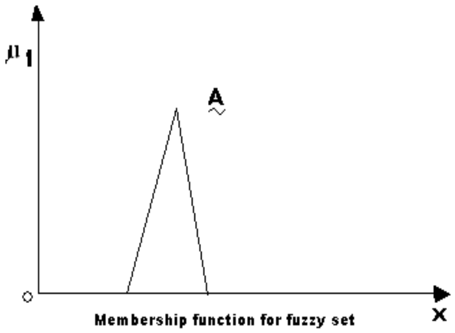
1.2.1 Membership functions

Let A be a crisp set defined on the universe X, then for any element $x \in X$, either $x \in A$ or $x \notin A$.In a fuzzy set theory this property is generalized. Therefore in a fuzzy set F, it is not necessary that either $x \in F$ or $x \notin F$. In the last few years there have been many theories that presented a generalization of membership property, but fuzzy set theory seems to be most intuitive among them.

It is probable to define a characteristic function for any set A

$\mu_A : X \rightarrow \{0,1\}$ as

$$\mu_A(x) = \begin{cases} 1, & \text{when } x \in A \\ 0, & \text{when } x \notin A \end{cases}$$



Membership function for fuzzy set

Definition 1.5 Fuzzy Set

In Fuzzy set theory the characteristic function is generalized to a membership function that assigns every $x \in X$, a value from the unit interval [0,1], instead from the two element set {0,1} . In short, $\mu_A(x)$ is a grade (degree) of membership of x in set A. The set that is defined on the basis of such an extended membership function is called a fuzzy set.

From the definition of fuzzy sets we can see that the fuzzy set theory is a generalized set theory that includes the classical set theory as a special case. Since $\{0,1\} \in [0,1]$, crisp sets are fuzzy sets. Membership function μ_A can also be viewed as a distribution of truth of a variable. In literature fuzzy set $\bar{A}$ is often presented as a set of ordered pairs.$\bar{A} = \{x, \mu_A(x) / x \in X\}$.Where the first part determines the element and the second part determines the grade of membership.

Definition 1.6

The universe of discourse X, if X is finite and discrete, the fuzzy set $\tilde{A}$ can be expressed in the for

$\tilde{A} = \{ \mu_A(x_1)/x_1 + \mu_A(x_2)/x_2 + \ldots \} = \{ \sum_i \mu_A(x_i)/x_i \}$

Example 1.3

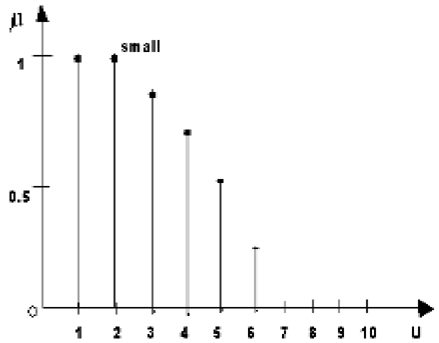
Let us consider a subset of natural numbers, say from 1 to 20, as the universe of discourse, U. we may define the terms small and medium by enumeration as follows. Given the discrete universe of discourse, U={1,2,3…,20} small and medium are fuzzy subsets of U.

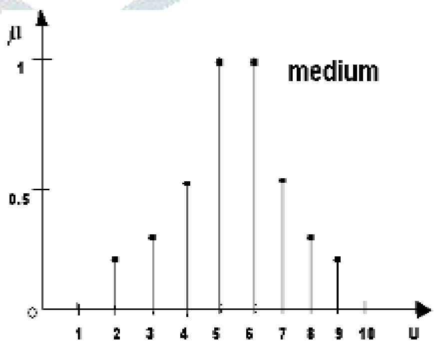
Characterize by the following membership functions.

Small ≡ 1/1+1/2+0.9/3+0.6/4+0.3/5+0.1/6.

Medium ≡ 0.1/2+0.3/3+0.7/4+1/5+1/6+0.7/7+0.5/8+0.2/9

The two fuzzy sets are also depicted graphically in Figure.





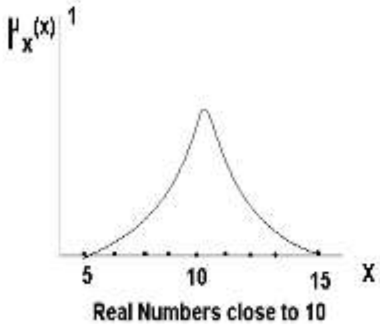
Example 1.4

A relator wants to classify the house he offers to his clients. One pointer of console of these houses is the how many numbers of bedrooms in it.Let X={1,2,3,5} be the set of obtainable types of houses described by x = number of available bed rooms in a house. Then it is calles and may be described as $\tilde{A}$={(1,.2), (2,5),(4,2), (5,1)} comfortable types of family members

Example 1.5

$\tilde{A}$ = "real numbers close to 10" $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) / \mu_{\tilde{A}}(x) = (1+(x-10)^{2)-1}\}$

Figure:



PROPERTIES OF FUZZY SETS

2.1.1 COMMON OPERATIONS

Define three fuzzy sets $\tilde{A}$,$\tilde{B}$, and $\tilde{C}$ on the universe X. For a given element x of the universe, the following function theoretic operations for the set theoretic operations of union, intersection, and complement are defined for$\tilde{A}$,$\tilde{B}$, and $\tilde{C}$ on X.

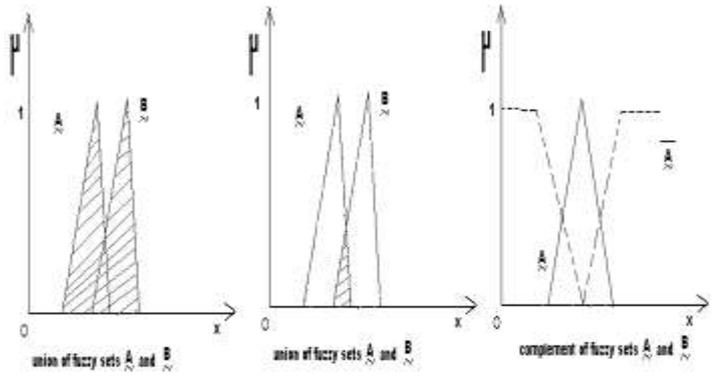
UNION $\mu_{\tilde{A}\cup\tilde{B}}$ (x) = $\mu_{\tilde{A}}$(x) ∨ $\mu_{\tilde{B}}$(x)

INTERSECTION $\mu_{\tilde{A}\cap\tilde{B}}$ (x) = $\mu_{\tilde{A}}$(x) ∧ $\mu_{\tilde{B}}$(x)

COMPLEMENT $\mu_{\bar{\tilde{A}}}$(x) = 1-$\mu_{\tilde{A}}$(x)

Figure 2.1

Venn diagrams for these operations,



Definition 2.1

Two fuzzy sets A,B are equal ($\tilde{A}$=$\tilde{B}$) if and only if ∀ x ϵ X. $\mu_{\tilde{A}}$(x) = $\mu_{\tilde{B}}$(x)

Definition 2.2

$\tilde{A}$ is a subset of $\tilde{B}$ ($\tilde{A} \subseteq \tilde{B}$) if and only if ∀ x ϵ X $\mu_{\tilde{A}}$(x) ≤ $\mu_{\tilde{B}}$(x).Any fuzzy set $\tilde{A}$ defined on X is a subset of that same set. As well by definition, just as with classical sets, the membership value of any element x in the null set Φ is 0, and the membership value of any element x in the whole set X is 1.

2.1.2 Demorgan's Law

Demorgan's laws for classical sets also hold for fuzzy sets, as denoted by these expressions.

$\overline{\tilde{A} \cap \tilde{B}} = A \cup B$

$\overline{\tilde{A} \cup \tilde{B}} = \text{A} \cap B$

2.1.3 Excluded middle laws

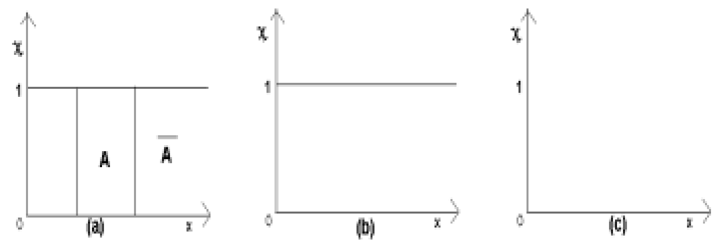
As enumerated before, all other operations on classical sets also hold for fuzzy sets, except for excluded middle laws. These two laws do not bold for fuzzy sets; since fuzzy sets can overlap, a set and its complement can also overlap. The excluded middle laws extended for fuzzy sets are expressed by

$\tilde{A}$∪$\tilde{A}$' ≠X

$\tilde{A} \cap \tilde{A}' \neq \Phi$

Figure 2.2

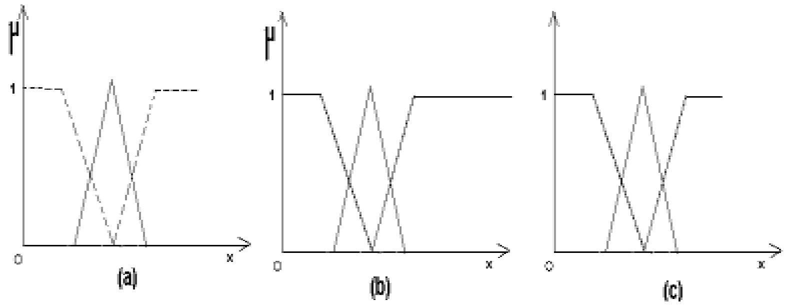
Venn diagrams;



Excluded middle laws for crisp sets:

a) Crisp set A and its complement A
b) Crisp set A∪$\bar{A}$ ≠ X (law of excluded middle)
c) Crisp set A ∩ $\bar{A}$ ≠ Φ (law of contradiction)

Figure2.3



Excluded middle laws for fuzzy sets

a) Fuzzy set A and its complement $\bar{A}$

b) Fuzzy A∪$\bar{A}$ ≠ X (law of excluded middle)

c) Fuzzy A ∩ $\bar{A}$ ≠ Φ (law of contradiction)

2.2 DEFINITIONS IN FUZZY SETS

Definition 2.3

The support of a fuzzy set $\tilde{A}$,S($\tilde{A}$) is the crisp set of all x∈X such that $\mu_{\tilde{A}}$(x) > 0.

Example 2.1

A relator wants to classify the house he offers to his clients. One indicator of comfort of these houses is the number of bedrooms in it. Let X = {1,2,3,…10} be the set of available types of houses described by x= number of bedrooms in a house. Then the fuzzy set "Comfortable type of house for a four-person family" may be described as $\tilde{A}$ ={(1,.2), (2,.5), (3,.8), (4,1), (5,.7), (6,.3)}.The support of S($\tilde{A}$)={1,2,3,4,5,6}. The elements (types of houses) {7,8,9,10} are not part of the support of $\tilde{A}$.

Definition 2.4

The (crisp) set of elements that belong to the fuzzy set $\tilde{A}$ atleast ∝ is called the $\propto -level\ set.$

$\boldsymbol{A}_{\propto}$= {x ϵX / $\mu_{\tilde{A}}$(x) ≥ ∝ }

$\overline{\mathrm{A}}_{\propto}$ = {x ϵX / $\mu_{\tilde{A}}$(x) ≥ α } is called "Strong α level set " or Strong $\alpha - cut.$

Example: 2.2

In the above example 2.1 the list of possible $\alpha -$ level sets,

$A_{.2}$={1,2,3,4,5,6}

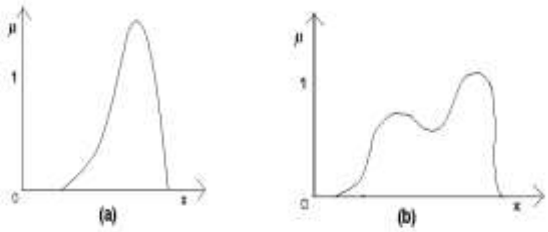
$A_{.5}$={2,3,4,5}

$A_{.8}$={3,4}, A_1={4}The strong level set for $\alpha = 0.8$ is $A'_{.8}$ = {4}.

Definition 2.5

A fuzzy set $\tilde{A}$ is non concave if $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}, x_1, x_2 \in X, \lambda \in [0,1]$. (ie) A fuzzy set is called convex if its function does not enclose 'dips'. This is membership function is, for example, increasing, decreasing, or bell-shaped.

Figure 2.4



Examples of (a) convex fuzzy set (b) Non – convex fuzzy set

Definition 2.6

For a finite fuzzy set Ã, the cardinality is defined as |Ã|=$\sum_{x\epsilon X} \mu_{\tilde{A}}$(x) ||Ã|| =| Ã |/|x| is called relative cardinality of Ã.

Example 2.3

For the fuzzy set "comfortable type of house for a four person family" from example 2.1 the cardinality is

|Ã| =.2+.5+.8+1+.7+.3=3.5

Its relative cardinality is ||Ã||=3.5/10 = 0.35

The relative cardinality can be interpreted as the fraction of elements of X being in Ã. weighted by their degrees of membership in Ã. For infinite X, the cardinality is defined by |Ã|= $\int_x \mu_{\tilde{A}}(x)dx$. |Ã| does not always exist.

2.5 NORMS IN FUZZY SETS:

t-Norm (Generalized Complements)

Let A be a fuzzy set on X. Then xϵA Let CA denote a fuzzy complement of A of type C.

Let a complement CA be function C:[0,1]→ [0,1],

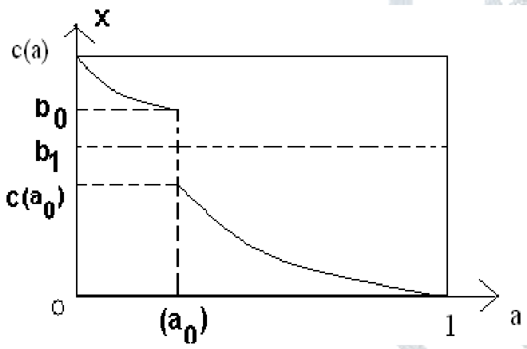
Then C satisfy the following axioms.

Axiom C1: C(0)=1 and C(1)=0 (boundary conditions)

Axiom C2: For all a,b ∈ [0,1], if a≤b, then c(a) ≥ c(b) (monotonicity)

Axiom C3: C is a continuous function
Axiom C4: C is involutive, which means that C(C(a))=a for each a∈ [0,1].
Let Axioms C1 and C2 be called the axiomatic skeleton for fuzzy complements.
The four axioms are not independent as articulated by the next theorem.

Theorem 2.3
Let a function C:[0,1] →[0,1] satisfy axioms C2 and C4. Then C also satisfies axioms C1 and C3 Moreover, C must be a objective function.
Proof:
(i) Since the range of C is [0,1],
C(0)≤1 and C(1)≥0
By axiom C2,
C(C(0)) ≥ C(1)
and by axiom C4,
0= C(C(0)) ≥ C(1)
Hence, C(1)=0
Now again by axiom C4 we have,
C(0)=C(C(1))=1
That is, function C satisfied Axiom C1
(ii) We have to prove C is bijective ,
observe that for all a ∈ [0,1] there exists
b=C(a) ∈ [0,1] such that
C(b) = C(C(a))= a
Hence, C is an onto function.
Assume now that $C(a_1) = C(a_2)$
Then by axiom C4,$a_1 = C(C(a_1))$
$= C(C(a_2)) = a_2$
That is ,C is also a one-to-one function;Consequently it is a bijective function.
(iii) Since C is bijective function and satisfied C2, it is not discontinuous points.
To show this, assume that c has a discontinuity at a_0. As illustrated in figure.



Then we have
$b_0= \lim_{a\to a0} C(a) > C(a_0)$
and clearly there must exist b_1 ∈ [0,1] such that
$b_0 > b_1 > C(a_0)$ for which no a_1 ∈ [0, 1]
exists such that $C(a_1) = b_1$
This contradicts the fact that C is bijective function.

Theorem 2.4
Every fuzzy complement has at most one equilibrium.
Proof:
Let C be an arbitrary fuzzy complement an equilibrium of c is a solution of the equation
C(a) –a =0 where a ∈ [0, 1]
We can demonstrate that any equation
C(a)-a = b where b is a real constant,
must have at most one solution, thus proving the theorem.
In order to do so , we assume that a_1 and a_2 are two different solutions of the equation.
C(a)-a = b such that $a_1 < a_2$
Then, since $C(a_1)-a_1 = b$ and
$C(a_2)-a_2 = b$,
we get,$C(a_1)-a_1 = C(a_2)-a_2$
However , because C is monotonic non increasing (by axiom C2)
$C(a_1) \geq C(a_2)$ and since $a_1<a_2$,$C(a_1)-a_1 > C(a_2) –a_2$
This inequality contradicts 1,
Thus demonstrating that the equation must have almost one solution.

t-Norm (Fuzzy intersections)

The intersection of two fuzzy sets A and B is specified in general by a binary operation on the unit interval; i.e. a function of the form i: $[0,1] \times [0,1] \rightarrow [0,1]$

Thus, $(A \cap B)(x) = i[A(x), B(x)]$ $\forall\ x \in X$

For all a, b, d $\in [0, 1]$ i is a binary operation on the unit interval.

Axiom i1: i (a,1) = a (boundary condition)

Axiom i2: $b \leq d$ implies $i(a, b) \leq i(a, d)$ (monotonicity)

Axiom i3: i (a, b) = i (b, a) (commutativity)

Axiom i4: i (a, i(b, d) = i (i (a, b), d) (associativity)

Let us call this set of axioms the axiomatic skeleton for fuzzy intersections / t-norms.

Axiom i5: i is a continuous function (continuity)

Axiom i6: i (a, a) < a (subidempotency)

Axiom i7: $a_1 < a_2$ and $b_1 < b_2 \Rightarrow i(a_1, b_1) < i(a_2, b_2)$ (Strict monotonicity).

Theorem 2.5

The fuzzy of standard intersection is the only idempotent t-norm.

Proof :

We know that, min(a,a) = a such that for all a $\in [0,1]$ there exists a t-norm such that

i (a,a) = a for all a $\in [0,1]$

Then for any a,b $\in [0,1]$

if $a \leq b$ then

$a = i(a, a) \leq i(a, b)$

$\leq i(a,1) = a$

By monotonicity and the boundary condition

Hence, i (a, b) = a = min(a, b)……………..1

Similarly,if $a \geq b$ then

$b = i(b, b) \leq i(a, b)$

$\leq i(1, b) = b$

and consequently,

i (a, b) = b = min(a, b) ……..2

Hence from 1 & 2 we get,

i (a, b) = min(a, b) for all a, b $\in [0, 1]$

Theorem 2.6

For all a, b $\in [0, 1]$ $i_{min}(a, b) \leq i(a, b) \leq \min(a, b)$ ……….1

Where i_{min} denotes the drastic intersection.

Proof:

Upper bound

By the boundary condition and monotonicity,

[i (a, 1) =a and $b \leq d \Rightarrow i(a, b) \leq i(a, d)$]

We have, $i(a, b) \leq i(a, 1) = a$ ……………………..…2

Since commutative, [i(a, b) = i(b, a)]

we have,i (a, b) = i(b, a)

$\leq i(b, 1) = b$ ………..….3

hence from 2 & 3 we get

$i(a, b) \leq a$ and $i(a, b) \leq b$

that is, $i(a, b) \leq \min(a, b)$

Lower bound

From the boundary condition

i (a, b) = a when b=1 and

i(a, b) =b when a=1 since

$i(a, b) \leq \min(a, b)$ and $i(a, b) \in [0,1]$

Clearly, i(a, 0) =i(0, b) =0

By monotonicity,$i(a, b) \geq i(a, 0)$

= i(0, b) = 0.

Hence the drastic intersection i_{min} (a, b) is the lower bound of i(a, b) for any a, b $\in [0,1]$.

t- co norms: (Fuzzy unions)

The sets A and B is specified expressed union of two fuzzy by it is define,

u: $[0,1] * [0,1] \rightarrow [0,1]$

Thus, $(A \cup B)(x) = u[A(x), B(x)]$ for all $x \in X$.

For all a, b, d $\in [0,1]$ and

u is a binary operation on the unit interval.

Axiom u1: u(a, 0) = a (boundary condition)

Axiom u2: $b \leq d$ implies $u(a, b) \leq u(a, d)$ (monotonicity)

Axiom u3: u(a, b) = u(b, a) (commutativity)

Axiom u4: u(a, u(b, d)) = u(u(a, b), d) (associativity).

We call it the axiomatic skeleton for fuzzy unions / t-conorms.

Axiom u5: u is a continuous function (continuity).

Axiom u6: u(a, a) > a (superidempotency)

Axiom u7: $a_1 < a_2$ and $b_1 < b_2$

$\Rightarrow u(a_1, b_1) < u(a_2, b_2)$ (strict monotonicity).

Theorem 2.7

The standard fuzzy union is the only idempotent t-conorm.

Proof:

Clearly, max(a, a) =a for all a∈ [0,1].

Assume that there exists a t-conorm such that

u(a, a) =a for all a ∈ [0,1]

Then for any a, b ∈ [0,1] , if $a \leq b$ then

$a = u(a,a) \leq u(a, b)$

$\leq u(a, 0) = a$

by monotonicity and the boundary condition

Hence, u(a, b) = a = max(a, b)

Similarly, if $a \geq b$ then

$b=u(b, b) \leq u(a, b)$

$\leq u(0, b) = b$

and consequently,u(a, b) = b = max(a, b)

Hence, u(a, b) = max(a, b) for a, b ∈ [0,1]

3.Boolean Algebra:

A Boolean algebra will generally be denoted by (B,-, ∧,∨,0,1) in which(B, ∧,∨) is a lattice with two binary operations ∧ and ∨ called the meet and join respectively.

A Boolean algebra ia a non-empty set B, together with two binary operations ∧,∨ and one unary operation, and two specific elements 0 ,1 satisfying the fllowing axioms for all a,b,c ϵ B

Commutative laws:

a∨b=b∨a and a∧b=b∧a

Associative Laws:

(a∨b)c=(a∨b)c and a∧ (b∧c)=(a∧b) ∧c

Distributive laws:

a∧ (b∨c)=(a∧b)(a∧c)

a(b∧c)=(a∨b) ∧ (a∨c)

Identity laws:

a∨0=0∨a=a=a and a∧1=1∧a=a

Complement laws:

a∨ a¯=1 and a∨ a¯=0

Idepotent laws:aa=a and a∧a=a

Null laws:

a∨1=1 and a∧0=0

Absorption laws:

a∨ (a∧b)=a and a∧ (a∨b)=a

Demorgan's laws:

(a∨b) ¯=a¯∧b¯ and (a∧b)

III CONCLUSION

A fuzzy set is determined by its indeterminate boundaries, there exists an uncertainty about the set boundaries. On the other hand, a crisp set is defined by crisp boundaries, and contain the precise location of the set boundaries. Fuzzy set elements are permitted to be partly accommodated by the set (exhibiting gradual membership degrees). Conversely, crisp set elements can have a total membership or non-membership. There are several applications of the crisp and fuzzy set theory, but both are driven towards the development of the efficient expert systems. The fuzzy set followed the infinite-valued logic whereas a crisp set is based on bi-valued logic. The fuzzy set theory is intended to introduce the imprecision and vagueness in order to attempt to model the human brain in artificial intelligence and significance of such theory is increasing day by day in the field of expert systems. However, the crisp set theory was very effective as the initial concept to model the digital and expert systems working on binary logic.

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